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Zhiyun (Frances) Xu

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ABSTRACT

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Zhiyun (Frances) Xu

Consumers' willingness-to-pay can change over time and can change differently for different consumers. Rational and forward-looking consumers incorporate this into their decision making. This dissertation examines the implication of such change in several dynamic IO settings.

The first chapter looks at a durable goods monopoly setting where consumers' valuations decline over time. The seller cannot commit to future prices, but she can commit to a best-price policy, a promise to give her customer a refund if she reduces her price after the customer's purchase. We characterize the optimal BP policy when the seller can control both the policy length (when the promise expires) and the refund scale (what portion of the price difference is refunded). We provide conditions under which the optimal policy length is finite. A finite-length BP allows the seller to commit not to lower her price too soon, but also allows her to capture some of the benefits of intertemporal price discrimination.

The second chapter looks at a setting where consumers purchase repeatedly and they have heterogeneous uncertainty in their future taste. Here, the seller can offer both long-term and short-term contracts and do history-based price discrimination. The equilibrium

involves firms offering both short and long-term contracts and leads to higher profits and higher second period prices, compared to the case when only short-term contracts are allowed. Long-term contracts serve the purpose of locking in consumers with more stable taste and thus reducing the poaching incentives in the second period. We show that efficiency is also increased when long-term contracts are allowed because products are better matched with consumer preferences in the second period.

The third chapter again looks at a setting where consumers purchase repeatedly. We allow switching cost and consumer taste change, and show that history-based price discrimination can increase the second period profit of both sellers, in contrast to existing results. However, this effect is countered by the increased demand elasticity in the first period which decreases first period profit. Unlike in the traditional model, price discrimination can increase efficiency if consumers' demand expands.

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CHAPTER 1

Optimal Best-Price Policy

1.1. Introduction

Retailers in the United States commonly use price protection policies that allow customers to claim a refund if an item is marked down within a certain period of time from the purchase date. These are promises to refund customers the difference between the price they pay and the lowest future price charged by the same firm on a particular item within some time horizon. We will call this practice "best-price policy" or simply BP. [1]

Examples of best-price policies can be found at many fashion retailers, electronic retailers, car manufacturers, and department stores. They are sometimes called "price adjustment" policies. A common feature of these retail industries is that consumers' willingness-to-pay for the product can decrease over time as the product becomes out of fashion or obsolete. This suggests that best-price policies are attractive in industries where it is difficult for firms to forecast the prices they will want to charge at different points in time. Hence this paper focuses on the use of the BP policy when the seller faces declining and uncertain demand for their products.

As is well-known, the inability to commit to future prices can have drastic consequences for a durable-goods monopolist. It is difficult for such a monopolist to keep prices high because her future self is tempted to cut prices to sell to those consumers unwilling to pay the earlier price. Forward-looking consumers foresee that the price will drop over time, so

[1] This follows the terminology used in Butz (1990).

they rationally wait for the price to drop. This can significantly hurt the seller's profit. In fact, Coase (1972) conjectured that the price will shrink to the lowest valuation in the market or the marginal cost, whichever is higher, if the seller can revise the price instantly. The Coase conjecture has been studied in a more formal, game-theoretic model by Gul, Sonnenschein and Wilson (1986), and many others. As observed by Bulow (1982), an implication of the Coase conjecture is that price commitment devices, like BP, will increase a seller's profit. However, the optimal design of such a policy remains an open question.

One stylized fact of BP policies cannot be explained within the standard model of a durable goods monopolist: in practice, the policy length is always finite and varies from seller to seller or even from product to product. For example: Jcrew, a fashion retailer, honors a 7-day period, while Crate and Barrel, a household item retailer, honors a 14-day period. Moreover, Best Buy, an electronic device retailer, specifies that the policy length is 30 days for its general items, but 14 days for a set of products including laptops and cameras, and 60 days for HDTVs. In the standard setup used in the literature on durable-goods monopolies, in which the valuation does not change over time, the optimal BP policy calls for an infinite horizon. As shown in Stokey (1979), the uniquely optimal outcome for a seller with full commitment is to sell only once to all the consumers whose willingness-to-pay is above a certain threshold and to never sell again. This can only be achieved by a BP policy of infinite length.

Aside from the policy length, sellers also control the refund scale. Most policies set the refund to be 100% of the price difference, and very few sellers set the refund to be 110% of the price difference. No seller, as far as we know, sets the refund to be less than 100%.

Our model allows us to address the question of what the optimal BP policy length and refund scale are, and allows us to explain the variation among the policies we observe. How should the seller choose the policy length given the product's demand structure? Should the monopolist only promise to pay the price difference, rather than, for instance, twice this price difference?

We wish to compare the profit of a seller who can commit to a BP policy to a seller who cannot commit, under the assumption that committing to future price is not possible. This allows us to determine the value of the BP policy. We are also interested in understanding whether and when BP allows the monopolist to achieve the profits under full commitment.

We consider a two-type infinite horizon model with demand uncertainty. We focus on the Markov Perfect Equilibria of this game. Because past prices may affect future profits through the BP policy, past but recent prices may be payoff-relevant for the future: this significantly complicates the analysis relative to the usual setup.

We show that a one-time irreversible drop in the willingness-to-pay can explain why, under certain conditions, it is optimal for the seller to choose a BP policy with a finite length.

This paper is not the first one to interpret fashion change and obsolescence as a factor that lowers consumers' willingness-to-pay. Lazear (1986) defined "fashion" and "obsolescence" as a deterministic drop in the willingness-to-pay in the second period in a two-period model.

As a benchmark, we consider the case in which the seller can commit to future prices that are contingent on the realization of the uncertainty ("full-commitment") as well as the case in which the seller cannot commit to future prices or to any refund policy ("no-commitment").

We show that, whenever the profit under full-commitment is higher than the profit under no-commitment, the optimal BP policy can always improve on the no-commitment profit, provided that the time period between successive prices is small enough (Figure 1.4). This improvement comes from two sources. As expected, BP has its value in mitigating the Coase Conjecture. A subtler benefit is that BP allows the seller to take advantage of high-valuation consumers' fear of a demand drop that will make the demand curve "flatter". In other words, a certain class of demand drop makes intertemporal price discrimination [2] profitable, and a BP policy enables the seller to actually do it. Without the BP policy, Coase Conjecture holds and the seller has no scope to do intertemporal price discrimination, even when it is profitable. With a BP policy that is optimally chosen to be of a limited length, the seller disciplines herself not to reduce price before the policy expires, but feels free to reduce price after the policy expires. Therefore, paradoxically, a contractual commitment with the consumers that promises *not* to intertemporally price discriminate (for a limited time), actually allows the seller to do intertemporal price discrimination.

One striking result of introducing demand uncertainty, and thus the opportunity for intertemporal price discrimination, is that the equilibrium profit under BP for a seller without any price commitment ability can be even higher than the equilibrium profit when demand does not decline and when the seller has full price commitment ability. This is because when willingness-to-pay falls more severely for high valuation consumers, a decline in demand facilitates intertemporal price discrimination, and increases a seller's profit when the seller has the commitment power provided by a BP policy. Even though demand drop makes consumers' willingness-to-pay evaporate, it can be something desirable for the seller once she has some commitment power.

[2] We take the definition from Stokey (1979) that intertemporal price discrimination means cutting price "for the purpose of exploiting differences in consumers' reservation prices".

However, BP also has limitations. For some parameter cases, the profit with an optimally designed BP policy fails to achieve the full-commitment/first-best profit (Figure 1.4). The reason is that, to achieve the first-best, the seller must have the flexibility of cutting the price as soon as demand drops, (but not otherwise, see Proposition 1 and Figure 1.1). In other words, the price should be contingent on the event of demand drop, which is not verifiable, although it is observable. In contrast, in any BP equilibrium, the seller is more reluctant to cut prices when demand has dropped than when demand has not dropped yet. When deciding whether or not to decrease the price and to sell more, the seller faces the trade-off between the refunds she has to pay to previous customers in the near past, and a delay in future revenues. If the seller decreases the price within the policy length, then she can cut it to a higher price under a high demand than under a low demand, thus incurring a lower refund under a high demand. On the other hand, if she keeps the price high for at least one more period, then her expected future profit is higher under a high demand than under a low demand, implying that the discounting cost of delay is higher under a high demand. Therefore, a seller facing a high demand is more tempted to cut price within the policy length and pay refunds than a seller facing a low demand. Therefore, the transaction timing under BP is always worse than if the seller has full price commitment.

We characterize when the optimal BP policy for the seller has a finite duration (Figure 1.5). This requires the high-valuation consumers to lose a larger fraction of their valuation than the low-valuation consumers upon a demand drop. This seems natural in the case of products that may go out of fashion or become obsolete. For instance, when a piece of clothing goes out of fashion, those who are style/design-conscious may lose considerable interest in the product, while consumers who are just interested in delaying laundry are less affected, if at all, by the fashion shift.

In the parameter case where the static monopoly outcome for the seller is to sell only to high-valuation consumers when the demand is high, but to all consumers when the demand is low, we get the following comparative statics: the optimal policy length decreases in the likelihood of a demand drop and the equilibrium profit on the optimal equilibrium given the optimal policy increases with the likelihood of a demand drop.

We show that the outcome of the optimal equilibrium at a finite optimal BP policy has the following simple feature: the seller sells to high-valuation consumers at the beginning, after which she increases the price and refrains from selling for the whole duration of the optimal BP policy, and then reduces price to sell to the low-valuation consumers when the policy expires. The realization of the demand drop does not affect the timing of the transaction with the low-valuation consumers, but does affect the price of the transactions. Refunding does not happen on the equilibrium path, even though the obligation to pay refunds is crucial for the seller to be willing to delay selling to the low-valuation consumers for a finite length of time. The model captures both the phenomena of moderate pre-sale and big final sale.

We also show that the refund scale does not matter, as long as it is above a critical value. This critical value is always below the commonly chosen level of 100%. If the seller sets the scale to be below the critical value while setting the policy length to be positive, then the equilibrium outcomes that follows such a policy commitment can only be one of the two cases: either she sells to all consumers at the beginning, or she sells only twice during the first two periods. This basically makes BP completely powerless, so we need a large enough refund scale to make a BP meaningful. The more profitable is intertemporal price discrimination versus selling to all at once, the less severe is the self-punishing scale that is necessary.

The related literature can be grouped into five strands: one about durable goods monopoly (Coase Conjecture), one about BP policy for a monopolist, one about durable goods oligopoly, one about intertemporal price discrimination, and one about dynamic pricing for the fashion industry.

(1) Literature on Coase Conjecture

Coase (1972) conjectured that rational consumers force the monopolist to lower the price to the competitive level right from the start. After Coase (1972), others have studied the problem of durable-goods monopolist rigorously, including notably Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989).

More broadly, this paper is related to the literature on the factors that mitigate the Coase Conjecture. If the product does not depreciate and is consumed over time, then it can be *rented* rather than sold. Provided that any rental price is applied to all consumers in the market no matter when they started renting, the monopolist can avoid the temptation to cut prices. However, many products cannot be rented because, for example, the process of consumption decreases the product's value to other consumers, or consumers may abuse the product if it is rented. Bulow (1982) shows that a monopolist might be led to produce a less durable good. Sometimes sellers can choose the *output* along with the price. Denicolò and Garella (1999) shows that allowing the seller to restrict output in the first period can help the sellers in a two-period model. However, this effect does not arise with an infinite horizon and vanishing time intervals. Cho (2007) considers a model where the stock of goods exogenously decreases over time, which he calls the process of *perishing*, and shows that this can help the seller even when the rate at which the good perishes is arbitrarily low. The seller has an incentive to delay transaction to allow the quantity available for sale to decrease, so in some sense this gives the seller some quantity commitment ability. Bond and Samuelson (1984) shows that an exogenous depreciation after consumers have bought the product helps the seller because the seller has an incentive to limit quantity to ensure the profitability of future replacement sales, but this effect disappears as the depreciation factor goes to zero. As shown in McAfee and Wiseman (2007), a one-time *capacity* commitment can increase the seller's profit and even a seller without capacity commitment can improve her profit, as long as any capacity augmentation is costly.

(2) Literature on BP policy for a monopolist

The literature on BP policy has defined the policy to be of infinite length and 100% refund scale, so the optimal design of the policy has not been studied.

Board (2005) studies the BP policy in a deterministic setting. He shows that a BP policy (of an infinite length) can implement the full commitment price path of his model with consumers entering sequentially. In contrast, the current paper can be viewed as highlighting the inability of a BP policy to implement a non-deterministic price path.

Butz (1990) and Png (1991) model the use of BP under demand uncertainty. The demand uncertainty they consider takes the following form: at the beginning everyone is not sure about the relative proportion of high types versus low types in the market, but it will become commonly known later. They show that a BP policy benefits the seller by allowing her to delay the pricing decision until more information is obtained about the demand. For example, after selling to high types, if the mass of low types turns out to be small, then the BP can successfully restrict the seller from selling more, and if the mass of low types turns out to be large, then the seller can choose to reduce the price and to pay refunds to previous customers. This ability to retrospectively "revise" the price when more information is available allows the seller to get to the level of static monopoly price (if there is no discounting) regardless of demand realization.[3] This theory for a BP policy is different from the one in our paper. Butz (1990)'s model is not game theoretic. Png (1991) considers a two-period model, so policy length is not an issue there. In contrast, in this paper, the demand uncertainty takes the form that no one knows when and whether the willingness-to-pay will drop in the future. Whether uncertainty lies in the mass or in the willingness-to-pay of the consumers has vastly different implications for the analysis. A BP policy in my model can no longer achieve first-best.

Spier (2003) studies how a BP reduces delay in settlements before litigation. There, the defendant is similar to a monopolist, offering settlement prices to plaintiffs before the litigation. She is tempted to offer prices that increase over time and BP helps the defendant resist such temptation. However, the setting there is different from the usual IO setting because, in finite time, litigation will fully reveal the types of the plaintiffs.

Overall, there has not been a previous theoretic setup that can be directly used to study the optimal design of a BP policy for a monopolist.

There was only one paper that paid some attention to the duration of the BP policy. In an empirical study of Low Price Guarantee (LPG), the policy that sellers use to match or beat the competitors' price, Arbatskaya, Hvvid and Shaffer

[3] Png (1991) allows rationing in his no-BP benchmark, so it can sometimes outperform BP.

(2004) notices that out of the 515 LPG they observe, 104 of them come with a most-favored-customer policy, i.e. a BP policy in this paper's terminology. The policy length of these BP policies is above 30 days. The paper conjectures that the intentions behind these policies is that "the firm is mostly concerned with inducing customers not to wait for a sale". However their paper's focus is to study the varieties of Low Price Guarantee, instead of BP policy, and it does not explore reasons why the policy length is not set to be even longer to induce consumers not to wait.

(3) Literature on durable goods oligopoly

Since the root of the Coase Conjecture problem is that the seller cannot punish her future self for reducing the price, a competitor that will punish her for reducing the price can actually help the seller. Therefore, in a repeated game setting, Gul (1987) and Ausubel and Deneckere (1987) show that monopoly profit can be achieved with tacit collusion under oligopoly (or with a potential entrant) if the sellers are patient enough.

When the game horizon is finite, the usual tacit collusion breaks down. There is a line of research that shows if both sellers commit to a BP policy, BP makes a certain level of tacit collusion possible even under finite horizon, because it reduces both sellers' incentive to reduce price to deviate from the collusive outcome. However, depending on the setting, sellers may or may not have incentives to commit to the BP policy. [4]

(4) Literature on intertemporal price discrimination

[4] See Cooper (1986), Nelson and Winter (1993) and Schnitzer (1994).

Stokey (1979) shows that if consumers have rational expectations, intertemporal price discrimination is not possible even for a full-commitment monopolist, unless we make one of the following assumptions: 1) consumers with higher valuation are also more impatient (Stokey, 1979), 2) the seller can use capacity to do rationing (Van Cayseele, 1991). Compared to this strand of literature, this paper can be viewed as proposing a situation where intertemporal price discrimination naturally arises for a seller with full commitment, or with partial commitment, such as a BP policy. More broadly, this paper is related to settings where price discrimination is profitable. This literature is nicely integrated by Anderson and Dana (2006), where the seller can choose different quality and price pairs. The demand condition in my model that makes intertemporal price discrimination profitable is similar to theirs, because a future product in an intertemporal setting is like a different level of quality in a static setting. However, my model takes into account the effect of uncertainty and allows the consumer type to be a function of time.

(5) Literature on dynamic pricing for fashion

Lazear (1986) uses the same definition for a fashion change or obsolescence as this paper does. Fashion product is a special case in his study of a seller who can learn about the underlying demand from the first period outcome in a two-period model. However, his model assumed that consumers are myopic and reached the comparative statics that the deeper is the valuation drop, the lower is the first period price in a two-period model, because the seller becomes more eager to sell. In my model, a deeper valuation drop or a larger likelihood of a drop can cause the first period price to increase, because high valuation consumers in my model are more eager to buy as well. Pashigian (1988) defined “fashion” in a different way: a seller is more in the “fashion” industry if she sells more variety. The paper used a uniform distribution of the prior on unobservable consumer valuation over the product line to approximate a seller in the fashion industry, while using a symmetric triangular distribution to approximate a seller in a non-fashion industry. They got the comparative statics that (if the marginal cost is 0) the fashion seller charges a higher first period price and a lower second period price, comparing to a non-fashion seller in a two period model.

There is also a strand of literature that takes obsolescence as endogenous and studies firms’ innovation behavior.[5] Instead, this paper takes obsolescence as exogenous. This can be interpreted as saying that the innovation is coming from an outside competitor, not the seller herself.

The remainder of the paper is organized as follows. We present the setup in Section 1.2. Section 1.3 analyzes our first benchmark model of full price commitment. Section 1.4 analyzes our second benchmark model of no commitment. (We present the benchmarks first because they will facilitate the understanding of the main model.) We solve for the optimal equilibrium given a BP policy, and for the optimal BP policy in Section 1.5. The main results are stated in Proposition 4 and Proposition 5, where comparisons are made with the results of the two benchmarks. Section 1.6 discusses issues, extensions, and explores alternative explanations for the adoption of BP policy. We conclude the paper in Section 1.7 by summarizing the main results and by discussing avenues for future research.

[5]See, for example, Waldman (1993).

1.2. The Model

The market for a durable good consists of a continuum of infinitely-lived consumers of total measure 1 and an infinitely-lived seller. We refer to the consumer as him and to the seller as her. The seller can only set a price at discrete moments in time $t = 0, \Delta, 2\Delta, \ldots$. Our focus will be on the limit as $\Delta \to 0$. All consumers and the seller have the same exogenous discount rate $r \in (0, 1)$. I.e., given Δ, the discount factor from one period to the next is $e^{-r\Delta}$.

Each consumer has an inelastic demand of one unit. There are two types of consumers: high types (or H types) and low types (or L types). The measure of high types is $h \in (0, 1)$ and that of low types is $1 - h$.

At the beginning of the game, H types have a willingness-to-pay of H and L types have a willingness-to-pay of L. Their willingness-to-pay may drop once over time, simultaneously, to H' and L', respectively. (We assume that $H > L > 0$, $H' > L' > 0$ and $H \geq H', L \geq L'$.) We interpret this as the product going out of fashion or becoming obsolete, so that this affects all consumers at the same time. We say that "demand is high at t" if the willingness-to-pay is H and L at t and "demand is low at t" if it is H' and L' at t. We refer to this simply as the "demand state" at time t: $d_t \in \{0, 1\}$, with 0 to be interpreted as low demand, and 1 to be interpreted as high demand. The demand state is observable. We emphasize that a demand drop refers to the drop in willingness-to-pay, not in the mass of the consumers demanding the good.

We assume that the demand drops probabilistically, according to a constant hazard rate $s > 0$. In other words, conditional on the demand being high at time t, the probability that the demand is high at time $t+\tau$ is given by $e^{-s\tau}$. Once the demand has dropped, the willingness-to-pay remains H' and L' forever after. [6]

We normalize the cost of producing one unit of the good to 0, i.e., we are considering the so-called "gap" case, where the marginal cost of production is below the lowest valuation in the market.

Within this set-up, we distinguish three different games:

(1) The game with a fixed BP policy, in which the seller cannot otherwise commit to prices. (Main model)

(2) The game in which there is no BP policy but the seller can commit to prices that are contingent on the state of demand (and on the entire history). (Benchmark 1)

(3) The game in which there is no BP policy and the seller cannot commit to prices. (Benchmark 2)

1.2.1. BP model Setup

A best-price (BP) policy is characterized by its "policy length" T and "refund scale" z, with $T \in \{0, \Delta, 2\Delta, \ldots\} \bigcup \{\infty\}$ and $z \geq 0$. A BP policy works as follows: if a consumer buys at time t at price P_t then he is entitled to a refund of scale z if the price drops within T periods of his purchase. Formally, given any $\tau \leq T$, if $P_{t+\tau} < \min_{0<v<\tau} P_{t+v}$, then the buyer receives $z(\min_{0\leq v<\tau} P_{t+v} - P_{t+\tau})$ at time $t+\tau$. Notice that the time window during which BP is effective for the particular consumer is counted from his initial time of purchase. Also, notice that z is independent of prices and time. In Section 1.6, we will consider more general forms of BP policy.

[6]We can think of H and L as present value of the consumption. So if consumers consume the product over time, all the analysis goes through because of the constant hazard rate assumption.

We emphasize that the BP policy is a commitment device for the seller: it is fixed throughout the game and it is common knowledge among all agents. A given BP policy defines a game. This game admits (potentially multiple) equilibria, defined more carefully below. The performance of the BP policy is measured by the seller's highest payoff across all these equilibria. The *optimal* BP policy, then, is defined to be the one that yields the highest performance.

More formally, let $G_{BP}(\Delta, T, z)$ be the game given Δ and the BP policy of length T and scale z. In each period (i.e., at $t = 0, \Delta, \ldots$), three events unfold in the following order: first, demand drops or not; second, after observing the state of demand the seller sets the price; third, all the consumers who have not yet bought decide whether to accept or to reject the price. Refunds are then executed and goods consumed.

The seller's payoff at time t is the expected total discounted future revenue minus the refunds to be paid. As for the consumer's flow payoff, 1) if he buys in this period, then it is his willingness-to-pay minus the price paid; 2) if he receives refunds in this period due to a BP policy, then it is the amount of refunds as described above [7]; 3) in all other contingencies, it is zero. The consumer's payoff at time t is the expected discounted sum of flow payoffs from period t onward.

The payoff-relevant variable, or *state*, can be described as follows: in addition to the residual demand, it includes the time, the price, and the mass of the consumers for each transaction that occurred within the last T length of time. These transactions are payoff-relevant because they determine the refund obligations for the periods to come. Formally,

[7] We assume here that the cost of claiming refund is 0. The result, however, will not change if we assume instead that everyone will incur the same cost $c < H - L$ if they claim the refund.

we define the *state* at time t to be composed of two variables if $T \geq \Delta$: 1) the residual demand curve, which in this two-type model is just the masses of consumers with willingness-to-pay at H, L, H' and L', 2) the sequence $\{(P_{t-\tau}, M_{t-\tau})\}_{\tau \in \{\Delta,\ldots,\min\{T,t\}\}}$, where P_u is the price at time u and $M_u \geq 0$ is the mass of consumers who bought at time $u \in \{0, \Delta, \ldots\}$. If $T = 0$, then the *state* only includes the residual demand. [Observe that the willingness-to-pay of these consumers who have bought is payoff-irrelevant.] Let the set of all *states* at time $t \geq \Delta$ be $S_T^t \subseteq [0,1]^4 \times [0,\infty)^{\min\{T,t\}} \times [0,1]^{\min\{T,t\}}$.[8]

Because we shall restrict attention to Markov Perfect equilibria, we immediately restrict the domain of the players' strategies accordingly. [9] Formally, given a BP policy, a strategy σ for the seller is a sequence of probability transition functions $\{\sigma_t\}_{\{t=0,\Delta,\ldots\}}$ where σ_t determines the monopolist's price at time t as a function of the state at t and the price in the previous period; i.e., if $T \geq \Delta$, then the seller's strategy is $\sigma_t : S_T^t \to \Phi_{[0,\infty)}$ where $\Phi_{[0,\infty)}$ is the set of all distributions on $[0,\infty)$; if $T = 0$, then the seller's strategy is $\sigma_t : S_0^t \times [0,\infty) \to \Phi_{[0,\infty)}$, which is a mapping from the state and the previous period price. A strategy κ for the consumers is a sequence of probability transition functions $\{\kappa_t\}_{\{t=0,\Delta,\ldots\}}$ where κ_t determines whether a consumer accepts or rejects a price at time t as a function of the state, the current period price and the current period willingness-to-pay, i.e., $\kappa_t : S_T^t \times [0,\infty) \times \{H, L, H', L'\} \to \Phi_{\{A,R\}}$, where $\Phi_{\{A,R\}}$ is the set of all distributions

[8]Formally, only those periods for which a positive mass of consumer bought (in the last $\min\{T,t\}$ periods) are payoff relevant. As it makes no difference for our result, we choose here the notational simpler specification in which all prices (in the last $\min\{T,t\}$ periods) are included into the state, whether some consumers bought or not.

[9]However, the Markov equilibrium we describe here is also an equilibrium for the game without the Markovian restriction: if all other players use Markovian strategies, then a player must have a best response that is Markovian. (If one player has a profitable deviation that depends on more than just the payoff-relevant part of history, then this player also has a profitable Markovian deviation.)

on $\{A, R\}$. [10] Decision A is to be interpreted as acceptance of the price and decision R is to be interpreted as rejection of it.

We further restrict attention to symmetric subgame-perfect equilibrium in Markovian strategies. Symmetry means that we restrict attention to profiles where consumers of the same type use the same strategy. This is WOLG because we allow mixed strategy. From now on, we refer to a symmetric subgame-perfect equilibrium in Markovian strategies simply as an "equilibrium". Equilibria where the prices only differ in periods with no transactions are identified. Among the equilibria for $G_{BP}(\Delta, T, z)$ (which we will show to exist), we pick one that attains the highest profit (which we will show to exist) and denote the profit level by $\Pi_{BP}(\Delta, T, z)$.

Next, we maximize $\Pi_{BP}(\Delta, T, z)$ over $(T, z) \in \{0, \Delta, \ldots\} \bigcup \{\infty\} \times [0, \infty)$, and denote the maximized value (which we will show to exist) by $\Pi_{BP}(\Delta)$ and call the maximizer(s) the optimal BP policy(ies), the maximizing T the optimal BP policy length(s) and the maximizing z the optimal BP policy scale(s).

1.2.2. Benchmark 1 – Full Price Commitment Model Setup

This model provides an upper bound that is the seller's "first-best" outcome. We call this game $G_{FPC}(\Delta)$. There are only two differences from the game $G_{BP}(\Delta, T, z)$. First, we do not impose sequential rationality on the seller's strategy. [11]

[10] We allow for mixed strategies, and allow the seller's strategy to depend on the last period's price (which is redundant for $T \geq \Delta$) to ensure existence. See Gul, Sonnenschein and Wilson (1986) (Section 2) for an example of non-existence otherwise.

[11] Notice that it is different from allowing the seller only to commit to a fixed sequence of future prices at $t = 0$, because here, since the event of demand drop is observable, a "full commitment" seller's future prices will depend on the demand realization.

Second, as there is no BP policy, there are no refunds, and the transactions in the past are no longer payoff-relevant. Therefore, the payoff-relevant *state* at time t is simply the residual demand at time t (i.e., the first component of the state for the BP game).

The definition of the (mixed) strategies for the seller and the consumers are the same as in the BP game. We emphasize that the difference from before is only that we drop the sequential rationality requirement on the seller's strategy in the definition of the equilibrium concept. (We still require the consumers' strategy to be sequentially rational.)

Let $\Pi_{FPC}(\Delta)$ be the highest profit level among that of the symmetric subgame equilibria of the game $G_{FPC}(\Delta)$ (which we will show to exist and to be unique).

1.2.3. Benchmark 2 – No Commitment Model Setup

This is a model where the seller has no power to commit to any future prices or to a BP policy. Formally, this is just a special case of the BP model with $T = 0$. We denote such a game by $G_{NC}(\Delta)$ and denote the highest profit of all symmetric Markov perfect equilibria given Δ by $\Pi_{NC}(\Delta)$ (which we will show to exist).

Next we will first analyze Benchmark 1 and 2, which will help with the analysis of our main model.

1.3. Benchmark 1 – Full Commitment model

We define three outcomes:

Definition 1.

If all H types buy at $t = 0$ and L types never buy, the outcome is said to be "Outcome-Only-H".

If all consumers buy at $t = 0$, the outcome is said to be "Outcome-All".

If all H types buy at $t = 0$ and all L types buy as soon as demand drops and never buy if demand remains high forever, the outcome is said to be "Outcome-Contingent".

Proposition 1. *(For fixed $\Delta > 0$) The space of parameters can be partitioned into three subsets. In the interior of each subset, the equilibrium outcome under full commitment is unique:*

(1) *If $L' < H'h$ and $L < Hh$, then it is Outcome-Only-H.*

(2) *If $L > Hh$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) < L - Hh$, then it is Outcome-All.*

(3) *If $L' > H'h$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$, then it is Outcome-Contingent.*[12]

Proof. See the Appendix. □

We summarize the Proposition 1 in a graph:

It is helpful to contrast this result when there is demand drop with the result when there is no demand drop (i.e., if $s = 0$):

Lemma 1. *(Stokey, 1979) When $s = 0$, i.e., when there is no demand drop, on the FPC equilibrium path, the seller charges the static monopoly price at $t = 0$ and no future transaction occurs after the first period.*

This is an instance of the familiar result stating that intertemporal price discrimination is not possible when the seller and consumers share the same discount rate. Let $\Pi_{s=0}$ denote the equilibrium profit if the seller has full commitment and there is no demand drop. We immediately have $\Pi_{s=0} = \max\{Hh, L\}$ from Lemma 1. (It is independent of Δ.) To parallel Figure 1.1, we present Lemma 1 in a figure as well:

[12] $L' > H'h$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$ is equivalent to ($L > Hh$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$) or ($L < Hh$ and $L' > H'h$). Therefore in Figure 1.1, this case is decomposed into two areas: I and II.

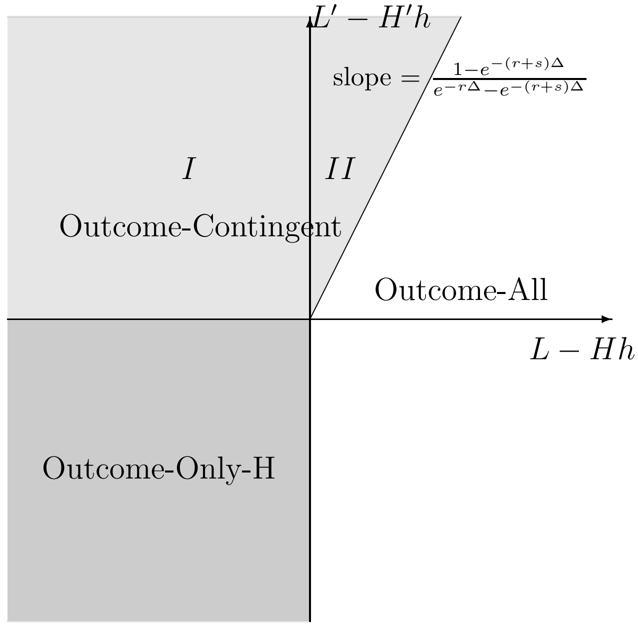


Figure 1.1. (Proposition 1) Equilibrium outcome under FPC.

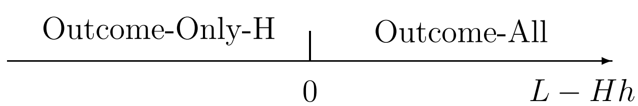


Figure 1.2. (Lemma 1) Equilibrium outcome under FPC without demand drop.

Remark: When demand can drop over time, if the seller sells to H types alone first and then to L types later, then the incentives of H types at the beginning are affected by both the future option they will have if the demand keeps high and the options they will have if the demand drops at different moments. H types at $t = 0$ applies to each future possibility the same discounting and the same probability in making their decisions, as that applied by the seller to each possibility for her profit. Therefore, in some sense, the case of demand drop can be separated out: what is optimal for the seller in the contingency of a demand drop only depends on the amount $L' - H'h$. If $L' - H'h > 0$, then it is optimal for the seller to sell to L types immediately at a demand drop because doing so does not depress too much the price H types are willing to accept at the beginning of the game. In other words, if the static monopoly price after the demand drop is L', it is best to sell to L types immediately after a drop; and if instead the static monopoly price after the demand drop is H', it is best to delay selling to L types forever after a demand drop.

We further look at two subcases given $L' - H'h > 0$:

First, consider the subcase $L' - H'h > 0$ and $L - Hh < 0$ (the area I in Figure 1.1). This is the case in which the static monopoly outcome is to sell to all types if demand is low, while the static monopoly outcome is to sell only to H types if demand is high. Thus, it is not surprising that the equilibrium outcome here is to sell to L types only if the demand drops, i.e., Outcome-Contingent.

Next, consider the subcase $L - Hh > 0$ and $\frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$ (the area II in Figure 1.1). There is something seemingly counter-intuitive here. Even though the static monopoly outcome calls for selling to all types no matter the demand is high or low (as $L - Hh > 0$ and $L' - H'h > 0$), it is still beneficial for the seller to delay selling to L types. This is because, when $L' - H'h$ is high enough (i.e., when the drop in the willingness-to-pay of H types as a fraction of H types' original willingness-to-pay is large enough relative to that of L types), the seller can take advantage of the concern of H types that their consumers surplus will diminish if demand drops (as the demand curve becomes "flatter" at a demand drop).

Both of the subcases share the feature that the demand drop is *disproportional*, in the sense that H types lose a bigger portion of their willingness-to-pay than L types. (If the drop is proportional, then 1) $L - Hh < 0$ implies that $L' - H'h < 0$, and 2) $L - Hh > 0$ implies that $L' - H'h < L - Hh$, both of which are ruled out by the definition of the subcases.)

We can calculate the expected profit for the three possible outcomes given Δ.

- Outcome-Contingent:

$$\Pi_{FPC}(\Delta) = Hh + \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h).$$

- Outcome-Only-H:

$$\Pi_{FPC}(\Delta) = Hh.$$

- Outcome-All:

$$\Pi_{FPC}(\Delta) = L.$$

Since $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}} \to \frac{s}{r+s}$ as $\Delta \to 0$, the following corollary is immediate:

Corollary 1. *(With $\Delta \to 0$)*

(1) *If $L' < H'h$ and $L < Hh$, then the equilibrium profit is Hh regardless of Δ.*

(2) *If $L > Hh$ and $\frac{s}{r+s}(L' - H'h) < L - Hh$, then the equilibrium profit is L, provided that Δ is small enough.*

(3) *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, then the equilibrium profit tends to $Hh + \frac{s}{r+s}(L' - H'h)$ as Δ tends to 0.*

The following proposition shows that, when the unique equilibrium outcome is Outcome-Contingent, the equilibrium profit of the seller is higher when there is a chance of a demand drop than when demand drop cannot happen. Again, this is because the seller can take advantage of the H types' concern about the demand drop if it has a chance to happen.

Proposition 2. *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, then when Δ is small enough:*

$$\Pi_{FPC}(\Delta) > \Pi_{s=0}$$

Proof. When Δ is small enough, we have $L' > H'h$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$, so by Proposition 1, we have:

$$\Pi_{FPC}(\Delta) = Hh + \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h).$$

Case 1. $L - Hh < 0$ (i.e., area I in Figure 1.1). In this case, $\Pi_{s=0} = Hh$.

$$\lim_{\Delta \to 0} \Pi_{FPC}(\Delta) - \Pi_{s=0} = \frac{s}{r+s}(L' - H'h) > 0.$$

Case 2. $L - Hh > 0$ (i.e., area II in Figure 1.1). In this case, $\Pi_{s=0} = L$.

$$\lim_{\Delta \to 0} \Pi_{FPC}(\Delta) - \Pi_{s=0} = -(L - Hh) + \frac{s}{r+s}(L' - H'h) > 0.$$

□

1.4. Benchmark 2 – No Commitment

"Coase conjecture" also holds when there is demand drop [13]:

Proposition 3. *Under no commitment, and in the absence of a BP policy,* $\Pi_{NC}(\Delta) \to L$ *as* $\Delta \to 0$.

Proof. See the Appendix. □

A comparison between the equilibrium profit under full-commitment and no-commitment gives us Figure 1.3.

It is immediate that when the full-commitment outcome is Outcome-All, the seller really doesn't need any commitment power.

[13] Notice that the demand drop here reduces the consumers' willingness-to-pay, but not the mass of consumers that value the product above the marginal cost.

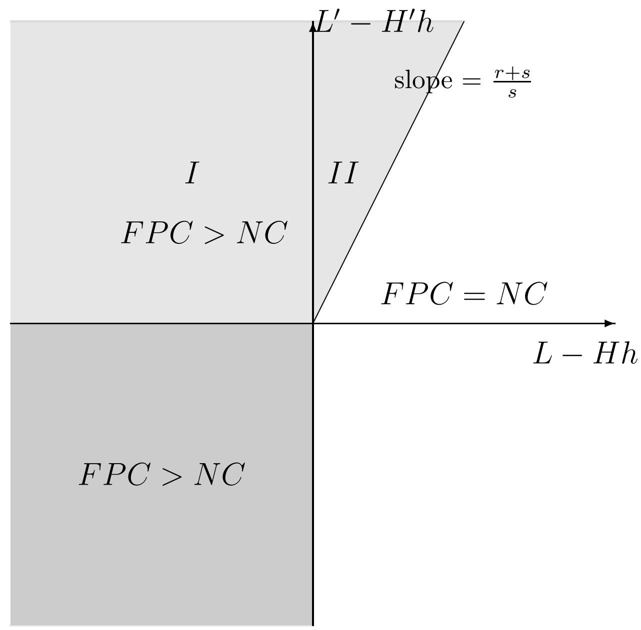


Figure 1.3. Equilibrium profit comparison between FPC and NC as $\Delta \to 0$.

1.5. BP model

We now consider the game $G_{BP}(\Delta, T, z)$, where T is the BP policy length and z is the refund scale. In this section, we will prove the three main results: the profit comparison across the three models, the characterization of the optimal policy, and the comparative statics. Since the analysis of backward induction will be lengthy, before proving them, we will first graphically present them and explain the intuitions.

1.5.1. Overview of the BP results

One result is about the positioning of the BP profit (at the optimal equilibrium given the optimal BP policy) relative to the benchmarks of FPC and NC. This is presented in Proposition 4 and Figure 1.4.

Proposition 4. *(With $\Delta \to 0$)*

(1) *If* $L > Hh$ *and* $\frac{s}{r+s}(L' - H'h) < L - Hh$*, then* $\Pi_{FPC}(\Delta) = \Pi_{BP}(\Delta) = \Pi_{NC}(\Delta)$ *and* $\Pi_{BP}(\Delta) = \Pi_{s=0}$*, provided that* Δ *is small enough.*

(2) *If* $L' > H'h$ *and* $\frac{s}{r+s}(L' - H'h) > L - Hh$*, then* $\Pi_{FPC}(\Delta) > \Pi_{BP}(\Delta) > \Pi_{NC}(\Delta)$ *and* $\Pi_{BP}(\Delta) > \Pi_{s=0}$*, provided that* Δ *is small enough.*

(3) *If* $L' < H'h$ *and* $L < Hh$*, then* $\Pi_{FPC}(\Delta) = \Pi_{BP}(\Delta) = \Pi_{s=0} > \Pi_{NC}(\Delta)$ *for any* Δ.

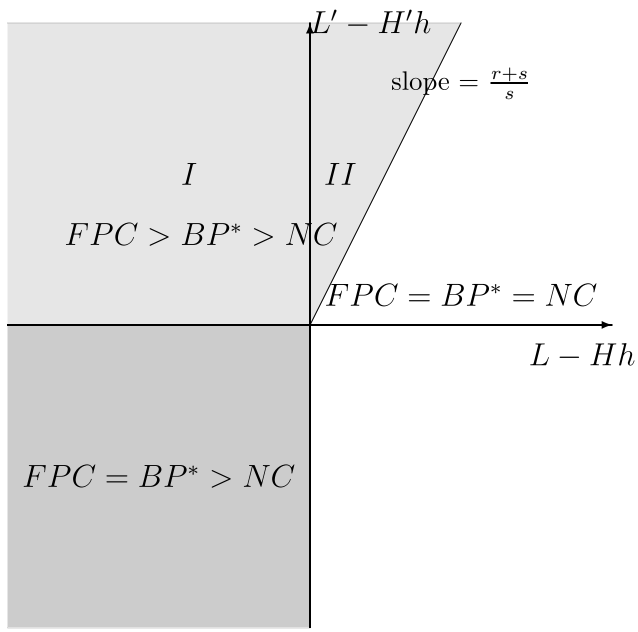


Figure 1.4. (Proposition 4) Equilibrium profit comparison as $\Delta \to 0$, where BP^* means the profit at the optimal equilibrium given the optimal BP length.

Even though the optimal BP policy gives a profit that is higher than the profit in a no-commitment model when the Coase conjecture is a problem (i.e., in the light grey and dark grey areas of in Figure 1.4), the BP profit fails to reach the full-commitment profit because in these cases, the full-commitment profit calls for an outcome that is Outcome-Contingent (recall Figure 1.1). Outcome-Contingent requires the seller to have the flexibility to cut price as soon as the demand drops, but to keep prices high as long as the demand is high.

We will show that such an outcome is impossible to duplicate if the seller can only commit to a BP policy. This is because subgame-perfection implies that when the refunds obligation is high enough to deter the seller from reducing the price when demand is high, then the seller will also refrain from reducing price when demand has dropped. This is caused by two effects: first, when demand is low, if the seller reduces price to satisfy the residual demand, she has to set a lower price, and thus has to pay higher refunds to previous customers than if the demand is high; second, when the demand is low, the seller is more willing to wait for the BP policy to expire, because the undiscounted future profit from selling to the residual demand is lower. Thus the seller loses less from discounting. Therefore, a BP commitment does not have the ability to create Outcome-Contingent. [14] Notice that it is not because a BP outcome fails to create outcomes that are contingent on the demand realization, but because the contingency goes the wrong way. [15]

When the seller's full-commitment outcome is Outcome-Contingent, the seller's optimal BP profit is still higher than under full commitment without demand drop (i.e., $\Pi_{BP}(\Delta) > \Pi_{s=0}$). So the demand drop is actually *desirable* for the seller if she can commit to her optimal BP policy! This is because the particular demand drop that gives rise to Outcome-Contingent under FPC makes the willingness-to-pay "flatter" among consumers, so that H types are actually afraid of buying at a demand drop. (Consider an extreme case that $H' = L'$, i.e., after the demand drop, H types and L types value the product the same: now when the demand is high, the seller has to leave H types some surplus to prevent them

[14]If the demand drop is deterministic instead of probabilistic, then BP can achieve the full commitment profit. See Section 1.6 for an alternative model of deterministic (and continuous) demand drop.

[15]Our "first-best" is a quite high standard, because it requires a commitment that can flexibly react to the realization of future uncertainty. Suppose we, instead, consider a different benchmark, allowing the seller only to commit to one future price sequence. Then this commitment benchmark will actually under-perform BP. See Section 1.6 for the detail.

from buying at the price intended for L types, however, when the demand is low, the seller can extract all the surplus from H types, just as she can do with L types, so H types is at a disadvantage if the demand curve is "flatter".) Therefore, the seller can take advantage of H types' fear of a demand drop to do intertemporal price discrimination profitably. The detail will be shown later.

The second main result in the BP model is that we characterize the optimal BP policy for each parameter case:

Proposition 5. (1) *If $L' < H'h$ and $L < Hh$, then the optimal BP policy must have a length of infinity, and any policy with infinite length and a refund scale $z \geq 1$ is optimal.*

(2) *If $L > Hh$ and $\frac{s}{r+s}(L' - H'h) < L - Hh$, then any BP policy is optimal, provided that Δ is small enough.*

(3) *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, then, when Δ is small enough, the optimal BP policy must have a finite and positive length of $T^*(\Delta)$, which tends to $T^* \equiv \frac{1}{s}\ln(\frac{r+s}{r}(1 - \frac{L-Hh}{L'-H'h}))$ as Δ tends to 0, and any policy with this length and a refund scale $z \geq 1$ is optimal. At $T^*(\Delta)$, there is an equilibrium whose outcome is uniquely optimal given $T^*(\Delta)$; on the equilibrium path, the seller sells to H types at the beginning and then sells to L types at $T^*(\Delta) + \Delta$ at L types' willingness-to-pay without paying any refunds.*

We highlight the optimal BP policy length in Figure 1.5:

Of particular interest is the parameter case that gives rise to positive and finite optimal BP length. Notice this is also the parameter case for which the equilibrium outcome under full commitment is Outcome-Contingent. Recall that Outcome-Contingent says that the

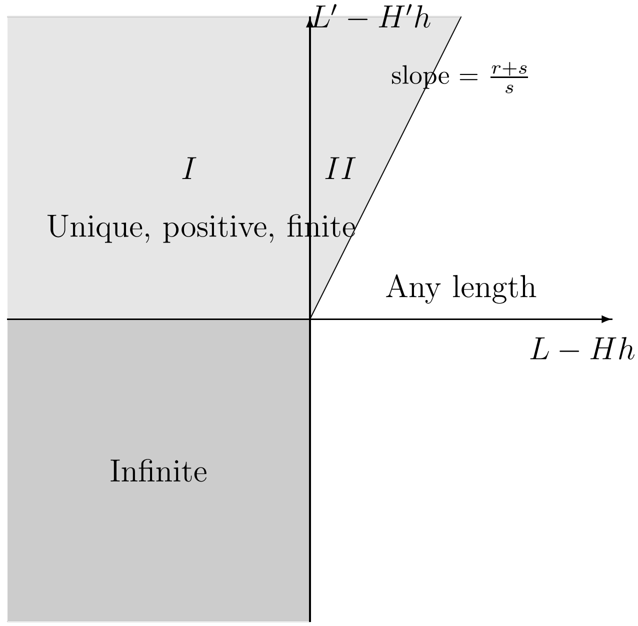


Figure 1.5. (Proposition 5) Optimal policy lengths as $\Delta \to 0$.

seller should not delay in selling to L types if demand drops, but should delay selling to L types forever if demand keeps high. A finite BP length creates a finite delay between selling to H types and selling to L types, which is like a compromise between the need of the different demand realizations. If the delay is set to be too small, then it is very likely that demand is still high by the end of the delay and the future seller will sell prematurely to L types. However, if the delay is too large, the demand is likely to have dropped long before the end of delay, while the future seller has to keep on discounting the future profit accruing from low types until after the delay. Therefore, the delay should be set to be finite and positive for the seller. Later, in the actually analysis, we will see why choosing an optimal length of BP is equivalent to choosing an optimal delay before selling to L types.

Thirdly, the following proposition does the comparative statics of the optimal policy length with respect to the parameter s (the likelihood of demand drop) for the parameter case where the optimal policy length has to be finite.

Proposition 6. *Suppose $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$. When Δ is small enough, we have the following comparative statics:*

(1) ***Policy length:*** *when $L' > H'h$ and $L < Hh$, the optimal policy length decreases in s.*

(2) ***Price:*** *when the fraction of H types is big enough, the first period price at the optimal equilibrium given the optimal equilibrium policy increases in s, otherwise it decreases in s.*

(3) ***Profit:*** *the equilibrium profit at the optimal equilibrium given the optimal policy increases in s.*

When s increases, there are two effects on the limit of the optimal policy, T^*. On one hand, the expected dropping time $\frac{1}{s}$ decreases, which decreases the optimal delay. On the other hand, higher s makes the change in delay have a smaller effect on the case of demand being high relative to the case of demand being low, which calls for a larger delay. These two effects pushes the optimal delay in the opposite direction. Which effect is larger depends on the parameter case. In particular, in region I in the parameter space, the first effect is always stronger and the optimal policy length decreases with s. Also, from the expression $T^* = \frac{1}{s}\ln(\frac{r+s}{r}(1 - \frac{L-Hh}{L'-H'h}))$, it is easy to see that all else equal, the higher H is, the higher T^* is. The intuition is that, when H is higher, it is more important to make H types not to wait for the discount after the delay, so the seller wants to make it less likely that the demand is high after the delay. The comparative statics are the same if we decrease L.

The price at the beginning is the price that H types pay on the equilibrium path. When s increases, the seller is more eager to sell to L types which pushes down the price, while H types are also more eager to buy the product which pushes up the price. When the fraction of H types is high, the first effect is weaker relative to the second, which makes the price go up in s.

The profit increases in s because the seller can benefit more from the demand drop which allows her to do intertemporal price discrimination.

The optimal equilibrium outcome at the optimal BP policy when it is finite is of a very simple form: at $t = 0$, the seller sells to all H types at a price that makes H types exactly indifferent between buying now and waiting to buy at $T^*(\Delta) + \Delta$ at L types' willingness-to-pay. Then the seller charges prices high enough such that there is no transaction or refunds until time $T^*(\Delta) + \Delta$ (regardless of demand realization), at which the seller sells to all L types at their willingness-to-pay. A major feature is that there is no refund on the equilibrium path. The basic intuition is that if it is better for the seller to sell to more consumers (which we call "group B") and pay refunds to some previous consumers (which we call "group A") than to wait until the optimal BP policy expires to sell to group B, then the seller should sell to group A and B together in the first place.

We shall also notice that on the equilibrium path the price in the second period is strictly higher than the price in the first period. The reason is the following. Starting from the second period until the policy expires, the seller is indifferent among any prices that discourages L types from buying and does not qualify H types for refund (i.e., any prices that are higher than or equal to the first period price). However, for the prices to be part of an equilibrium outcome for the whole game, they have to be high enough so that H types at $t = 0$ do not want to deviate to not buying and waiting to buy at $t > 0$. This in particular implies that price in the second period when demand turns out to be strictly high in the second period must be higher than the first period price. Otherwise, such a deviation is attractive because deviating at $t = 0$ to buy just one period later allows an infinitesimally small H type to be eligible for refunds when the price is reduced later at $t = T^*(\Delta) + \Delta$. We show that a price great or equal to H is sufficient to deter deviations for the second periods and onward until the policy expires. This price jump in the second period corresponds to the phenomenon that the seller offers a discount only applicable to new products or "full price items", which is also sometimes called "introductory offers". [16]

Next, we will go through the following steps to establish Proposition 4 and Proposition 5:

1. We show that the optimal BP policy is easy to identify for parameter cases for which Outcome-Contingent is not the equilibrium outcome under FPC, by trying to duplicate the equilibrium outcome under full commitment. So the parameter case where the equilibrium outcome under FPC is Outcome-Contingent remains as the only "interesting" case. (Section 1.5.2)

2. Then, for each policy, we narrow down the set of outcomes that are potentially optimal to four classes. (Section 1.5.3)

3. We identify the BP policy that maximizes the profit for one particular type of outcome, and show that this outcome given this BP policy dominates all other outcomes at any BP policy when Δ is small enough and when the parameter case is the "interesting" one. (Section 1.5.4)

[16] This price jump would disappear under two alterations. First, if instead of setting a policy length counting from the initial time of purchase for each transaction, the seller specifies a particular time (a date) after which the policy expires (regardless of when the consumers bought). The analysis would not change, except that prices do not need to be higher after the first period to discourage purchases from H types because H types cannot get refund by waiting to the second period and so on. For example, Chrysler had a BP policy in 1989 that specifies an end date for the policy to be the last day of 1990 (Brandenburger and Nalebuff, 1997), and Ford used a BP policy in 1999 that specifies an end date for the policy to be the last day of 1999. Second, if the seller can specify that the policy only applies to those consumers who bought at time $t = 0$ (but not later), then the prices do not need to be higher after the first period as well.

4. We show that this outcome can be supported by an equilibrium in the game given its optimal BP policy. (Section 1.5.4)

1.5.2. The easier parameter cases

Lemma 2. *If an equilibrium is optimal given a BP policy and on the equilibrium path the seller offers price $P_0 = L$ at $t = 0$, then all the consumers must accept it and the game must end in the first period on the equilibrium path.*

Remark: This lemma implies that, to identify the optimal equilibrium, among those in which not all sales occur in the first period, we need only consider those that have a first period price above L.

Let us first address the easy parameter cases:

Lemma 3. *(The easy cases)*
If $L' < H'h$ and $L < Hh$, then the optimal BP policy must have a length of infinity, and any policy with infinite length and a refund scale $z \geq 1$ is optimal.
If $L > Hh$ and $\frac{s}{r+s}(L' - H'h) < L - Hh$, then any BP policy is optimal, provided that Δ is small enough.

Proof. Notice that a seller with full commitment ability (i.e., in the FPC model) can at least achieve the profit of an equilibrium of any BP model. Suppose the BP equilibrium outcome is such that for each demand realization $u \in \{0, \Delta, 2\Delta, \ldots\}$ (where u denotes the time of demand drop for $u > 0$ and $u = 0$ denotes no demand drop), H types buy at time t^u_H and pay Q^u_H, which is the price minus the (expected) future refunds, and L types buy at time t^u_L and pay Q^u_L, which is the price minus the (expected) future refunds discounted to that moment. Since refunds do not depend on consumers' type, if H and L types buy at the same time given some u, then $Q^u_H = Q^u_L$. Also, notice that refunds are always lower than the initial price because future prices cannot be negative. So $Q^u_H \geq 0$ and $Q^u_L \geq 0$. A seller with commitment power can commit to a contingent price path that charges Q^u_H at t^u_H and Q^u_L at t^u_L for all u and charges an infinite price elsewhere. Therefore, the profit level under FPC gives an upper bound of the profit under BP.

Recall that a FPC equilibrium can only have three outcomes: Outcome-Contingent, Outcome-Only-H and Outcome-All (Proposition 1).

Case 1. Parameters are such that Outcome-Only-H is the equilibrium outcome, i.e., $L < Hh$ and $L' < H'h$.

We first claim that Outcome-Only-H can be an equilibrium outcome under a BP game with $T = \infty$ and $z = 1$. The following is an equilibrium: 1) L types only accept a price below or equal to their willingness-to-pay 2) H types only accept a price below or equal to their willingness-to-pay, 3) the seller charges price H and then an infinite price forever.

After selling to H types at H, if the seller sells to L types, the price has to be at most L or L' depending on whether demand has dropped or not. We have $L(1-h) - (H-L)h = L - Hh < 0$ and $L'(1-h) - (H-L')h = L' - H'h < 0$. Therefore the seller will not sell again after selling to H types. (This is also true for $z > 1$, because the refund becomes even bigger.)

On the other hand, no finite BP length can be optimal no matter what the refund scale is. This is because the seller will for sure sell after the policy expires and the profit of Outcome-Only-H cannot be achieved as a result.

Case 2. Parameters are such that Outcome-All is the equilibrium outcome under FPC, i.e., $L > Hh$ and $\frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) < L - Hh$.

First, we claim that given any $T = 0, \Delta, \ldots$ and $z = 0$, the following is a BP equilibrium: 1) L types accept with probability one any price below or equal to their willingness-to-pay; 2) H types accept any price below or equal to $H - e^{-(r+s)\Delta}(H-L) - e^{-r\Delta}(1-e^{-s\Delta})(H'-L')$ with probability one if L types are still in the market and demand is high, and accept any price below or equal to $H' - e^{-r\Delta}(H' - L')$ with probability one if L types are still in the market and demand is low, and accept any price below or equal to their willingness-to-pay if L types are not in the market; 3) the seller's strategy is as follows: if the static monopoly price given the residual demand in the market is L types' willingness-to-pay, then the seller charges this price; if instead the static monopoly price of the residual demand is H type's willingness-to-pay, then the seller charges a price that will make H types indifferent between buying now and waiting to buy a period later at L types' willingness-to-pay. This implies that on the proposed equilibrium path, the seller charges price L at $t = 0$ and all consumers accept it there.

The seller does not want to increase the price at $t = 0$: if she deviates to a price in $(L, H - e^{-(r+s)\Delta}(H - L) - e^{-r\Delta}(1 - e^{-s\Delta})(H' - L')]$, then H types buy at $t = 0$, but none of the L types buy. This means that, in the next period, she will set a price equal to L types' willingness-to-pay and get all the L types to buy there. The profit at $t = 0$ is thus at most $(H-e^{-(r+s)\Delta}(H-L)-e^{-r\Delta}(1-e^{-s\Delta})(H'-L'))h+e^{-(r+s)\Delta}L(1-h)+e^{-r\Delta}(1-e^{-s\Delta})L'(1-h)$, which is less than L precisely because $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) < L - Hh$; if she deviates to a price in $(H - e^{-(r+s)\Delta}(H - L) - e^{-r\Delta}(1 - e^{-s\Delta})(H' - L'), \infty)$, then no consumers buy in the first period, and the seller charges (i) L in the next period if demand is high there (after which all consumers buy in the next period), or (ii) L' if the demand is low and $L' \geq H'h$ (after which all consumers buy in the next period), or (iii) $H' - e^{-r\Delta}(H' - L')$ if the demand is low and $L' < H'h$ (after which only H types will buy in the next period and afterwards the seller will sell to L types at L'. None of these three possible outcomes gives the seller a profit higher than the equilibrium profit. The seller will not decrease the price at $t = 0$ either, because all consumers accept L.

Since this BP equilibrium achieves the highest profit possible given any T and any z (the profit under FPC), this policy is optimal. The construction of equilibria when $z > 0$ is similar but more convoluted.

□

In other words, the only parameter case of interest (where the optimal BP policy is not obvious) is when Outcome-Contingent is the equilibrium outcome under FPC, i.e., when we have:

$$L' > H'h \text{ and } \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) > L - Hh.$$

or its limit version:

$$L' > H'h \text{ and } \frac{s}{r+s}(L' - H'h) > L - Hh.$$

We re-emphasize that this case implies that the demand drop is disproportional in the sense that H types lose a bigger portion of their willingness-to-pay than L types.

In our search for the optimal equilibrium given a BP policy, we can safely disregard equilibria such that no consumers buy in the first period, because if one exists, then there must also exist an equilibria where everything gets shifted in time such that some consumers buy in the first period. This "shifted" equilibrium yields the seller strictly higher profit because of positive discounting.

Definition 2. *Given a BP policy, an equilibrium is called a **pure-action** equilibrium if consumers do not randomize on the equilibrium path given any demand realization.*

Notice that this definition does not rule out randomization off the equilibrium path. We will first focus on the set of pure-action equilibria, then Lemma 9, to be discussed later, will show that the optimal equilibrium (across all BP policies) among the set of pure-action equilibria outperforms the optimal equilibrium (across all BP policies) among the set of non-pure-action equilibria.

It is immediate that, on any potentially optimal pure-action equilibrium, by definition, transactions happen at most twice on equilibrium (even though the exact time and price of the transactions may depend on the demand realization).

1.5.3. Four classes of outcomes

We now solve the game $G_{BP}(\Delta, T, z)$ by backward induction.

1.5.3.1. Subgame after H types have all bought at $t = 0$ at $P_H > L$. In this section, we consider the subgame starting at time Δ, given that 1) all H types have bought at $t = 0$ at a price denoted by $P_H > L$ and L types all rejected the price at $t = 0$ and 2) the seller has committed to a policy with $0 < T < \infty$, $z > 0$ and 3) the demand for $t = \Delta$ has not been revealed.

Lemma 4. *At any time t at which all H types have already bought, the equilibrium strategy of L types must specify that they will accept any price below or equal to their willingness-to-pay with probability one.*

Proof. See the Appendix. □

By Lemma 4, we know that the outcome at time t (if L types are all in the market) is "binary" in the sense that either the seller charges a price equal to L types' willingness-to-pay (and all L types buy and H types possibly get refunds) or there is no transaction. At time $t = T + \Delta$ because if all L types are still in the market at that time, the seller must sell to all of them at their willingness-to-pay immediately (by Lemma 4), because there is no longer any refund obligations. Therefore, the game effectively ends in finite periods if T is finite. This enables us to do backward induction to get the following result:

Proposition 7. *We first define three outcomes for the subgame:*

- ***Subgame-Outcome-Wait:*** *the seller does not sell to L types until $t = T + \Delta$ regardless of demand.*
- ***Subgame-Outcome-Semi-Wait:*** *the seller sells to L types at $t = \Delta$ if demand is high at $t = \Delta$ but does not sell to L types until $t = T + \Delta$ if demand is low at $t = \Delta$.*
- ***Subgame-Outcome-No-Wait:*** *the seller sells to L types at $t = \Delta$ regardless of demand.*

For the subgame starting at Δ given $P_H > L$, the equilibrium outcome must be one of the three: Subgame-Outcome-Wait, Subgame-Outcome-Semi-Wait and Subgame-Outcome-No-Wait. Moreover,

(1) *If $P_H > \overline{P_H}(T, z)$, then the unique equilibrium outcome is Subgame-Outcome-Wait.*

(2) *If $\underline{P_H}(T, z) < P_H < \overline{P_H}(T, z)$, then the unique equilibrium outcome is Subgame-Outcome-Semi-Wait.*

(3) *If $L < P_H < \underline{P_H}(T, z)$, then the unique equilibrium outcome is Subgame-Outcome-No-Wait.*

where for any $t = \Delta, 2\Delta, \ldots$ and any $z > 0$:

$$\overline{P_H}(t,z) \equiv L + \frac{1}{zh}[L(1-h) - e^{-rt}((1-e^{-st})L' + e^{-st}L)(1-h)]$$
$$\underline{P_H}(t,z) \equiv L' + \frac{1}{zh}[L'(1-h) - e^{-rt}L'(1-h)]$$

Remark: 1) it is intuitive that the higher the price at which H types bought, the higher the subsequent liability imposed on the seller. Therefore, a higher P_H gives the seller stronger incentives to wait for the refund policy to expire before selling to L types.

2) Among the three possible subgame outcomes, we see that if the demand is low the seller sells to L types later than (or at the same as) if the demand keeps high. The basic intuition is that when the demand is low, the seller has to incur a bigger refund to H types when selling to L types, but the cost of delaying selling to L types is lower because the discounting applies to a smaller amount of profit.

3) Transaction with L types happens either at $t = \Delta$ or $t = T + \Delta$ in the subgame, because considering the positive discounting and positive chance of drop in L types' valuation from one period to the next, a seller will choose to sell now when she realizes that her future self will sell and pay refunds anyway if she does not sell now.

4) A higher z leads to lower cutoffs $\overline{P_H}(T,z)$ and $\underline{P_H}(T,z)$ because a higher refund scale allows the seller to charge a lower price to H types and still be very disciplined in the future.

Proof. See the Appendix. It is a process of backward induction starting from $t = T$. $\square$

1.5.3.2. The whole game.

Definition 3. *We define four classes of pure-action outcomes for the whole game $G_{BP}(\Delta, T, z)$:*

(1) ***BP-Outcome-1*** *The seller sells to H types at $t = 0$ at a price $P_H \geq \overline{P_H}(T, z)$ and then Subgame-Outcome-Wait follows from $t = \Delta$.*

(2) ***BP-Outcome-2*** *The seller sells to H types at $t = 0$ at a price $P_H \in [\max\{\underline{P_H}(T, z), L\}, \overline{P_H}(T, z)]$ and then Subgame-Outcome-Semi-Wait follows from $t = \Delta$.*

(3) ***BP-Outcome-3*** *The seller sells to H types at $t = 0$ at a price $P_H \in (L, \underline{P_H}(T, z)]$ and then Subgame-Outcome-No-Wait follows from $t = \Delta$.*

(4) ***BP-Outcome-4*** *The seller sells to H types at $P_H = L$ and all consumers buy at once.*

Lemma 5. *The optimal equilibrium outcome among pure-action equilibrium outcomes given a BP policy must be one of the four classes of outcomes: BP-Outcome-1, BP-Outcome-2, BP-Outcome-3, BP-Outcome-4*[17].

Proof. This is directly implied by Lemma 2 and Proposition 7. □

Definition 4. *Let $Q_1(\Delta, T)$ be the price that makes H types indifferent at $t = 0$ between accepting and rejecting on a BP-Outcome-1 equilibrium path. Let $Q_n(\Delta, T, z)$ be the price that makes H types indifferent at $t = 0$ between accepting and rejecting when the equilibrium outcome is BP-Outcome-n for $n = 2, 3$.* [18]

It immediately follows that:

[17] We will establish existence later.

[18] Notice that *BP-Outcome-1* does not involve refunds. Therefore, the price that makes H types indifferent does not depend on z.

$$H - Q_1(\Delta, T) = e^{-(r+s)(T+\Delta)}(H - L) + e^{-r(T+\Delta)}(1 - e^{-s(T+\Delta)})(H' - L'),$$

$$H - Q_2(\Delta, T, z) + e^{-(r+s)\Delta} z(Q_2(\Delta, T, z) - L) =$$
$$e^{-(r+s)\Delta}(H - L) + e^{-r(T+\Delta)}(1 - e^{-s\Delta})(H' - L'),$$

$$H - Q_3(\Delta, T, z) + e^{-r\Delta} z(Q_3(\Delta, T, z) - (e^{-s\Delta} L + (1 - e^{-s\Delta})L')) =$$
$$e^{-(r+s)\Delta}(H - L) + e^{-r\Delta}(1 - e^{-s\Delta})(H' - L').$$

It also immediately follows that we have three profit "ceilings" for outcome classes BP-Outcome-1, BP-Outcome-2, and BP-Outcome-3 (if they are equilibrium outcomes). These are the profits when the seller sells to H types at a price that makes H types exactly indifferent, i.e., at prices $Q_1(\Delta, T)$, $Q_2(\Delta, T, z)$, and $Q_3(\Delta, T, z)$. Notice that the profit ceiling for BP-Outcome-2 does not depend on z any more and the upper bound for BP-Outcome-3 does not depend on T or z:

$$\Pi_1^*(\Delta, T) \equiv L + (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})(L' - H'h)$$
$$- (1 - e^{-(r+s)(T+\Delta)})(L - Hh),$$

$$\Pi_2^*(\Delta, T) \equiv Q_2(\Delta, T, z)h + e^{-(r+s)\Delta}(L - Q_2(\Delta, T, z)h)$$
$$+ e^{-r(T+\Delta)}(1 - e^{-s\Delta})L'(1 - h)$$
$$= L + e^{-r(T+\Delta)}(1 - e^{-s\Delta})(L' - H'h) - (1 - e^{-(r+s)\Delta})(L - Hh),$$

$$\Pi_3^*(\Delta) \equiv Q_3(\Delta, T, z)h + e^{-(r+s)\Delta}(L - Q_3(\Delta, T, z)h)$$
$$+ (e^{-r\Delta} - e^{-(r+s)\Delta})(L' - Q_3(\Delta, T, z)h)$$
$$= L + (e^{-r\Delta} - e^{-(r+s)\Delta})(L' - H'h) - (1 - e^{-(r+s)\Delta})(L - Hh).$$

We denote the profit from BP-Outcome-4 by Π_4, so we have $\Pi_4 = L$. It does not depend on Δ, T or z.

Among the class of BP-Outcome-1, we are particularly interested in one outcome:

Definition 5. ***BP-Outcome-1**** *is a BP-Outcome-1 with* $P_H = Q_1(\Delta, T)$.

1.5.4. Optimal BP policy

The following two lemmas show us that, under the interesting parameter case, the profit of an outcome within BP-Outcome-2 and BP-Outcome-3 is lower than that of BP-Outcome-1*, when the BP policy is such that it maximizes the profit level of BP-Outcome-1*.

Lemma 6. *If $L' > H'h$, then $\Pi_3^*(\Delta) > \Pi_2^*(\Delta, T)$ for any $T = \Delta, \ldots$.*

Proof. Since $L' > H'h$, we have:

$$\Pi_3^*(\Delta) - \Pi_2^*(\Delta, T) = (e^{-r\Delta} - e^{-r(T+\Delta)})(1 - e^{-s\Delta})(L' - H'h) > 0.$$

$\square$

Definition 6. *Let $\Gamma_1(\Delta)$ be the set of maximizers for the following problem, which we will refer to as the Original Problem:*

$$Max_{T\in\{0,\Delta,\ldots\}} \quad \Pi_1^*(\Delta, T)$$

We denote the element(s) of $\Gamma_1(\Delta)$ by $T_1^*(\Delta)$.

The following lemma establishes that $T_1^*(\Delta)$ exists and the maximized profit for BP-Outcome-1* exceeds that of BP-Outcome-2, -3, and -4 at $T_1^*(\Delta)$.

Lemma 7. *If $L' > H'h$ and $\frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$, then $T_1^*(\Delta)$ exist(s) and is/are finite.*

If $L' > H'h$ and $\frac{s}{s+r}(L' - H'h) > L - Hh$, then as $\Delta \to 0$, $T_1^*(\Delta)$ converge to a finite and positive number:

$$\frac{1}{s}\ln(\frac{r+s}{r}(1 - \frac{L - Hh}{L' - H'h})).$$

Moreover, when Δ is small enough, $\Pi_1^*(\Delta, T_1^*(\Delta))$ is strictly greater than $\Pi_2^*(\Delta, T_1^*(\Delta))$, $\Pi_3^*(\Delta)$ and Π_4.

Proof. See the Appendix. $\square$

Remark: it is obvious that $\Pi_1^*(\Delta, T_1^*(\Delta)) \geq \Pi_3^*(\Delta)$ just by the definition of $T_1^*(\Delta)$ and the fact that $\Pi_3^*(\Delta) = \Pi_1^*(\Delta, 0)$. What Lemma 7 does is that it shows that actually the difference does not shrink to zero as $\Delta \to 0$.

The following is a necessary (but not sufficient) condition for BP-Outcome-1* to be an equilibrium outcome. It says that the price H types accepted must be high enough so that it is credible that the future seller will wait until the policy expires before selling to L types.

Definition 7. *We call BP-Outcome-1* **feasible** if $Q_1(\Delta, T) \geq \overline{P_H}(T, z)$.*

Lemma 8. *At $T_1^*(\Delta)$, BP-Outcome-1* is feasible for any $z \geq 1$.*

Proof. Take any T such that $\Pi_1^*(\Delta, T) > \Pi_4 = L$, we have:

$$
\begin{aligned}
&\Pi_1^*(\Delta, T) > L \\
\Leftrightarrow & Q_1(\Delta, T) > \overline{P_H}(T + \Delta, 1) \\
\Rightarrow & Q_1(\Delta, T) > \overline{P_H}(T, 1)
\end{aligned}
$$

This exactly means that BP-Outcome-1* is feasible at such a T for $z = 1$, so in particular it holds at $T = T_1^*(\Delta)$ for $z = 1$. Since $\overline{P_H}(T, z)$ decreases in z, we know that $Q_1(\Delta, T_1^*(\Delta)) \geq \overline{P_H}(T_1^*(\Delta), z)$ for any $z \geq 1$. □

Remark: 1) To see the intuition, notice that another way of saying that BP-Outcome-1* is better than BP-Outcome-4 is to say that after high types have accepted a price of $Q_1(\Delta, T)$, the seller is willing to wait for $T + \Delta$ time to avoid paying refunds if $z = 1$. The feasibility condition given $z = 1$ simply means that after high types have accepted a price of $Q_1(\Delta, T)$, the seller is willing to wait for T time to avoid paying refunds. Therefore the feasibility condition is weaker than the condition that BP-Outcome-1* is more profitable than BP-Outcome-4. 2) $z \geq 1$ is a sufficient but not necessary condition. We can have a lower z to make BP-Outcome-1* feasible at $T_1^*(\Delta)$. The exact cutoff depends on the parameters $(\Delta, r, s, H, L, H', L', h)$, but it is always less than $z = 1$. (It is always strictly positive, because $z = 0$ is equivalent to having no BP policy, so for $z = 0$, BP-Outcome-1* is not feasible at any T.) The more the seller prefers the profit $\Pi_1^*(\Delta, T)$ versus L, the less punishment the seller need to discipline herself from cutting price too soon later, so the lower the critical cutoff is. Therefore, when $\Pi_1^*(\Delta, T) = L$, the critical cutoff is the highest given a Δ.

We can plot the profit of BP-Outcome-1* and -4 as functions of the policy length T. Figure 1.6 and Figure 1.7 show how such profit functions would look like when Δ is close to zero (for two different parameter cases). The figures are not drawn for any specific values of parameters, but are drawn to show the general shape of the functions. Notice that, in Figure 1.7, the line that stands for the profit of BP-Outcome-1* is absent for large T. This is because BP-Outcome-1* is no longer feasible when T is too large.

So far, we have only considered outcomes that are pure-action, the next lemma will show that it is without loss of generality to do so if the best outcome among pure-action outcomes can indeed be an equilibrium outcome. Recall that Lemma 4 have already shown that if the actions of H types are “pure” on the equilibrium path, then the actions of L types must also be “pure”, so we only need to show that on the equilibrium path of the optimal equilibrium the actions of H types must be “pure”.

Lemma 9. *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, then the profit attained by the optimal BP policy for the set of equilibria where H types randomize on the equilibrium*

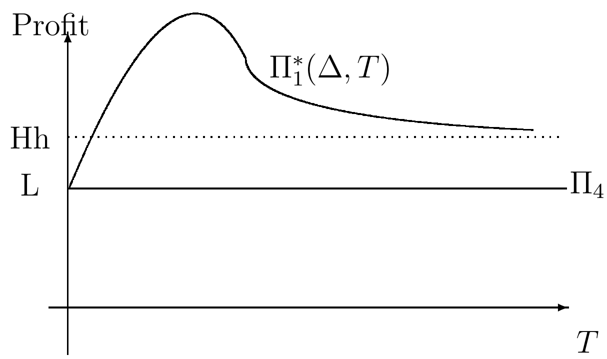


Figure 1.6. (Lemma 7) $L' > H'h, L < Hh$.

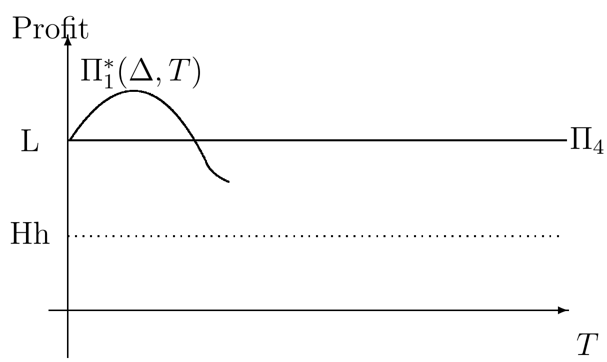


Figure 1.7. (Lemma 7) $L' > H'h, L > Hh, \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$.

path is less than that attained by BP-Outcome-1 at its optimal BP policy when* Δ *is small enough.*

Proof. See the Appendix. □

Remark: Even though having H types buying gradually at different moments can help the seller in a NC model for a positive Δ (because it allows the seller to delay selling to L types credibly), such randomization by the consumers does not help the seller when she has the ability to commit to a BP policy. This is because she can simply set a BP policy with a severe enough refund scale to obtain the necessary self-discipline.

Lemma 10. *There exists a threshold $\tilde{\Delta} > 0$ such that for any $\Delta < \tilde{\Delta}$ and any $z \geq 1$:*

The optimal length $T^*(\Delta)$ exist(s) and equal(s) to $T_1^*(\Delta)$ (defined in Definition 6). It takes at most two values (in which case they are Δ apart), which is/are strictly positive and finite; at $T^*(\Delta)$, there is an equilibrium for the game $G_{BP}(\Delta, T^*(\Delta), z)$, whose outcome is uniquely optimal among all the equilibrium outcomes of the game; on the equilibrium path, the seller sells to H types at the beginning and then sells to L types at $T^*(\Delta)+\Delta$ at L types' willingness-to-pay without paying any refunds if and only if

$$L' > H'h \text{ and } \frac{s}{r+s}(L' - H'h) > L - Hh.$$

Proof. *"If" part:* Lemma 7 has already shown that BP-Outcome-1* dominates BP-Outcome-2, -3, and -4 for a BP policy length $T_1^*(\Delta)$. We further notice that the profit $\Pi_1^*(\Delta, T_1^*(\Delta))$ strictly exceeds the profit of BP-Outcome-3 and -4 not only at $T_1^*(\Delta)$, but also at all other possible $T = 0, \Delta, \ldots$ simply because the profits of BP-Outcome-3 and -4 do not depend on T. Also, BP-Outcome-3 dominates BP-Outcome-2 for all T (Lemma 6), so BP-Outcome-1* dominates BP-Outcome-2, -3 and -4 for all T. Therefore, if we can show that BP-Outcome-1* with $T_1^*(\Delta)$ can indeed be supported by an equilibrium, then we have $T^*(\Delta) = T_1^*(\Delta)$.

Next we show that BP-Outcome-1* with $T_1^*(\Delta)$ can indeed be supported by an equilibrium.

We develop some notations here, let $\bar{P}(\Delta)$ be defined by the following inequality:

$$H - \bar{P}(\Delta) = e^{-(r+s)\Delta}(H - Q_1(\Delta, T_1^*(\Delta))) + e^{-r\Delta}(1 - e^{-s\Delta})(H' - L')$$

Simple algebra shows that $\bar{P}(\Delta) > Q_1(\Delta, T_1^*(\Delta))$.

If the seller deviates to a price $P' \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$, it is no longer sequentially rational for H types to reject the price with probability one by the definition of $\bar{P}(\Delta)$. In this case, the equilibrium involves both the seller and the H types randomizing off the equilibrium path. The detail of the equilibrium is shown in the Appendix.

"Only if" part: It follows directly from Lemma 3. □

The following corollary is immediate:

Corollary 2. *When $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > (L - Hh)$, as $\Delta \to 0$, the optimal BP length(s) converge to T^* (defined in Proposition 5, which is just the limit of $T^*(\Delta) = T_1^*(\Delta)$ as $\Delta \to 0$) and the optimal BP profit $\Pi_{BP}(\Delta)$ converges to $\Pi_1^*(0, T^*)$.*

Now our second main result **Proposition 5**, presented in the overview, is just a limit version of a summary of Lemma 3 and Lemma 10. Proposition 6 is based on Proposition 5. It is proved in the Appendix.

Our first main result **Proposition 4** also follows. To see that, we know that the profit under NC converges to L (Proposition 3), but $\Pi_1^*(0, T^*) > L$ (Lemma 7). Thus if $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, when Δ is small enough, at optimal BP length $T^*(\Delta)$, we have

$$\Pi_{BP}(\Delta) > \Pi_{NC}(\Delta).$$

The profit level at the optimal BP length is still strictly less than that at the FPC equilibrium because we know that only the transaction timing of Outcome-Contingent can achieve the first-best profit for our interesting parameter case, but the optimal BP results in a different transaction timing. Therefore, if $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, when Δ is small enough, at the optimal BP length $T^*(\Delta)$, we have

$$\Pi_{BP}(\Delta) < \Pi_{FPC}(\Delta).$$

The next proposition shows that, following the optimal BP policy, the equilibrium is however not unique.

Proposition 8. *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, when Δ is small enough, given any $T > 0$ and $z = 1$, there exists an equilibrium with the outcome that all consumers buy at $t = 0$ at L.*

Proof. See the Appendix. □

Remark: Let's first look at why the equilibrium is unique in a setting without BP policy. If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, when Δ is small enough, pricing at L at $t = 0$ fails to be part of an equilibrium, because the seller can deviate to a price slightly above L and all H types will accept it because they cannot expect a lower price later and there is positive discounting, and such a deviation gives the seller a higher profit. However, such a deviation is no longer profitable when a positive length BP policy has been committed to because the deviation price is so low that after selling to H types, the seller cannot resist the temptation to sell to L types in the second period and pay refunds to H types, which makes such a deviation not profitable. This also implies that if we have a large enough punishment scale, we can eliminate this "bad" equilibrium. Therefore, if we re-define "optimal" best-price policy by considering the lowest-profit equilibrium given each policy as a measurement of the policy performance, then the optimal policy length still does not change. However, we need a larger refund scale than $z = 1$.

Next, we compare the level of efficiency across the three benchmarks for our interesting parameter case.

	Consumer welfare (W^C)	Seller welfare/profit (W^S)	Total welfare (W^T)
NC	$(H-L)h$	$Hh+(L-Hh)$	$Hh+L(1-h)$
FPC	$\frac{s}{r+s}(H'-L')h$	$Hh+\frac{s}{r+s}(L'-H'h)$	$Hh+\frac{s}{r+s}L'(1-h)$
Optimal BP	$e^{-(r+s)T^*}(H-L)h+$ $e^{-rT^*}(1-e^{-sT^*})(H'-L')h$	$Hh+e^{-(r+s)T^*}(L-Hh)+$ $e^{-rT^*}(1-e^{-sT^*})(L'-H'h)$	$Hh+e^{-(r+s)T^*}L(1-h)+$ $e^{-rT^*}(1-e^{-sT^*})L'(1-h)$

Table 1.1. Limit of the welfare as $\Delta \to 0$ if $L' > H'h$ and $\frac{s}{r+s}(L'-H'h) > L - Hh$.

BP model's comparison with the No Commitment Benchmark (NC) is clear: $W^C_{BP} < W^C_{NC}$, $W^S_{BP} > W^S_{NC}$ and $W^T_{BP} < W^T_{NC}$. The intuition is that allowing the seller some commitment ability increases the seller's profit but hurt the consumers' welfare, and the total welfare is lower because now some consumers' purchase is delayed.

BP model's comparison with the Full Commitment Benchmark (FPC) is somehow ambiguous. Allowing even more commitment ability, i.e. moving from BP to FPC, the consumers' welfare may or may not be hurt and the total welfare may or may not decrease depending on the parameters. This is because even though some consumers' purchase is delayed under FPC (compared to BP) if demand does not drop before T^*, it actually becomes earlier if demand drops before T^*.

1.6. Discussion

In this section, we discuss some of the assumptions and alternative theories for BP policies.

1.6.1. More Complex BP policy

So far we have only considered BP policies where the seller can control only two things: policy length and refund scale. How about more complex policies? In general, a policy can specify a refund that depends on the whole history and the price to be charged at each discrete moment. Will more complex policies help the seller achieve the first best profit?

Here we obtain the result that the optimal BP policy when we allow more general forms of policies can achieve the first-best if the optimal equilibrium emerges after this policy has been committed to, however, this optimal equilibrium requires the seller to choose the "right" action when she is indifferent between two actions in each period after the very first. Therefore, such an equilibrium is less convincing and harder to communicate to the consumers.

Definition 8. *A* ***general BP policy*** *given Δ is characterized by a sequence of functions $r_t : [0, \infty)^{\frac{t}{\Delta}} \times \{\Delta, 2\Delta, \ldots, t - \Delta\} \to [0, \infty)$ for all $t \in \{\Delta, 2\Delta, \ldots\}$.*

Here the interpretation of r_t is that it is the refund to be paid (at time t) to previous consumers who bought at a particular time in the past given the history of prices before time t. Notice that we still do not let the refund to depend on the event of demand drop, because the event of demand drop is non-verifiable.

To achieve the first best profit, we do not need to exactly duplicate the equilibrium outcome under full commitment. We only need to (and have to) duplicate the transaction timing and make sure H types' incentive constraints are binding and L types' individual rationality constraints are binding on the equilibrium path, but it does not affect the seller's profit if, for example, the seller will pay H types a refund when selling to L types but charges H types a higher price at the beginning that exactly gets back the present value of the "promised" future refunds.

Consider the second period, after H types bought in the first period. The first-best outcome specifies that the seller should sell to L types at price L' if the demand drops, but restrain from selling if demand keeps high. Suppose such an equilibrium outcome exists under a general BP policy. This implies that when the demand drops, for the seller to be willing to sell more, the total refunds to be paid to H types, given the history and the new price being L', must be less than or equal to $L'(1-h)$. On the equilibrium path, the expected future profit when the demand is high is zero because there will be no more transaction. However, the seller also has the option of selling at L' to all the low types when the demand is high because L types can never expect a lower price than L' or any future refunds. Selling at L' gives a seller a non-negative profit, while not selling gives the seller a zero profit, therefore for the seller not to sell, it must be that the profit of selling at L' is also zero, i.e., the total refunds equal $L'(1-h)$. In other words, when choosing to sell to more L types when the demand is low at $t = \Delta$, the seller is indifferent between selling and not selling. The same logic applies to all later periods. Therefore, we have the following necessary condition for the optimal general BP policy and its optimal equilibrium:

Proposition 9. *If an optimal general BP policy exists, then its optimal equilibrium satisfies that on the equilibrium path H types all buy at $t = 0$. At $t = \Delta, 2\Delta, \ldots$, if the demand drops, the seller sells to all L types at price L' and pay a total refund of $L'(1-h)$*

to H types; if the demand does not drop, the seller charges a price so high that no more transaction happens.

The existence of such a general BP policy is established by finding a general policy and an optimal equilibrium for it such that the first-best profit is achieved. The following proposition is obvious.

Proposition 10. *The following BP policy is an optimal BP policy:*

Regardless what price a consumer bought at, the seller has to refund to each of the previous customers who bought at $t = 0$ a refund equal to the selling price times $\frac{1-h}{h}$ if the selling price is less or equal to L. Consumers who bought at other times are not entitled to refunds.

The optimal equilibrium for this policy has the following outcome on the equilibrium path:

The seller sells to H types at $t = 0$ at following price:

$$P_0 \equiv H + \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(\frac{L'}{h} - H')$$

and then the seller sells to L types at price L' as soon as the demand drops and pay refunds $L'\frac{1-h}{h}$ to each H types; the seller charges a price higher than L and makes no transaction if the demand keeps high.

It is an equilibrium because after selling to H types, the seller is exactly indifferent between selling to L types or not at each point in time.

1.6.2. Quantity Commitment v.s. BP Commitment

For many sellers, it is nowadays common to outsource the production to a remote country and ship the product to the U.S. market. A casual look at the labels of the products, from clothing to electronics, reveals that they come mostly from China, Sri Lanka, Cambodia or Philippines. Given the time sensitivity of the products and the transportation time, quantity becomes a somewhat credible commitment for the seller. If the seller find the market has unsatisfied demand and want to meet it by more production, the factory in China will have the problem producing the product in time for the U.S. market. Will a quantity commitment perform better than a BP commitment?

A commitment to a total quantity can achieve the outcome of either only selling to H types, or selling to all almost instantly, both of which are outcomes achievable by a BP policy. Therefore, quantity commitment under-performs the BP policy.

1.6.3. Gradual drop in the willingness-to-pay

The driving force behind a finite optimal BP length is that the drop in the willingness-to-pay is proportionally higher for H types than for L types. We can allow the drop to be gradual over time, and still obtain that the optimal BP length is finite.

Let H types and L types start with willingness-to-pay at H and L respectively, with $L > Hh$. At time t, the willingness-to-pays are $e^{-s_H t}H$ and $e^{-s_L t}L$ with $s_H > s_L$, i.e., H types' willingness-to-pay drops faster than L types.

Under full commitment, the seller solves the following problem:

$$
\begin{aligned}
&Max_{P_0,t_L} \quad P_0 h + e^{-(r+s_L)t_L} L(1-h) \\
&\qquad s.t. H - P_0 \geq e^{-r+s_H} t_L H - e^{-r+s_L} t_L L \\
&\qquad L < P_0 \\
&\qquad t_L \in \{0, \Delta, 2\Delta, \ldots\}
\end{aligned}
$$

which is equivalent to:

$$
\begin{aligned}
&Max_{t_L} \quad (1 - e^{-(r+s_H)t_L}) Hh + e^{-(r+s_L)t_L} L \\
&\quad s.t. \quad t_L \in \{0, \Delta, 2\Delta, \ldots\}
\end{aligned}
$$

The solution t_L converges to $\frac{1}{s_H - s_L} \frac{Hh(r+s_H)}{L(r+s_L)} > 0$ when $\Delta \to 0$ [19]. That is, the equilibrium outcome under FPC is to sell to H types at the beginning and wait to sell to L types some time later. Since this outcome is not contingent on any realization of uncertainty, it can be achieved under a BP model if the seller commits to a BP policy with length equal to t_L and a refund scale that is big enough.

1.6.4. Alternative commitment benchmark

In this paper, our "full commitment" model allows the seller to commit to future prices that are contingent on the demand realization. A less "perfect" benchmark is to allow the seller to commit to one price sequence for the future regardless of demand realization. Here

[19] Suppose that at such a time, both types' willingness-to-pay are still above zero.

in this section, we will show that such a benchmark, which we call "non-contingent commitment", under-performs BP. Let $\Pi_C(\Delta)$ be the equilibrium profit under non-contingent commitment. It is obvious that $\Pi_C(\Delta) = \Pi_{BP}(\Delta)$ for our "easier" parameter cases, i.e., the white and dark grey regions in Figure 1.4. The following proposition shows the comparison for our "interesting" parameter case.

Proposition 11. *If $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$, then when Δ is small enough,*

$$\Pi_C(\Delta) < \Pi_{BP}(\Delta).$$

The basic intuition is that with BP, the seller can charge two different prices at one time depending on whether demand is high or low there, however, with a deterministic price path, the seller can charge only one price at one time. We outline the proof here.

Under non-contingent commitment, the seller basically chooses only two timings of transaction, τ, τ', referring to the timings of the transaction with L types if demand is high and if demand is low respectively. A skimming property argument gives us $\tau \leq \tau'$. (Due to our parameter restriction $H - L > H' - L'$, H types always buy before L types and thus at the very beginning on the equilibrium path.) The seller solves the following problem:

$$\max_{\tau,\tau'} \quad Hh + e^{-(r+s)\tau}[L - Hh] + e^{-r\tau'}(1 - e^{-s\tau})[L' - H'h] - e^{-(r+s)\tau'}[L - L']$$
$$s.t. \quad \tau \leq \tau'$$

Next we claim that for all parameter cases except for possibly one hairline case, the optimal deterministic price path will result in a profit level dominated (at least weakly) by the optimal BP policy.

Case 1. $L' < H'h$. Recall that the first-best in this case is either selling only to H types (which can be achieved by a BP policy of length infinity) or selling to all at the beginning (which can be achieved by a BP policy of zero length). Therefore the optimal BP policy dominates any optimal deterministic price path.

Case 2. $L' > H'h$. This implies that on the optimal deterministic price path, $\tau' < \infty$. First, if $\tau = 0$ then all consumers buy at the beginning, which is an outcome duplicable by a BP policy of length zero. We further analyze the subcases when $\tau > 0$ and $\tau' < \infty$. Subcase 2a. $\tau = \tau' > 0$. This means the price at $\tau = \tau'$ is L'. This option is clearly worse than what can be achieved under a BP policy, because the seller can commit to a BP policy of length $T = \tau - \Delta$, but sell at a higher price L if the demand is high at τ. Therefore, compared to committing to a deterministic price path, BP has the benefit of extra flexibility. Subcase 2b. $0 < \tau < \tau' < \infty$. This implies that both first order conditions must hold at equality, which gives us a hairline case for the parameters.

1.6.5. Alternative theory – within the period price discrimination

A BP policy reminds one of the coupons that sellers regularly hand out. The basic theory for coupons is that they are a tool for price discrimination in the following sense: H types do not have the time and energy to cut out the coupon and go through all the trouble to redeem it, while L types are willing to do it. In other words, a coupon is more costly for H types to use. Can there be a similar story for the BP policy?

The first issue is about what we actually mean by H types having a higher cost of claiming refund. If it means that H types have a higher cost of making a physical trip to the store in the future, then this cost also makes H types less willing to wait for price reduction because this is also a form of cost for shopping. Then the seller can do intertemporal price discrimination without a BP policy or anything. To make the matter interesting, let's suppose that H types have a higher chance of losing the receipt, which is necessary for claiming refunds but is not a cost of making a purchase itself.

The second issue is whether a BP policy is actually a benefit for some consumers. While a coupon provides a sure benefit, whether BP policy is a benefit or not depends on whether the seller has the subsequent incentive to cut prices and pay the refunds. Therefore, for the within-period price discrimination story to hold, refunds has to happen on the equilibrium path and some consumers must buy with the correct expectation that they will get refunds later. We can generate inventive for the seller to subsequently cut price by introducing sequential entry of consumers.

Consider the following simple example of a model of three types. There is no demand drop. Suppose H and M types exist in the market from the beginning, but a large amount of L types enter after 10 days. H types will not be able to find the purchase receipt after 9 days, but M type will always have the receipt around. In this setting, the seller benefits from adopting a BP policy of 11 days, sells to H and M types at the beginning and then refunds to M types when selling to L types after 10 days. However, any policy of length above 11 days is also optimal because the seller just need the policy to be long enough so that it is credible to the M types that she will pay refunds to them before the policy expires. Therefore, such within-period price discrimination story still fails to give a convincing answer about why the policy is not set to be of infinite length.

A testable implication to distinguish this alternative theory from the main theory of this paper is whether consumers get refunds on the equilibrium path. If no consumer or few gets refund on the equilibrium path, then the seller is not doing any within-period price discrimination. Casual information obtained from sales personnel at the stores suggests that few refund is executed.

1.6.6. Alternative theory – cost decline

Declining cost can be another reason for profitable intertemporal price discrimination (Stokey 1979). Therefore, if the seller expects the cost to decline over time, then she optimally commits to a policy of finite policy length to allow herself a price cut later but not too soon. However, this theory implies that the earlier a cost reduction will be, the shorter the policy length should be. This prediction does not seem to fit the stylized facts we observe, because the marginal cost of clothes does not seem to be declining faster than the marginal cost of electronics.

1.6.7. Alternative theory – behavioral explanation

Even though a full exploration of behavioral explanations are beyond the scope of this paper, we want to suggest two possible theories along that line:

First, consumers may over-estimate their likelihood to remember to check prices for chances to claim refunds after purchase. If they delay the purchase, because they have not fulfilled their consumption needs they will remember to re-check the item price. However once they have bought the item, their attention will be shifted to new desires and wants. Consumers may not be able to foresee that, so the BP policy can induce these consumers to purchase when the item is considered too “expensive” at the time of purchase on a false hope to get refunds later.

Second, consumers may want to avoid “regrets”. Buying something whose price is reduced subsequently within a short period time (say 14 days) causes consumers to have dis-utility from having made a “mis-judged” decision. However if the price turned out to get reduced much later, the consumer will not feel much guilt for not having waited.

1.6.8. Return Policy

Return policies can sometimes be used by consumers to achieve the same price matching effect by returning and then re-buying the product at the reduced price. In real life, we observe that sellers typically set a much longer return policy period than the BP policy period. For example, Ann Taylor, a women’s fashion retailer, has a BP policy length of 14 days, but a return policy of unlimited length. Does that mean that there is an unlimited effective BP length? This is not the case, because these sellers have a limited selling period of a certain product. When a product of last season is returned, it simply will never be sold again. Also, sellers can prevent consumers from using the return policy to match prices by requiring at least a day’s wait before it can be re-bought, by which time a product (especially clothes of a particular scale) might have been bought by other customers. In addition, a return policy has strict requirements on the condition of the product returned, while a BP policy has no requirement on the product. In fact, when claiming a refund, a consumer only needs to show a receipt, but not the original product.

1.7. Concluding Remarks

This paper has argued that a finite and positive best-price policy can be the optimal choice for a durable-goods monopoly, whose product may go out of fashion or become obsolete and who has control over the policy length and the refund scale. The seller's equilibrium profit under the optimal best-price policy is between the first-best profit and the no-commitment profit. It cannot achieve the first-best because the demand drop is uncertain and BP does not have the flexibility to be contingent on the event of demand drop which is unverifiable and non-contractible. We show that the seller can take advantage of high valuation consumers' concern about a demand drop that makes demand curve "flatter" to get a profit beyond the static monopoly profit before a demand drop. As a result, BP not only improves the seller's commitment ability, but also allows the seller to profit from intertemporal price discrimination. We also show that refunds do not happen on the equilibrium path of the BP equilibrium optimal for the seller, and that refund scales higher than one do not improve seller's profit.

To keep the analysis simple, the present model has restricted attention to two types and made the assumption that the product goes out of fashion and becomes obsolete exogenously.

In future research, we would like to generalize the model to continuous type, and allow the seller to choose the pace of obsolescence by making product choices as well.

CHAPTER 2

Long-Term Contracts Soften Competition

2.1. Introduction

When consumers repeatedly purchase a service/product, it is typical that firms compete by offering both long-term and short-term contracts and it is up to the consumers to choose which contract of which firm they want to accept. In making their decisions on whether or not to lock themselves into a long-term contract, consumers will take into account their private information on their own future taste.

In repeated purchase literature, consumers' preferences over time have been modeled mostly in one of two ways. They are either completely fixed (Fudenberg and Tirole (2000)) or completely random from one period to another (Caminal and Matutes (1989)), i.e., consumers' preferences are either completely dependent or completely independent across periods. In other words, either consumers perfectly know their preference in the future, or they have absolutely no idea. However, in this paper, we believe that consumers' preference may change in the future, and consumers have heterogenous private information on how it will change. Incorporating such heterogeneity is important because it affects consumers' decision in choosing between long-term and short-term contracts.

One exception in the literature is Chen and Pearcy (2007) which considers the case that there is an exogenous probability that consumers' preference is completely random and with the complementary probability it is fixed. However, in their model, this exogenous probability of preference change is the same for all consumers, so we think this model is still unable to provide full sights into why some consumers would choose long-term contracts and some would choose short-term contracts from the same seller.

Retailers who can offer both long-term and short-term contracts can also know, in most cases, whether a consumer has bought from them before. Therefore, the price of the short-term and the long-term contract can further depend on whether a consumer is a new one or an old one. Following the literature, we call the price offered to new consumers the "poaching price". Fudenberg and Tirole (2000) shows that the sellers' ability to price discriminate with respect to consumers' purchase history reduces their profit and the ability to offer long-term contracts further exacerbates the sellers' problem by making the competition even more fierce in the second period of a two period model in a setting where consumers' preferences are fixed from the first period to the second. The intuition is that by locking in a set of consumers with the highest valuation for one's product, a seller commits herself to more aggressive pricing in the second period of a two period model, and it is tempting for this seller to do the lock-in in the first period because this commitment induces the opponent to lower its second period poaching price, which makes it more attractive for consumers to buy from this seller in the first period. The use of the long-term contracts is a story of prisoner's dilemma in Fudenberg and Tirole (2000). Long-term contracts leave the sellers with a second period market consisting of consumers who have lower valuation for the products and thus give rise to lower second period prices than if long-term contracts are not feasible. This story hinges on a special feature of the Hotelling model. In a Hotelling model, those consumers who have more extreme preference are also those who have higher willingness-to-pay, therefore when the extreme-preference consumers are not locked in by long-term contracts, each seller will not compete too much on price as they have many valuable consumers to exploit. However, we think that there is no reason to believe that consumers with weaker preference for one seller over the other are also those with lower valuations. This unnatural (but very over-looked) assumption of the Hotelling setup causes long-term contract to lock in consumers with higher valuations and as a result hurt the sellers' profit. In this paper, we will show that the prediction about long-term contract will change if we separate the taste element and the level of willingness-to-pay by using a modified Hotelling setup.

In our simple two period model where consumers have private information about the level of uncertainty of their taste in the future, long-term contracts can lock in consumers with more stable preference and make taste more balanced in the pool of the consumers who can be poached in the future. The key difference from Fudenberg and Tirole (2000) is that here long-term contracts does not leave out to the second period those consumers of lower valuations, instead they leave out consumers of more uncertain valuation (from the point of view of the first period). This reduces the aggression of a poaching competitor in the second period, because the competitor knows that they face consumers that are very likely to have a high valuation for her product any way. The higher profit achievable when long-term contract is available also comes from higher first period prices. The sellers' incentive to reduce price to grab more consumers in the first period is also lowered because the benefit of doing that in increasing the symmetry among consumers in the second period is lowered.

The intuition that symmetry helps a seller when she is facing a poaching competitor has escaped the attention of the literature of history dependent price discrimination [1], but the idea that it is in the interest of a firm leave a competitor some advantage so that the competitor does not become too aggressive has been suggested in other IO literature,

[1] For a literature review on price discrimination by purchase history, see Stole (2003).

notably Fudenberg and Tirole (1984). Whenever information on purchase history is readily available (e.g. the simple ability to recognize new versus old consumers), history-dependent price discrimination is inevitable. First, long-term contract can be a device that increases the symmetry on the second period market to which the competitor can target with a poaching price. Second, it also reduces first period competition by reducing sellers' incentive to grab more consumers to increase the second period symmetry among one's old customers. This paper shows that sellers do use long-term contracts on the equilibrium if they are available and the equilibrium profit is higher compared to a model there long-term contracts are not an option due to the above two effects.

In the basic model, the consumers' preference/location from one period to the other is either fixed or completely random, which we call respectively the "certain" type and the "uncertain" type. Since there can be multiple equilibria, we focus on the equilibrium that gives the seller the highest profit among symmetric equilibria in pure strategy, which we will call the "best" equilibrium. In a section to check the robustness of the model, we allow all consumers to have some degree of certainty in their preference, but some have more than the other. We show that the qualitative results do not change in this slightly model general model. In addition, we get that on the "best" equilibrium when both LT and ST contracts are allowed, we see the phenomenon of "poaching" on the equilibrium, and we also see that the sum of the short-term prices of two periods is lower than the long-term price, representing a discount that the seller gives to people who are willing to lock themselves into long-term contracts.

This paper also shows that long-term contracts can also increase efficiency by reducing poaching in the second period, which is a form of mis-matching between the sellers and the consumers.

In the literature, other alternative explanations for the use of long-term contracts have been explored. Another name for long-term contract is "intertemporal bundling". Dana and Fong (2005) studied it in the setting of infinitely repeated game and showed that it facilitates tacit collusion. This paper instead looks at a finite dynamic game and the non-collusive behavior of the firms.

In behavioral economics, Loewenstein, Donoghue and Rabin (2003) argues that consumers possess "projection bias" and tend to reply too much on their current tastes to estimate their future preferences, and therefore sellers can take advantage of that by offering long-term contracts. This paper differs in assuming that consumers have rational expectation.

The paper is organized as follows: Section 2.2 presents some motivating examples, followed by Section 2.3 that lays out the model. Some useful preliminary analysis is done in Section 2.4. Then we study the benchmark where sellers are restricted to offering only short-term contracts in Section 2.5. Then Section 2.6 studies the case when sellers can offer both long-term and short-term contracts, after which we study the welfare implications in Section 2.7. Section 2.8 studies a variation of the model where all consumers have some uncertainty in their preference. We explain why results in Fudenberg and Tirole (2000) is different in Section 2.9. Finally we conclude in Section 2.10.

2.2. Motivating Examples

We show a few examples that share the features that contracts of different lengths are offered (our equilibrium outcome) and there is also price discrimination by purchase history in the sense that new consumers are offered a better deal (our basic assumption).

Magazine subscription The Economist has an "introductory offer for new US subscribers only" at \$98 per year, on the other hand the renewal price for a year is \$129. It also offers longer contracts at \$219 for two year, and \$299 for three years. [2]

Cell phone network subscription A typical introductory offer is bonus minutes/airtime for new subscribers. For example, Verizon Wireless has "BONUS \$10 of airtime when you activate an INpulse phone". [3] Cell phone providers' long-term contracts may not involve a "discount" in the price itself compared to their short-term contracts, but they typically involve incentives in other forms like free or reduced price cell phones.

Athletic Club subscription Evanston Athletic Club offers to full time students one month contract at \$65 per month, 6 month contract at \$55 per month, and 1 year contract at \$40 per month. The prices for non-students exhibit the same pattern: one month contract at \$85 per month, 6 month contract at \$80 per month and 1 year contract at \$65 per month. It also offers a one time "first visit bonus" of \$50. [4]

2.3. The Model

There are two periods. Consumers have inelastic demand of one unit of a good in each period and there are two sellers they can buy from: A and B. We also call their products respectively A and B. Total mass of consumers is one.

Consider a Hotelling line of length one forming the interval $[-\frac{1}{2}, \frac{1}{2}]$ in each period, with seller A located at the right end (i.e., at $\frac{1}{2}$) and seller B located at the left end (i.e., at $-\frac{1}{2}$). The transportation cost is $t > 0$ and the consumer located at $\theta \in [-\frac{1}{2}, \frac{1}{2}]$ has willingness-to-pay $1 + \theta t$ for product A and willingness-to-pay $1 - \theta t$ for product B. We assume $0 < t < 1$,

[2] http://www.economist.com, as of Apr 03 2007.
[3] http://www.verizonwireless.com, as of Mar 18 2007.
[4] Price information is obtained through sales personnel, as of Apr 03 2007.

which ensures that in a one period Hotelling model, if a consumer is located at the middle, then he buys on the equilibrium.

In the first period, before consumption, the consumers do not know exactly where they are located on the line, but they know which seller they prefer. There are only two cases: either their distribution is uniform over $[0, \frac{1}{2}]$ (i.e. they prefer A), or their distribution is uniform over $[-\frac{1}{2}, 0]$ (i.e. they prefer B) [5]. For each case, the consumers can be further distinguished by their information on their future preference. Some consumers will have "uncertain" future taste, meaning that their second period location will be uniformly distributed over the whole Hotelling line. These consumers are of mass $e \in (0, \frac{1}{2})$ for each case. The rest of the consumers will have a stable future taste, meaning that their second period location will be the same as their first period location, but since their first period location is not exactly known before consumption, their future location is a half-interval distribution from the viewpoint of the first period, the same as the one in their first period. In other words, consumers in the first period have two dimensions of private information: his first period distribution and his second period distribution. Here, since both dimensions are binary, we have four types and we label them BC, BU, AU, AC, where A or B denotes which seller the consumer prefers in the first period and U stands for "uncertain" future taste and C stands for "certain" future taste. Let τ denote the type in the first period, i.e. $\tau \in \{AC, AU, BC, BU\}$. Their masses are respectively $\{\frac{1}{2} - e, e, \frac{1}{2} - e, e\}$.

The timing of the game when only short-term contracts are allowed is as follows:

[5] Restricting their information in the first period regarding their first period period helps ensuring the existence of second period subgame equilibrium in pure strategy. We will discuss this assumption more in Section 2.9.

1. The sellers set their first period prices P_i^S simultaneously with $i = A, B$. S superscript denotes short-term contract price. Consumers decide which contract to buy. Consumers consume and then they find out their first period location.

2. In the second period, consumers' exact location on the Hotelling line for their second period are revealed to them privately. After observing all the history, each seller sets two prices simultaneously, one for the set of consumers who bought from her before, and one for the set of consumers who didn't buy from her before. We denote these prices P_i^1 (for previous/old customers) and P_i^2 (for new customers) where $i = A, B$. Consumers decide on which contract to buy. Consumers consume.

The timing of the game when both long-term and short-term contracts are allowed is as follows:

1. The sellers set first period prices Q_i^S and Q_i^L simultaneously with $i = A, B$ for short-term and long-term contracts. S superscript denotes short-term contract price and L denotes long term contract price. Consumers decide which contract to buy. Consumers consume and then they find out their first period location.

2. In the second period, consumers' exact location on the Hotelling line for their second period are revealed to them. After observing all the history, each seller sets two prices simultaneously, one for the set of consumers who bought from her before, and one for the set of consumers who didn't buy from her before. We denote these prices Q_i^1 (for previous/old customers) and Q_i^2 (for new customers) where $i = A, B$. Consumers who have bought short-term contracts in the first period decide on which contract to buy. Consumers consume.

We study long-term contract as if it is just a bundling of products/ services over time.

We study symmetric subgame perfect equilibrium in pure strategies (We can think of the sellers as knowing all the information, but not being able to depend their prices on individual consumers' valuation.) Players all maximize their expected future utility. Consumers' utility is valuation minus price paid and the sellers' utility is the profit. We assume marginal cost of production is 0. There is no discounting. By symmetry, we mean that sellers' strategies are the same.

2.4. Preliminaries

We develop a useful notation: let $U_\tau(P_A, P_B)$ be the second period utility of a consumer with first period type τ, if he belongs to a turf to which A offers price P_A and B offers price P_B in the second period and if he rationally chooses to accept one of these two prices. Therefore, for P_A, P_B such that $P_A - P_B \in [-t, t]$, we have:

$$U_{AC}(P_A, P_B) = \int_{\frac{P_A - P_B}{2t}}^{\frac{1}{2}} (1 + xt - P_A)2\,dx + \int_0^{\max\{\frac{P_A - P_B}{2t},0\}} (1 - xt - P_B)2\,dx$$

$$U_{BC}(P_A, P_B) = \int_{\min\{\frac{P_A - P_B}{2t},0\}}^{0} (1 + xt - P_A)2\,dx + \int_{-\frac{1}{2}}^{\frac{P_A - P_B}{2t}} (1 - xt - P_B)2\,dx$$

$$U_{AU}(P_A, P_B) = U_{BU}(P_A, P_B) = \int_{\frac{P_A - P_B}{2t}}^{\frac{1}{2}} (1 + xt - P_A)\,dx + \int_{-\frac{1}{2}}^{\frac{P_A - P_B}{2t}} (1 - xt - P_B)\,dx$$

Lemma 11. *Suppose in the beginning of second period, turf i (for $i = A, B$) has mass $\frac{1}{2}$ of consumers, m mass of the whom has uncertain taste (i.e., with location uniformly* *distributed on the whole Hotelling line* $[-\frac{1}{2}, \frac{1}{2}]$*) and* $\frac{1}{2} - m$ *mass of whom has certain taste biased to seller* i *(i.e., with location uniformly distributed on the section* $[0, \frac{1}{2}]$ *if* $i = A$ *and on the section* $[-\frac{1}{2}, 0]$ *if* $i = B$*), with* $e \in [0, \frac{1}{2}]$*, then the subgame equilibrium of pricing (after consumers learned their second period exact location) on that turf has prices* $P_i^1 = \frac{2-m}{3(1-m)}t$ *and* $P_{-i}^2 = \frac{1+m}{3(1-m)}t$*.*

Proof. WLOG, we will take $i = A$. The analysis is standard Hotelling analysis, so the reaction curves of the two sellers are:

$$P_A^1 = \frac{P_B^2}{2} + \frac{t}{2} \quad P_B^2 = \frac{P_A^1}{2} + \frac{m}{1-m}\frac{t}{2}$$

$$\Rightarrow P_A^1 = \frac{2-m}{3(1-m)}t \quad P_B^2 = \frac{1+m}{3(1-m)}t$$

□

Remark: 1) Notice that $\frac{m}{1-m} \leq 1$, so from their reaction curves, we can see that seller B is more aggressive on turf A than seller A. This is because preference is more biased to A on A's turf, so we can say that the price set by B is a "poaching price" because her price is lower than A's: $\frac{2-m}{3(1-m)}t > \frac{1+m}{3(1-m)}t$ for any $m < \frac{1}{2}$. 2) Also notice that both prices increase in m. m is in some sense a measure of how symmetric a turf is. When $m = 0$, the turf A is the most asymmetric in the sense that the distribution of preferences is the most tilted to A. When $m = \frac{1}{2}$, the turf A is the most symmetric in the sense that the distribution of preference is exactly uniform over the whole Hotelling line, so the prices also reach their maximum at $P_A^1 = P_B^2 = t$ when $m = \frac{1}{2}$.

2.5. Benchmark – ST Contracts Only

Lemma 12. *On any symmetric equilibrium in pure strategies, it must be that AU and AC consumers buy from A and BU and BC consumers buy from B in the first period.*

Proof. See the Appendix. $\square$

Proposition 12. *When the seller can only offer ST contracts, if $t \leq \frac{4}{5}$,* [6] *then the symmetric equilibrium in pure strategies that gives the seller the highest profit has the following features: in the first period, the sellers charge $P_A^S = P_B^S = \frac{2e^2-2e+5}{9(1-e)}t$ and all consumers buy from the seller they prefer; then in the second period the sellers charge $P_A^1 = P_B^1 = \frac{2-e}{3(1-e)}t$ to old customers and $P_A^2 = P_B^2 = \frac{1+e}{3(1-e)}t$ to switchers. The equilibrium profit of the sellers is* [7]*:*

$$\Pi_b = \frac{2e^2 - 2e + 5}{9(1-e)}t$$

Remark: Profit increases with e and reaches the highest level at $e = \frac{1}{2}$. This is intuitive. Higher e means that the second period turfs are more symmetric and thus the second period prices are higher. More symmetric turfs in the second period also reduces incentive to reduce price in the first period, thus allows a higher price to be sustained at first period equilibrium prices.

Proof. If an equilibrium is symmetric where all consumers buy, then Lemma 11 and Lemma 12 imply that the equilibrium prices in the second period are $P_A^1 = P_B^1 = \frac{2-e}{3(1-e)}t$ and $P_A^2 = P_B^2 = \frac{1+e}{3(1-e)}t$. Now we can characterize a symmetric equilibrium by a single price

[6]When this condition fails, there will be no equilibrium in pure strategies. The reason is similar to the reason why if two masses of consumers are located at two points on a Hotelling line, a Bertrand competition equilibrium fails to exist when the transportation cost is too high.

[7]"b" stands for benchmark.

$P^S = P_A^S = P_B^S$. We need to find the necessary and sufficient condition on this price to make it an equilibrium.

Let's first consider the seller's first period incentive.

For any symmetric pair of price $P_A^S = P_B^S$, a necessary condition is that the seller does not have incentive to raise the price by just a little. There are two possibilities, either $P_A^S = P_B^S$ extracts all of the consumers' first period surplus, which is $1 + \frac{t}{4}$ so that any increase in price will cause consumers not to buy, or it is lower. Suppose $P_A^S = P_B^S < 1 + \frac{t}{4}$ and consider the deviation that A raises the price by ϵ, there is a subgame equilibrium after the deviation where all AU and AC consumers will go to buy from B in the first period, believing that if they buy from A they will be charged a very huge price in the second period that makes them not wanting to buy in the second period. This is sequentially rational for the seller to charge huge price because each consumer is infinitesimally small. By buying from B in the first period, these consumer get the chance to buy in the second period, where they will be charged t by both sellers because B's turf will be symmetric, which gives both AU and AC consumers a second period surplus of $1 - \frac{3}{4}t$. Therefore, for such decision to be rational for AU and AC, it must be that:

$$\frac{t}{2} \leq 1 - \frac{3}{4}t \Leftrightarrow t \leq \frac{4}{5}$$

Therefore, if on the equilibrium $P_A^S = P_B^S < 1 + \frac{t}{4}$, then we must have $t \leq \frac{4}{5}$.

If $P_A^S = P_B^S = 1 + \frac{t}{4}$, then seller A has no incentive to raise the price unilaterally, because if she does, there is a subgame equilibrium such that all consumers do not buy from anyone in the first period and only buy in the second period where both sellers charge t. Consumers will do that because if any one consumer deviates to buy from any seller in the first period (given no other consumer buys), then she will get charged t as well, but currently prices in the first period are higher than or equal to his valuations. Therefore, any deviation to raise price unilaterally will give seller A a profit $\frac{1}{2}t$.

Now we consider deviations where sellers reduce prices. Suppose seller A in the first period deviates by decreasing price to $[P_A^S - \frac{t}{2} + \frac{8e^2-14e+5}{18(1-e)^2}t, P_A^S]$, then there is an equilibrium where the subgame does not change from the proposed equilibrium, because here BU and BC consumers still do not have incentive to deviate to buying from seller A in the first period. (Notice that buying from A in the first period gives a BC consumer a second period benefit of $\frac{8e^2-14e+5}{18(1-e)^2}t$, but costs him a first period loss of $\frac{t}{2}$. When price is dropped to the interval $[P_A^S - \frac{t}{2}, P_A^S - \frac{t}{2} + \frac{8e^2-14e+5}{18(1-e)^2}t]$, there exists a subgame equilibrium where a fraction of BC consumers buy from seller A and the profit is maximized when price offered by seller A is as low as $P_A^S - \frac{t}{2}$, at which there is a subgame equilibrium where all BC consumers buy from seller A while BU consumers still buy from B. Seller A's deviation profit is:

$$\Pi'_A = (P_A^S - \frac{t}{2})(1-e) + \frac{t}{2}$$

If the seller deviates to a price just sightly below $P_A^S - \frac{t}{2}$, then the unique subgame equilibrium is that all consumers buy from seller A in the first period, and there is competition in the second period where both sellers' prices are t. (There does not exist a subgame equilibrium after the deviation where only BU consumers switch to seller A. This is because if that happens, both sellers' prices on turf A will be higher than those on turf B by the same amount, which means that BU should prefer buying from B, which is a contradiction.) This deviation gives her a profit that can be arbitrarily close to:

$$\Pi''_A = (P_A^S - \frac{t}{2})1 + \frac{t}{2}$$

Therefore, this is the best deviation for seller A among all possible deviations of lowering price.

The profit on the proposed equilibrium as a function of P_A^S is:

$$\frac{1}{2}P_A^S + \frac{2e^2 - 2e + 5}{18(1-e)}t$$

Therefore, a necessary condition for A not wanting to cut first period price is that:

$$(P_A^S - \frac{t}{2}) + \frac{t}{2} \leq \frac{1}{2}P_A^S + \frac{2e^2 - 2e + 5}{18(1-e)}t \Leftrightarrow P_A^S \leq \frac{2e^2 - 2e + 5}{9(1-e)}t \quad (*)$$

Since $\frac{2e^2-2e+5}{9(1-e)}t < 1 + \frac{t}{4}$, the above condition satisfies the necessary condition for A not to raise her price. Therefore, we have the following necessary condition:

$$P_A^S \leq \frac{2e^2 - 2e + 5}{9(1-e)}t$$

Notice that this first period price is less than t, so the rationality constraints of the consumers are met.

Considering only the second period, consumers with uncertain preference do not care which turf he belongs to because of the symmetry of the prices between turf A and turf B, but consumers with certain preference will prefer to buy from the seller they dislike in the first period, because then they have a lot to gain from the lower price offered to switcher from his preferred seller. To see this, the utility of AC in the second period on the proposed equilibrium is:

$$U_{AC}(\frac{2-e}{3(1-e)}t, \frac{1+e}{3(1-e)}t) = 1 + \frac{5e^2+10e-13}{36(1-e)^2}t$$

If an AC consumer deviates to buy from seller B in the first period, his second period utility will be:

$$U'_{AC}(\frac{1+e}{3(1-e)}t, \frac{2-e}{3(1-e)}t) = 1 - \frac{1+7e}{12(1-e)}t$$

The difference is:

$$U'_{AC}(\frac{1+e}{3(1-e)}t, \frac{2-e}{3(1-e)}t) - U_{AC}(\frac{2-e}{3(1-e)}t, \frac{1+e}{3(1-e)}t) = \frac{8e^2-14e+5}{18(1-e)^2}t > 0$$

However, by deviating to buying from B, AC consumers loses a first period payoff of

$$\int_0^{\frac{1}{2}} (1+xt-P_A^S)2\,dx - \int_0^{\frac{1}{2}} (1-xt-P_B^S)2\,dx = \frac{t}{2}$$

Since $\frac{t}{2} > \frac{8e^2-14e+5}{18(1-e)^2}t$ for any $e \in (0, \frac{1}{2})$. Therefore, this is not a profitable deviation for AC consumers.

Sufficiency follows directly from the process of reaching the necessary conditions.

□

2.6. LT and ST Contracts Both Allowed

Lemma 13. *When LT and ST contracts are both allowed, on any symmetric equilibrium in pure strategies, it must be that in the first period AU and BU consumers buy A and B's ST contracts respectively, and AC and BC consumers buy A and B's LT contracts respectively.*

Remark: The basic intuition is that if only ST contracts are being accepted, the seller has the incentive to lower LT price to lock in consumers with certain preference so that the second period turf is more symmetric.

Proof. See the Appendix. □

Proposition 13. *When the sellers can offer both ST and LT contracts, if $t \leq \frac{4}{5}$, then the equilibrium that gives the sellers highest profit among symmetric equilibria in pure strategies has the following features: in the first period, the sellers charge $Q_A^S = Q_B^S = t$ and $Q_A^L = Q_B^L = 2t$, AC consumers buy A's LT contract, AU consumers buy A's ST contract and BC consumers buy from B's LT contract and BU consumers buy from B's ST contract; and then in the second period the sellers charge $Q_A^1 = Q_A^2 = Q_B^1 = Q_B^2 = t$ to old customers and switchers alike. The equilibrium profit of the sellers are:*

$$\Pi = t$$

Proof. If an equilibrium is symmetric where all consumers buy, then Lemma 11 and Lemma 13 imply that the equilibrium prices in the second period are $Q_A^1 = Q_B^1 = Q_A^2 = Q_B^2 = t$. Now we can characterize a symmetric equilibrium by a two prices (Q_A^S, Q_A^L). We need to find the necessary and sufficient condition on these prices to make it an equilibrium.

Now we study seller A's incentives. We will consider all kinds of possible price deviations, as presented in Figure 2.1, where the center is the proposed equilibrium price pair, and derive necessary conditions for A not wanting to deviate. Since A is maximizing profit when setting the LT contract price, a symmetric equilibrium implies that $Q_A^L = Q_A^S + t$, i.e. A makes AC consumers indifferent between ST and LT contracts.

The seller will not deviate to $Q_A'^S > Q_A^S$ and $Q_A'^L > Q_A^L$ when $t \leq \frac{4}{5}$. The reasoning is the same as in the Benchmark proposition. After such a deviation, there exists a subgame where all consumers buy from B's short-term contract. This covers deviations to prices in the area I in Figure 2.1.

If seller A deviates to $Q_A'^S \in [0, Q_A^S - \frac{t}{2})$ and $Q_A'^L > Q_A'^S + t$, then the unique subgame equilibrium is that all consumers buy from seller A's ST contract. Then such a deviation can at most get A a profit of $(Q_A^S - \frac{t}{2}) + \frac{t}{2}$. The proposed equilibrium profit is $\frac{1}{2}Q_A^S + \frac{t}{2}$. For the deviation not to be strictly profitable, we must have:

$$(**) \qquad (Q_A^S - \frac{t}{2}) + \frac{t}{2} \leq \frac{1}{2}Q_A^S + \frac{t}{2} \Leftrightarrow Q_A^S \leq t$$

If seller deviates to $Q_A'^S \in (Q_A^S - \frac{t}{2}, Q_A^S)$ and $Q_A'^L > Q_A'^S + t$, then the unique subgame equilibrium is the same as in the proposed equilibrium, which is thus not a profitable deviation. It covers deviations to prices in the area II in Figure 2.1. And the conclusion from this region is that a necessary condition is that $Q_A^S \leq t$.

If seller A deviates to $Q_A'^S \in [Q_A^S - \frac{t}{2}, Q_A^S)$ and $Q_A'^L \in [Q_A'^S + t - \frac{t}{4}, Q_A'^S + t]$, then there is a subgame equilibrium that is the same as that in the proposed equilibrium. The seller gets a lower profit by such a deviation because $Q_A'^S < Q_A^S$ and $Q_A'^L \leq Q_A^S + t = Q_A^L$. If seller

A deviates to $Q'^S_A \in [0, Q^S_A - \frac{t}{2})$ and $Q'^L_A \in [Q'^S_A + t - \frac{t}{4}, Q'^S_A + t]$, then there is a unique subgame equilibrium where AC buys A's LT/ST contract and the rest of the consumers buy A's ST contract, which is weakly worse than a deviation where $Q'^L_A > Q'^S_A + t$, already studied in area II. If seller A deviates to $Q'^S_A \in [0, Q^S_A)$ and $Q'^L_A \in [0, Q'^S_A + t - \frac{t}{4})$, then there exists a subgame equilibrium where BU and BC still buy from B's ST and LT contracts, and both AU and AC consumers buy A's LT contract, which is not a profitable deviation for A. BU and BC are not attracted to A's ST contract despite its low price is because they believe that they will be charged price that leaves them zero surplus in the second period. This means that they will lose $\frac{t}{2}$ current surplus, $1 - \frac{3}{4}t$ future surplus if they buy A's ST contract. Since $\frac{t}{2} + 1 - \frac{3}{4}t > t$, it is not good for them to buy A's ST contract even if it is free. These two cases cover deviations to prices in the area III in Figure 2.1.

If seller A deviates to $Q'^S_A \geq Q^S_A$ and $Q'^L_A \in [Q^L_A - \frac{3}{4}t, Q^L_A)$, then there is a subgame equilibrium such that BU and BC keep on buying from B, while AU and AC both buy from seller A's LT contract. Such an outcome gives seller A less profit because $Q'^L_A < Q^L_A = Q^S_A + t$. If seller A deviates to $Q'^S_A \geq Q^S_A$ and $Q'^L_A \in [Q^L_A - \frac{3}{4}t - \frac{40e^2-58e+19}{36(1-e)^2}, Q^L_A - \frac{3}{4}t)$, then there is a subgame equilibrium where AU, AC consumers all buy A's LT contract, and BU and BC consumers buy B's ST contract, which is not a profitable deviation for A. BC and BU now wants to buy B's ST contract because future prices will be lower there on B's turf. If If seller A deviates to $Q'^S_A \geq Q^S_A$ and $Q'^L_A \in [Q^L_A - \frac{50}{36}t, Q^L_A - \frac{3}{4}t - \frac{40e^2-58e+19}{36(1-e)^2})$, then there is a subgame equilibrium where BU, AU, AC all buy A's LT contract and BC buys B's ST contract. By buying B's ST contract BC consumers get $1 + \frac{t}{4} - Q^S_B + 1 - \frac{13}{36}t$ and by buying A's LT contract they get $2(1 - \frac{t}{4}) - Q'^L_A$, so $Q'^L_A \geq Q^L_A - \frac{50}{36}t$ implies that it is rational for BC consumers to choose B's ST contract. If $Q'^S_A \geq Q^S_A$ and $Q'^L_A < Q^L_A - \frac{50}{36}t$, then there is a unique subgame equilibrium where all consumers buy from A's LT contract, which gives A a deviation profit of $Q_A^L - \frac{50}{36}t$. Therefore, a necessary condition for the equilibrium is:

$$Q_A^L - \frac{50}{36}t \leq \frac{1}{2}Q_A^L \Leftrightarrow Q_A^L \leq \frac{25}{9}t \Rightarrow Q_A^S \leq \frac{16}{9}t$$

It covers deviations to prices in the area IV in Figure 2.1.

Since $Q_A^S \leq \frac{16}{9}t$ is implied by $Q_A^S \leq t$ from deviations in area II, we take the latter condition. Therefore, we have exhausted all possible deviations for the sellers and because we always used the most unprofitable subgame after each possible deviation, the condition identifies the biggest set of Q_A^S possible in an symmetric equilibrium.

AC and BC consumers have no incentive to deviate to buying from ST contracts instead because the prices are such that they are exactly indifferent between the LT and ST contract offered by the seller they prefer. AU and BU consumers have no incentive to deviate to buying LT contract because by doing that they will be charged a second period price just as high, but with one half probability they will be forced to buy from a seller they do not prefer. The rationality constraints for AU and AC to be willing to buy contracts in the first period is the following:

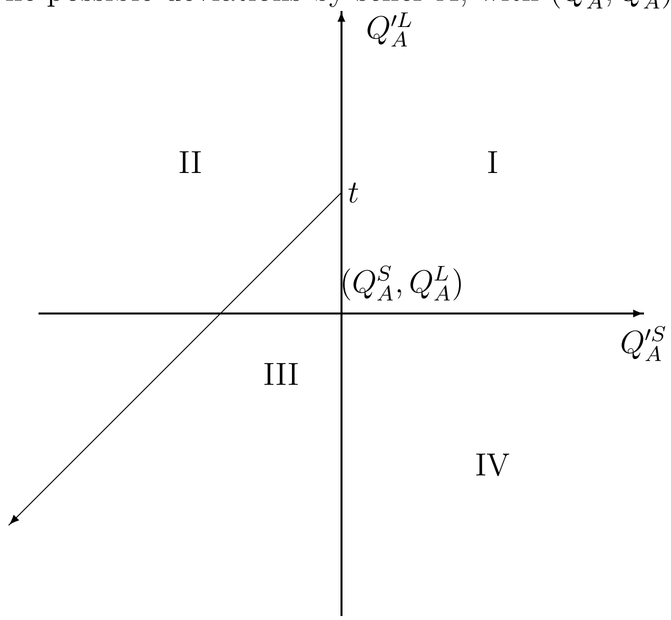
$$(1 + \frac{t}{4}) - Q_A^S + 1 - \frac{3}{4}t \geq 0 \Leftrightarrow Q_A^S \leq 2 - \frac{t}{2}$$

$$2(1 + \frac{t}{4}) - Q_A^L \geq 0 \Leftrightarrow Q_A^L \leq 2 + \frac{t}{2}$$

Therefore, $Q_A^S \leq t$ meets AU's rationality constraint. Also, $Q_A^L \leq 2t$ implies that it meets AC's rationality constraint.

Sufficiency of the equilibrium follows.

Figure 2.1. The possible deviations by seller A, with (Q_A^S, Q_A^L) as the origin.



$\square$

The following corollary is immediate from Proposition 12 and Proposition 13.

Corollary 3. *On the "best" symmetric equilibrium, 1) the first period short term contract price, 2) the second period prices by both sellers, and 3) equilibrium profit are all higher when both LT and ST contracts are allowed than when only ST contracts are allowed.*

Remark: The reason why the second prices are higher with LT contract, compared to the benchmark is clear. This is because the LT contracts locked in consumers with stable preferences and as a result both turfs are symmetric in the second period. The reason why the first period prices are higher is more subtle. In both models, the first period prices have to be low enough such that there is not too big an incentive to cut short-term price and win over all the competitor's consumers in the first period. This is the binding constraint for both models. We re-write the two constraints there:

$$(*) \qquad (P_A^S - \frac{t}{2}) + \frac{t}{2} \leq \frac{1}{2}P_A^S + \frac{2e^2 - 2e + 5}{18(1-e)}t$$

$$(**) \qquad (Q_A^S - \frac{t}{2}) + \frac{t}{2} \leq \frac{1}{2}Q_A^S + \frac{t}{2}$$

The stronger the incentive, the lower the prices an equilibrium can sustain. In the benchmark, the incentive to cut price is stronger because if a seller wins over all the consumers, she will face a more symmetric turf in the future, and thus lower price competition in the future, so winning consumers in the first period can also benefit the second period. This effect disappears when LT contracts are allowed, because future turf is already symmetric on the equilibrium.

Proof. For any $e \in (0, \frac{1}{2})$, we have the following.

First period short-term prices:

$$\frac{2e^2 - 2e + 5}{9(1-e)}t < t$$

Second period prices:

$$\frac{2-e}{3(1-e)}t < t \quad \frac{1+e}{3(1-e)}t < t$$

Profit:

$$\Pi_b = \frac{2e^2 - 2e + 5}{9(1-e)}t < t = \Pi$$

When $e = \frac{1}{2}$, i.e. when all consumers' preference is uncertain, all the above inequalities become equality. □

2.7. Efficiency

The transfer between the sellers and the consumers do not affect efficiency, so if an outcome matches each consumer to his preferred seller in each period, then it is the most efficient outcome. Efficiency loss comes only when consumers are mis-matched.

In the equilibrium when only ST contracts are allowed. There is efficiency loss in the second period: AU and AC consumers whose second period location turns out to be in the interval $[0, \frac{P_A^1 - P_B^2}{2t}]$ are mis-matched to seller B because seller B's price is lower than that of A; BU and BC consumers whose second period location turns out to be in the interval $[\frac{P_A^2 - P_B^1}{2t}, 0]$ are mis-matched to seller A because seller A's price is lower than that of B. The total efficiency loss is thus:

$$2[\int_0^{\frac{1-2e}{6(1-e)}} (2xte + 2xt(1-2e))\, dx] = \frac{(1-2e)^2}{18(1-e)}t$$

Notice that asymmetry causes poaching price, which in turn causes mis-matching. Therefore, when $e = \frac{1}{2}$, i.e. where there is no asymmetry in the preferences, the efficiency loss is zero.

In the equilibrium when both ST and LT contracts are allowed, the second period prices offer to AU and BU consumers are exactly symmetric, and AC and BC consumers also buy from the seller they prefer, so there is no efficiency loss.

However, the profit increase is higher than the efficiency increase. For any $e \in (0, \frac{1}{2})$, we have:

$$t - \frac{2e^2 - 2e + 5}{9(1-e)}t > \frac{(1-2e)^2}{18(1-e)}t$$

Therefore, in fact consumer welfare decreases when LT contracts are allowed. Even though they are matched with the right seller, the increase in the prices they pay more than offsets that effect.

Proposition 14. *The "best" equilibrium is more efficient when both LT and ST contracts are allowed than when only ST contracts are allowed. However, consumer welfare is lower when both LT and ST contracts are allowed.*

2.8. A Variation of the Model

In this section we check the robustness of the result by allowing the "uncertain" preference consumers to have some level of certainty in their preference. We will see that the quantitative results of profit comparison and price comparison will not change from this modification of the model. In addition, we get that on the "best" equilibrium when both LT and ST contracts are allowed, we see the phenomenon of "poaching" on the equilibrium, and we also see that the sum of the short-term prices of two periods is lower than the long-term price, representing a discount that the seller gives to people who are willing to lock themselves into long-term contracts.

Now suppose the uncertain consumers have $1-a$ probability to have their preference fixed from period one to period two, with $a \in (0,1)$. ($a = 1$ is thus the special case we studied before.) The rest of the model remains the same.

2.8.1. Benchmark

In the benchmark model, the proof goes through as before, except that we need to replace e by ea. Therefore, the "best" symmetric equilibrium is the following: in the first period, the sellers charge $P_A^S = P_B^S = \frac{2ae^2-2ae+5}{9(1-ae)}t$ and all consumers buy from the seller they prefer;

then in the second period the sellers charge $P_A^1 = P_B^1 = \frac{2-ae}{3(1-ae)}t$ to old customers and $P_A^2 = P_B^2 = \frac{1+ae}{3(1-ae)}t$ to switchers. The equilibrium profit of the sellers is:

$$\Pi_b = \frac{2ae^2 - 2ae + 5}{9(1-ae)}t$$

The important constraint (*) has become the following:

$$(P_A^S - \frac{t}{2}) + \frac{t}{2} \leq \frac{1}{2}P_A^S + \frac{2ae^2 - 2ae + 5}{18(1-ae)}t \quad \text{(*')}$$

2.8.2. Main model

In the model when both LT and ST contracts are allowed. In a symmetric equilibrium where "certain" preference consumers buy the LT contracts and "uncertain" preference consumers buy the ST contracts, the second period turfs are no longer completely symmetric because the "uncertain" consumers' preference is a uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$. Instead, it is a distribution with density a over $[-\frac{1}{2}, 0]$ and density $2-a$ over $[0, \frac{1}{2}]$. Therefore, standard Hotelling analysis implies the following second period prices: prices $Q_i^1 = \frac{4-a}{3(2-a)}t$ are offered to old customers and prices $Q_i^2 = \frac{2+a}{3(2-a)}t$ are offered to new customers for $i = A, B$. This means that poaching exists in the second period: the price for new customers (i.e., the competitor's old customer) is lower than that for one's own old customers. The second period profit per seller is therefore:

$$[\frac{4-a}{3(2-a)}t\frac{4-a}{6}]e + [\frac{2+a}{3(2-a)}t\frac{2+a}{6}]e = \frac{a^2 - 2a + 10}{9(2-a)}te$$

On the other hand, to make certain preference consumers to be willing to be locked into the long-term contract, the seller needs to set the long-term price long enough relative to short-term price:

$$2(1+\frac{t}{4}) - Q_A^L = (1+\frac{t}{4}) - Q_A^S + U_{AC}(\frac{4-a}{3(2-a)}t, \frac{2+a}{3(2-a)}t)$$
$$\Rightarrow Q_A^L = Q_A^S + \frac{a^2-14a+22}{9(2-a)^2}t$$

Notice that $\frac{a^2-14a+22}{9(2-a)^2}t < \frac{4-a}{3(2-a)}t$ for any $a \in [0,1)$, so the sum of the two prices seller A charges a consumer who keeps on buying the short-term contracts of A for two periods is more than the long term contract price A charges. The difference represents the discount A yields to consumers who sacrifices the possible opportunities to take advantage of poaching price later:

$$Q_A^S + \frac{4-a}{3(2-a)}t - (Q_A^S + \frac{a^2-14a+22}{9(2-a)^2}t) = \frac{2(1-a)^2}{9(2-a)^2}$$

The important constraint (**) will become:

$$(Q_A^S - \frac{t}{2}) + \frac{t}{2} \leq Q_A^S e + (Q_A^S + \frac{a^2-14a+22}{9(2-a)^2}t)(\frac{1}{2}-e) + \frac{a^2-2a+10}{9(2-a)}te$$

(**')
$$\Rightarrow Q_A^S \leq 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e) + \frac{a^2-2a+10}{9(2-a)}te]$$

Therefore the highest prices sustainable on a symmetric equilibrium are:

$$Q_A^S = 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e)+\frac{a^2-2a+10}{9(2-a)}te]$$
$$Q_A^L = 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e)+\frac{a^2-2a+10}{9(2-a)}te]+\frac{a^2-14a+22}{9(2-a)^2}t$$

Notice that both Q_A^S and $Q_A^L - Q_A^S$ are reduced when $a \in (0,1)$ compared to the case when $a = 1$ at the "best" symmetric equilibrium for the sellers. The profit now is:

$$\Pi = 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e)+\frac{a^2-2a+10}{9(2-a)}te]$$

2.8.3. Comparison

The qualitative comparison between the main model and the benchmark remain the same for their "best" symmetric equilibrium. For $a \in (0,1)$, $e \in (0,\frac{1}{2})$ and $i = A, B$, we have the following.

First period short-term prices are higher when LT is allowed:

$$P_i^S = \frac{2ae^2-2ae+5}{9(1-ae)}t < 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e)+\frac{a^2-2a+10}{9(2-a)}te] = Q_i^S$$

Second period prices are higher when LT is allowed:

$$P_i^1 = \frac{2-ae}{3(1-ae)}t < \frac{4-a}{3(2-a)}t = Q_i^1 \quad P_i^2 = \frac{1+ae}{3(1-ae)}t < \frac{2+a}{3(2-a)}t = Q_i^2$$

Profit is higher then LT is allowed:

$$\Pi_b = \frac{2ae^2-2ae+5}{9(1-ae)}t < 2[\frac{a^2-14a+22}{9(2-a)^2}t(\frac{1}{2}-e)+\frac{a^2-2a+10}{9(2-a)}te] = \Pi$$

2.9. Compare with Fudenberg and Tirole (2000)

Fudenberg and Tirole (2000) gave an opposite prediction in terms of how prices and profit will change when LT contracts are allowed. They predict that first period price, second period prices and the profit are all lower when LT contracts are allowed. The reason is that among a segment of consumers that are indifferent between LT and ST in the first period, they make the assumption that those to closer to the end of the Hotelling line choose LT contract and those closer to the middle of the Hotelling line choose St contract. Such "tie breaking" assumption ensures that subgame equilibrium in pure strategy exists in the second period. If they make other assumptions regarding how these consumers choose between LT and ST contracts when indifferent, there may not exists a subgame equilibrium in pure strategy. Even though the assumption was made for the existence of an equilibrium, it means that LT contracts lock in those consumers at the end of the Hotelling line, i.e., those who have higher valuation for a seller's product. This makes the competition more severe in the second period within each turf, and as a result reduces prices and profit. Therefore, we think that it is unsatisfactory that the prediction about LT contract came from an assumption that has no strong economic justification.

However, we recognize that it is a modeling difficulty to ensure the existence of second period subgame equilibrium in pure strategies. The way we got around the problem in this model is to assume that the first period preference of each consumer is an distribution, rather than a single point on the Hotelling line.

We also want to point out here that, unlike in the variation of our model, the Fudenberg and Tirole (2000) model is unable to capture the phenomenon that long-term contract presents a “saving” for the consumers on the equilibrium. This is because their model is deterministic, so two same flow of consumption must cost the same to the consumers.

2.10. Concluding Remarks

This paper shows that in a simple setup of a two period model, where in the first period consumers not only differ in taste, but also in information about their future taste, and where sellers can offer purchase-history-dependent prices, sellers will choose to sell long-term contracts together with short-term contracts in the first period. The profit for the sellers in the unique symmetric equilibrium when long-term contracts are allowed is higher than our benchmark where long-term contracts are banned. The profit increase is due to the fact that consumers with more certain taste are locked in by the long-term contracts, which makes the second period competition less intense. The increased profit also comes from reduced incentive to grab more consumers with the short-term contract in the first period, the benefit of doing that in increasing the symmetry of the second period is reduced. The efficiency also increases relative to the benchmark where long-term contracts are banned, because less intense competition in the second period means that consumers are more likely to be matched with the seller whose product they prefer.

CHAPTER 3

Price Discrimination under Taste Change and Switching Cost

3.1. Introduction

With the advance of technology in gathering data and organizing date about ones' customers, nowadays sellers can offer different prices to consumers' with different purchase history at a very low administrative cost. Therefore an interesting question to ask is how the ability to use consumers' purchase history to price discriminate (PD) affect the seller's profit and the level of efficiency.

The poaching model of Fudenberg and Tirole (2000) answers that question under fixed taste and no switching cost.

In these poaching models, PD decreases profit in the second period. This is because the taste does not change from period to period, so the purchase behavior in the first period reveals information about the preferences of the consumers in the second period and this makes the sellers compete more aggressively in the second period. However, by this logic, PD will hurt second period profit less if the taste of consumers change from period to period. In fact, if the taste is completely random and there is switching cost, PD even increases the second period profit because there is no information revealed about the consumers' taste by their purchase history, and in addition PD allows a seller to take advantage of the loyalty created by the switching cost among consumers who have bought from her before. Therefore, the total effect of PD on sellers' profit is not ex ante obvious.

Here I study a set-up where the consumers' taste are random across periods and there is positive switching cost. The positive switching cost gives the sellers the incentives to price discriminate in the second period. The result differs from traditional poaching model in the following aspects:

First, in the poaching model, relative to the benchmark of no PD, PD increases first period profit (demand becomes more inelastic), but depresses second period profit (there is more competition) and the second period effect dominates the first period effect.

However, in the model here, PD decreases first period profit (demand becomes more elastic), but increases second period profit (switching cost softens competition more) and the first period effect dominates the second period effect.

Therefore, even though the overall effect for both models is lower profit for the sellers, the inner working is different.

Also, we show that whether PD increases efficiency or not depends on on whether the demand is expanding or shrinking. Because switching cost is incurred once for each switch, while utility from product depends on the quantity each consumers buys. PD induces more inefficient switching, but at the same time reduces inefficient mis-matching between consumers and the seller's profit.

Chen (1997) is similar to this paper in the first period, but different in the second period. PD decreases profit in its second period, while in my model's second period, PD increases the second period profit. This is due to the presence of taste change in my model. Taste change makes consumers' taste more evenly distributed in the second period among those consumers who bought from the same seller in the previous period, which makes a poaching seller much less aggressive.

The literature on purchase-history price discrimination was nicely surveyed in Stole (2003).

The paper is also related to the literature of exogenous switching cost. Closest to the current set-up is Klemperer (1987b), which restricts sellers to charging one price per period.

The study of the impact of price discrimination when there is competition was also done, albeit in a static setting, by Corts (1998). It shows that price discrimination may lower sellers' profit and increase efficiency. The type of price discrimination he studied was a third-degree one.

Introductory sales (discounts to new consumers) was also studied as a signaling of unknown qualities or cost by an monopolist (Bagwell (1985a), Bagwell (1987)).

The paper is organized as follows: Section 3.2 gives some examples of real world applications. Section 3.3 sets up the model. Section 3.4 is the benchmark of no price discrimination. Section 3.5 allows the sellers to price discriminate, followed by Section 3.6, which compares the result with the benchmark in terms of both profit and welfare. We also contrast these comparison with the result in Fudenberg and Tirole (2000). Concluding remarks and discussion of some assumptions are in Section 3.7.

3.2. Motivating Examples

Here we will describe a set of examples that share the following features:

- Repeated purchase, nondurable goods
- Switching cost
- Changing taste
- Purchase history observable to the sellers

- Prices are lower at the early stages of a consumer's consumption but go up eventually

Apartment renters: They typically increase the old tenants' rent every year, while offering lower prices together with incentives like one-to-two-month-free to potential customers. Preference for apartments do tend to change over time due to lots of unforeseeable circumstances in life. People may get busier in work so they want to reduce the commute time, or they have new members adding to the household so they would like larger space, or their roommates got married and moved out or their PhD stipends have decreased and they cannot afford the same place anymore, etc.

Internet service providers: Comcast and SBC (at&t Yahoo! DSL) are the two major competitors for the Chicago Greater Area. Their services are respectively bundled (by the nature of the technology) with Cable TV service and land-line telephone service, which presents significant switching cost for consumers, as they would also have to change telephone or cable TV service providers if they want to change their Internet providers in addition to learning about how the new set of equipments work. Consumers' preference for the two may change over time due to the uncertainty of the conditions of the facilities and the simultaneous usage of other people in the neighborhood which affects cable service's speed. Both providers' offers are of the same pattern: promising a lower price for the first few months and a much higher price later. For example, Comcast (as of March 11, 2007) has an offer of $19.99 per month for the first 6 month (plus free modem and 100 dollars cash back). It is stated in the fine prints of its terms and conditions that

"After promotional period, regular monthly service charge of \$42.95 and equipment charges (if applicable) will apply". [1].

Credit card companies: Typically credit cards offer a low introductory APR and/or zero annual fee for a limited time. Consumer's taste for credit card may change because of their changing circumstances: if someone starts to travel a lot, a mileage accumulating credit card starts to be appealing; if someone starts to go to foreign countries a lot, a card with low foreign currency surcharge becomes desirable; if someone starts to drive a SUV, a credit card that has cash back on gasoline fills starts to make sense, so consumers' preference for credit card can be constantly changing.

Cell phone network providers: Consumers' preference may change because their main area of activities may change and the quality of the cell phone signals largely depend on the locations. Switching cost involves the pain of re-entering the contact numbers (for CDMA phones). A typical introductory offer is bonus minutes/airtime for new subscribers. For example, Verizon Wireless has "BONUS \$10 of airtime when you activate an INpulse phone" [2].

3.3. The Model

Two firms, A and B, sell horizontally differentiated products, A and B, with marginal cost 0.

There are two periods. In each period, a continuum of consumers with mass 1 each demands one unit of either A or B or nothing. [3]

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[1] http://www.comcastoffers.com
[2] http://www.verizonwireless.com
[3] In Section 3.6, we will discuss what happens if demand expands or shrinks.

Their "taste" (θ) are located along a unit line $[-\frac{1}{2}, \frac{1}{2}]$. Consumers learn of their private taste at the beginning of each period before making the purchase decision, and taste in the second period is independent of the first period. The taste parameter in both periods are distributed uniformly [4] on $[-\frac{1}{2}, \frac{1}{2}]$.

Consumer θ's willingness-to-pay for A's product is $v + t(\frac{1}{2} + \theta)$ and for B's product is $v + t(\frac{1}{2} - \theta)$. We assume the base willingness-to-pay v is large enough that all consumers buy in any period. If in the second period consumers choose to buy from a seller different from the previous seller, then they will incur an exogenous switching cost $s > 0$. We assume $s < t$, i.e. switching cost is relatively small compared to differentiation. The consumers are forward looking and maximize their expected discounted total future utility by choosing in each period whether or not to buy and which firm to buy from. Their per period utility is their willingness-to-pay less the price and the switching cost. There is no switching cost in period 1.

Firms maximize the expected discounted total future profit. Both consumers and the firms use the same discount factor $\delta \in (0, 1)$.

The solution concept is perfect Bayesian equilibrium.

3.4. Benchmark: No Price Discrimination

As a Benchmark, here we allow firms only to charge one price in the second period.

Using a standard revealed preference argument, we know that in any equilibrium, there exists a first period cutoff $\tilde{\theta}$ such that consumers to the right of it buys from A in period one, and those to the left of it buys from B. Following Fudenberg and Tirole (2000), we

[4] This draft only considers uniform distribution. I ran into problem with existence of second period price equilibrium with general distribution: it seems that MHR is not strong enough to guarantee existence for a second period following extreme first period cutoff.

call them respectively A's "turf" and B's "turf". Notice that the turf refers to a segment of consumers, not the types, as their types will change from the first to the second period.

3.4.1. Consumers' purchase decision in period 2

In this period consumers are not only distinguished by their taste, but also by their history, i.e. whether they belong to A's turf, or to B's turf. Let P^A and P^B be the second prices. If a consumer of taste θ belongs to A's turf, he buys from A if $\theta > \frac{P^A - P^B - s}{2t}$, from B if $\theta < \frac{P^A - P^B - s}{2t}$. If a consumer of taste θ belongs to B's turf, he buys from A if $\theta > \frac{P^A - P^B + s}{2t}$, from B if $\theta < \frac{P^A - P^B + s}{2t}$. The indifferent type is of measure 0, so we can ignore his behavior.

3.4.2. Firms' pricing in period 2

The next lemma establishes the existence and uniqueness of second period price equilibrium.

Lemma 14. *Given any first period cutoff $\tilde{\theta} \in [-\frac{1}{2}, \frac{1}{2}]$, there exists a unique second period price equilibrium:*

$$P^A(\tilde{\theta}) = t - \frac{2}{3}\tilde{\theta}s \quad P^B(\tilde{\theta}) = t + \frac{2}{3}\tilde{\theta}s$$

Proof. First we show that the second period price equilibrium (if it exists) must have $|P^A - P^B| \leq t - s$, i.e. there exists an indifferent type on both turfs. We prove this by contradiction. There are two alternative cases: 1) all consumers have strict preference for one seller; 2) all consumers of only one turf has strict preference for one seller.

It is easy to see that case 1) is impossible. It leaves one seller with zero second period profit, but for any $\tilde{\theta}$ the seller can guarantee herself a positive profit by charging the same price as her competitor because $s < t$.

WLOG, we now suppose there exists $\tilde{\theta} \in [-\frac{1}{2}, \frac{1}{2}]$ and a price equilibrium following it such that, all the consumers on A's turf strictly prefers A to B, which implies $P^A < P^B - (t-s)$. Since B only gets profit from B's turf, P^B must satisfy the first order condition locally, which implies

$$P^B = \frac{P^A + t + s}{2}$$

Also A's objective function over the range $[P^B - (t+s), P^B - (t-s)]$ must have an interior optimizer over that range, which implies:

$$P^A = \frac{t + P^B + \frac{\frac{1}{2}-\tilde{\theta}}{\frac{1}{2}+\tilde{\theta}} 2t - s}{2}$$

These two equalities have already pinned down for us:

$$P^A = t - \frac{s}{3} + \frac{4}{3}\frac{\frac{1}{2}-\tilde{\theta}}{\frac{1}{2}+\tilde{\theta}}t \quad P^B = t + \frac{s}{3} + \frac{2}{3}\frac{\frac{1}{2}-\tilde{\theta}}{\frac{1}{2}+\tilde{\theta}}t$$

Now we show that this cannot be an equilibrium because there is a strictly more profitable deviation for B.

Here B's profit is

$$\Pi_2^B(\tilde{\theta}) = \frac{1}{2t}(t + \frac{s}{3} + \frac{2}{3}\frac{\frac{1}{2}-\tilde{\theta}}{\frac{1}{2}+\tilde{\theta}}t)^2(\frac{1}{2} + \tilde{\theta})$$

Alternatively, B can deviate to a lower price: $P'^B = \frac{t+P^A+2\tilde{\theta}s}{2}$ such that B gets some profit from A's turf as well. Now B's profit is:

$$\Pi_2'^B(\tilde{\theta}) = \frac{1}{2t}(t + \frac{s}{3} + \frac{2}{3}\frac{\frac{1}{2}-\tilde{\theta}}{\frac{1}{2}+\tilde{\theta}}t - (\frac{1}{2} - \tilde{\theta})s)^2$$

$$\Pi_2'^B(\tilde{\theta}) - \Pi_2^B(\tilde{\theta}) = \frac{1}{2t}(t + \frac{s}{3} + \frac{2}{3}\frac{\frac{1}{2} - \tilde{\theta}}{\frac{1}{2} + \tilde{\theta}}t - s)^2(\frac{1}{2} - \tilde{\theta}) > 0$$

Therefore, if a price equilibrium exists it can be characterized [5] by the First Order Conditions derived from objective functions assuming that the cutoff consumers on turf A and B are $\theta_A \equiv \frac{P^A - P^B - s}{2t}, \theta_B \equiv \frac{P^A - P^B + s}{2t}$ respectively.

Given the competitor's price, each seller chooses her own price to maximize:

$$P^A \frac{t + P^B - 2\tilde{\theta}s - P^A}{2t}$$
$$P^B \frac{t + P^A + 2\tilde{\theta}s - P^B}{2t}$$

Simple algebra yields:

$$P^A(\tilde{\theta}) = t - \frac{2}{3}\tilde{\theta}s \quad P^B(\tilde{\theta}) = t + \frac{2}{3}\tilde{\theta}s$$
$$\theta_A(\tilde{\theta}) = \frac{-\frac{4}{3}\tilde{\theta}s - s}{2t} < 0 \quad \theta_B(\tilde{\theta}) = \frac{-\frac{4}{3}\tilde{\theta}s + s}{2t} > 0$$
$$\Pi_2^A(\tilde{\theta}) = \frac{1}{2t}(t - \frac{2}{3}\tilde{\theta}s)^2 \quad \Pi_2^B(\tilde{\theta}) = \frac{1}{2t}(t + \frac{2}{3}\tilde{\theta}s)^2$$

It is easy to check that they satisfy $|P^A(\tilde{\theta}) - P^B(\tilde{\theta})| < t - s$ □

A few quick comparative statics can be drawn here: second period price increases with one's first period market share (i.e. decreases with $\tilde{\theta}$ for seller A); profit increases with one's first period market share (i.e. decreases with $\tilde{\theta}$ for seller A).

[5] Second order conditions always hold locally. When v is big enough, the local maximum is also global. See footnote 8 of Klemperer (1987).

3.4.3. Consumers' purchase decision in period 1

Consumers have rational expectations, so in the first period the indifferent type, $\tilde{\theta}$, must satisfy:

$$v+t(\frac{1}{2}+\tilde{\theta})-p^A+\delta\int_{\theta_A(\tilde{\theta})}^{\frac{1}{2}} v+t(\frac{1}{2}+\tau)-P^A(\tilde{\theta})\,d\tau+\delta\int_{-\frac{1}{2}}^{\theta_A(\tilde{\theta})} v+t(\frac{1}{2}-\tau)-P^B(\tilde{\theta})-s\,d\tau$$

$$=v+t(\frac{1}{2}-\tilde{\theta})-p^B+\delta\int_{\theta_B(\tilde{\theta})}^{\frac{1}{2}} v+t(\frac{1}{2}+\tau)-P^A(\tilde{\theta})-s\,d\tau+\delta\int_{-\frac{1}{2}}^{\theta_B(\tilde{\theta})} v+t(\frac{1}{2}-\tau)-P^B(\tilde{\theta})\,d\tau$$

where lower case letters p^A, p^B denote the first period prices. So:

(†)
$$\tilde{\theta}(p^A, p^B) = \phi(\frac{p^A - p^B}{2t + \frac{4}{3}\delta\frac{s^2}{t}})$$

where $\phi(\cdot)$ is just the function that binds the argument into the interval $[-\frac{1}{2}, \frac{1}{2}]$. [6]

Compared to a model without switching cost, we have here an extra term $\frac{4}{3}\delta\frac{s^2}{t}$ which decreases the sensitivity of the cutoff type to prices. This shows how switching cost makes demand more inelastic in the first period. The intuition is that if a seller cuts the price in the first period, the consumers can foresee that by gaining a larger market share this seller will charge a higher price in the second period and buying from this seller makes one more glued to her in the second period and thus more likely to be accepting a higher price in the second period, which reduces the attractiveness of this price cut in the minds of first period consumers.

6

Formally $\phi(x) = \begin{cases} x, & \text{if } x \in [-\frac{1}{2}, \frac{1}{2}]; \\ -\frac{1}{2}, & \text{if } x < -\frac{1}{2}; \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$

3.4.4. Firms' first period problem

In the first period, seller A chooses p^A to maximize:

$$\Pi^A(p^A) = p^A(\frac{1}{2} - \tilde{\theta}(p^A, p^B)) + \delta\Pi_2^A(\tilde{\theta}(p^A, p^B))$$

where $\tilde{\theta}(p^A, p^B)$ is given by equation (†).

Notice that on equilibrium, we must have $|\frac{p^A - p^B}{2t + \frac{4}{3}\delta\frac{s^2}{t}}| \leq \frac{1}{2}$, because otherwise, one seller can locally increase her price by a very small amount and get strictly more profit. Therefore, $\tilde{\theta}(p^A, p^B) = \frac{p^A - p^B}{2t + \frac{4}{3}\delta\frac{s^2}{t}}$ and the function is strictly concave [7] .

We first consider possible equilibrium where both prices are positive. The set of first order conditions are (after some reorganizing):

$$(\frac{1}{2} - \tilde{\theta}(p^A, p^B))(2t + \frac{4}{3}\delta\frac{s^2}{t}) - \frac{2}{3}\delta(s - \frac{2}{3}\tilde{\theta}(p^A, p^B)\frac{s^2}{t}) = p^A$$
$$(\frac{1}{2} + \tilde{\theta}(p^A, p^B))(2t + \frac{4}{3}\delta\frac{s^2}{t}) - \frac{2}{3}\delta(s + \frac{2}{3}\tilde{\theta}(p^A, p^B)\frac{s^2}{t}) = p^B$$

Summing up the two first order conditions, we get:

$$\tilde{\theta}(p^A, p^B) = 0 \Rightarrow p^A = p^B$$

Now we consider the case where one price is zero. WLOG, assume $p^A = 0, p^B > 0$. The first order condition of A's problem implies that:

$$(\frac{1}{2} - \tilde{\theta}(p^A, p^B))(2t + \frac{4}{3}\delta\frac{s^2}{t}) - \frac{2}{3}\delta(s - \frac{2}{3}\tilde{\theta}(p^A, p^B)\frac{s^2}{t}) \leq 0$$

[7] SOC is $(\frac{4}{9}\delta\frac{s^2}{t} - 1)\frac{\partial\tilde{\theta}(p^A, p^B)}{\partial p^A} < 0$.

Rearranging the left-hand-side:

$$t + \frac{2}{3}\delta\frac{s^2}{t} + \frac{2}{3}\delta s - (2t + \frac{2}{3}\delta\frac{s^2}{t})\tilde{\theta}(p^A, p^B) > 0$$

which forms a contradiction.

Therefore, the unique equilibrium is the symmetric equilibrium where $p^A = p^B > 0$:

$$p^A = p^B = t + \frac{2}{3}\delta\frac{s^2}{t} - \frac{2}{3}\delta s > 0$$

The second positive term in the expression for p^A comes from the inelasticity of demand in the first period, while the third positive term shows the sellers' desire to lower price in the first period to grab more market share as market share increases second period profits.

Proposition 15. *When the sellers cannot price discriminate, the equilibrium is unique, for which the first period prices are* $p^A = p^B = t + \frac{2}{3}\delta\frac{s^2}{t} - \frac{2}{3}\delta s$ *and the second period prices are* $P^A = P^B = t$. *The symmetric equilibrium profits are:*

$$\Pi_B = \frac{t}{2} + \frac{1}{3}\delta\frac{s^2}{t} - \frac{1}{3}\delta s + \delta\frac{t}{2}$$

3.5. Price Discrimination in the Second Period

Now we allow sellers to charge history dependent prices in the second period.

Given $\tilde{\theta}$, the cutoff consumers in the first period as in the Benchmark [8], sellers competes in A's turf with prices $P_O^A(\tilde{\theta})$ and $P_R^B(\tilde{\theta})$; in B's turf with prices $P_R^A(\tilde{\theta})$ and $P_O^B(\tilde{\theta})$. Here, we use subscripts O ("own") and R ("rival's") to denote prices charged to one's old customers and new customers.

[8] Whenever it applies, the notation within this section follows that in the Benchmark to form a parallel. We will use subscripts "B" and "PD" to distinguish the two models once we are out of this section.

3.5.1. Consumers' purchase decision in period 2

In this period consumers are not only distinguished by their taste, but also by their history, i.e. whether they belong to A's turf, or to B's turf. If a consumer of taste θ belongs to A's turf, he buys from A if $\theta > \frac{P_O^A - P_R^B - s}{2t}$, from B if $\theta < \frac{P_O^A - P_R^B - s}{2t}$. If a consumer of taste θ belongs to B's turf, he buys from A if $\theta > \frac{P_R^A - P_O^B + s}{2t}$, from B if $\theta < \frac{P_R^A - P_O^B + s}{2t}$. The indifferent type is of measure 0, so we can ignore him.

3.5.2. Firms' pricing decision in period 2

Now prices are determined by competition within each turf. As a result, the size of the turfs do not affect the second period prices anymore [9], so actually the prices are independent of $\tilde{\theta}$. Simple algebra gives us:

$$P_O^A = P_O^B = t + \frac{s}{3} \quad P_R^A = P_R^B = t - \frac{s}{3}$$

$$\theta_A = -\frac{s}{6t} < 0 \quad \theta_B = \frac{s}{6t} > 0$$

$$\Pi_2^A(\tilde{\theta}) = \frac{1}{2t}[(\frac{1}{2} - \tilde{\theta})(t + \frac{s}{3})^2 + (\frac{1}{2} + \tilde{\theta})(t - \frac{s}{3})^2] \quad \Pi_2^B(\tilde{\theta}) = \frac{1}{2t}[(\frac{1}{2} - \tilde{\theta})(t - \frac{s}{3})^2 + (\frac{1}{2} + \tilde{\theta})(t + \frac{s}{3})^2]$$

As in the Benchmark, the profit increases with the market share and the second period cutoff in each turf is biased: cutoff on turf A is biased to the left-hand-side, the the cutoff

[9] The result holds even when consumers have heterogeneous switching cost. The average switching costs of the two turfs will be the same and thus the prices.

on turf B is biased to the right-hand-side. However, these second period cutoffs and prices are independent of first period market share.

3.5.3. Consumers' purchase decision in period 1

Consumers have rational expectations, so in the first period indifferent type, $\tilde{\theta}$, must satisfy:

$$v + t(\frac{1}{2} + \tilde{\theta}) - p^A + \delta \int_{\theta_A}^{\frac{1}{2}} v + t(\frac{1}{2} + \tau) - P_O^A \, d\tau + \delta \int_{-\frac{1}{2}}^{\theta_A} v + t(\frac{1}{2} - \tau) - P_R^B - s \, d\tau$$

$$= v + t(\frac{1}{2} - \tilde{\theta}) - p^B + \delta \int_{\theta_B}^{\frac{1}{2}} v + t(\frac{1}{2} + \tau) - P_R^A - s \, d\tau + \delta \int_{-\frac{1}{2}}^{\theta_B} v + t(\frac{1}{2} - \tau) - P_O^B \, d\tau$$

Simple algebra yields:

$$\tilde{\theta}(p^A, p^B) = \phi(\frac{p^A - p^B}{2t}) \tag{‡}$$

Here the demand elasticity is the same as in a model without switching cost. The result of no reduction in demand elasticity is not surprising, because here market share does not affect the future prices and consumers foresee that no matter which seller they buy from, they will be subject to the same pair of prices in the second period so they act "as if" they are myopic.

3.5.4. Firms' first period problem

Just as in the Benchmark, in the first period, seller A chooses p^A to maximize:

$$\Pi^A(p^A) = p^A(\frac{1}{2} - \tilde{\theta}(p^A, p^B)) + \delta\Pi_2^A(\tilde{\theta}(p^A, p^B))$$

where $\tilde{\theta}(p^A, p^B)$ is given by equation (‡). By the same method as in the Benchmark analysis, we can show that the unique equilibrium is symmetric: [10]

$$p^A = p^B = t - \frac{2}{3}\delta s > 0$$

Compared to the Benchmark, the positive term from the inelasticity of demand in the first period disappears here, while the term that is due to the sellers' desire to lower price in the first period to grab more market share remains the same (even though second period profit is a different function of $\tilde{\theta}$).

Proposition 16. *When firms can charge history dependent prices in the second period, the equilibrium is unique, at which the first period prices are $p^A = p^B = t - \frac{2}{3}\delta s$ and the second period prices are $P_O^A = P_O^B = t + \frac{s}{3}$; $P_R^A = P_R^B = t - \frac{s}{3}$. The symmetric equilibrium profits are:*

$$\Pi_{PD} = \frac{t}{2} - \frac{1}{3}\delta s + \delta\frac{1}{2}(t + \frac{s^2}{9t})$$

[10]The first order conditions are:

$$(\frac{1}{2} - \tilde{\theta}(p^A, p^B))2t - \frac{2}{3}\delta s = p^A$$
$$(\frac{1}{2} + \tilde{\theta}(p^A, p^B))2t - \frac{2}{3}\delta s = p^B$$

3.6. Comparison: No PD (Benchmark) vs. PD

3.6.1. First period price

$$p_B = t + \frac{2}{3}\delta\frac{s^2}{t} - \frac{2}{3}\delta s > t - \frac{2}{3}\delta s = p_{PD}$$

Price discrimination in the second period decreases the first period price because consumers no longer has inertia towards price change in the first period.

3.6.2. Profit

We rewrite the profits of the two models as follows:

$$\begin{aligned}
\Pi_B &= \frac{t}{2} + \frac{1}{3}\delta\frac{s^2}{t} - \frac{1}{3}\delta s + \qquad \delta\frac{t}{2} \\
\Pi_{PD} &= \frac{t}{2} + \qquad 0 - \frac{1}{3}\delta s + \delta\frac{1}{2}(t + \frac{s^2}{9t})
\end{aligned}$$

Here the *first* period profit is decomposed to three parts: the positive part due to product differentiation, the positive part due to decrease in elasticity (which is 0 for PD), and the negative part representing more aggressive competition for market share (all relative to the no switching cost baseline). We see that PD hurts profit because its increased second period profit (relative to the Benchmark of no PD), $\delta\frac{1}{2}\frac{s^2}{9t}$ cannot compensate the decreased first period profit due to higher demand elasticity, $\frac{1}{3}\delta\frac{s^2}{t}$. The dominance of the elasticity effect is not surprising. This is because it comes from the fact that when a seller gains a larger market share in the second period she also gains advantage on the switching cost. A larger market share for one seller means smaller market share for the other seller, so the switching cost advantage is in some sense "doubled" because each seller charges only one price in the second period. However, the switching cost advantage when the sellers can price discriminate is limited to each turf, so it is not "doubled".

The comparison of price and profit does not change if we let the demand expand or shrink in the second period, i.e. if we let the consumers each demand more than one unit or less than one unit. This is because such a change is equivalent to changing the discount factor δ and allow δ to be any positive value.

Proposition 17. *Suppose the consumers' taste are random from period to period and they have homogenous switching cost. If the consumers' taste is uniformly distributed, then when sellers are allowed to charge history dependent prices in the second period, the first period profit is decreased, the second period profit is increased, the total equilibrium profit is decreased.*

3.6.3. Efficiency

In this set-up, there are potentially two sources of efficiency loss: 1) switching cost and 2) mismatching between the consumers and the sellers' products.

In the benchmark of no PD, consumers whose second period taste θ located in $[-\frac{1}{2}, -\frac{s}{2t}]$ on A's turf, and in $[\frac{s}{2t}, \frac{1}{2}]$ on B's turf switch, so the total switching cost loss is:

$$LS_B \equiv \frac{s}{2} - \frac{s^2}{2t}$$

In the benchmark of no PD, consumers whose second period taste θ located in $[-\frac{s}{2t}, 0]$ on A's turf and $[0, \frac{s}{2t}]$ on B's turf are mismatched to the seller whose product they prefer less. Therefore, the total loss due to mismatching is:

$$LM_B \equiv 2\int_0^{\frac{s}{2t}} 2t\theta\frac{1}{2}\,d\theta = \frac{s^2}{4t}$$

In the main model with PD, consumers whose second period taste θ located in $[-\frac{1}{2}, -\frac{s}{6t}]$ on A's turf, and in $[\frac{s}{6t}, \frac{1}{2}]$ on B's turf switch, so the total switching cost loss is:

$$LS_{PD} \equiv \frac{s}{2} - \frac{s^2}{6t} < LS_B$$

Since there are wider ranges of consumers switching here than in the benchmark, it is unsurprising that loss due to switching cost is higher here.

In the benchmark of no PD, consumers whose second period taste θ located in $[-\frac{s}{6t}, 0]$ on A's turf and $[0, \frac{s}{6t}]$ on B's turf are mismatched to the seller whose product they prefer less. Therefore, the total loss due to mismatching is:

$$LM_{PD} \equiv 2\int_0^{\frac{s}{6t}} 2t\theta\frac{1}{2}\,d\theta = \frac{s^2}{36t} > LM_B$$

Since there are narrower ranges of consumers being mismatched here than in the benchmark, it is unsurprising that loss due to mismatching is lower here.

In this set-up, we see that the change in loss due to switching cost $\frac{1}{3}\frac{s^2}{t}$ dominates the change in loss due to mismatching $-\frac{2}{9}\frac{s^2}{t}$, therefore, if switching cost is a social loss, then PD decreases efficiency. Examples of switching cost that is a social loss includes cost of learning about the new product and service.

However, the above comparison can change if we let the demand expand or shrink in the second period. This is because the switching cost is associated with each consumer that switches, while the mismatching cost is associate with each unit that the consumers consume in the second period. Therefore, when demand expands, the efficiency gain due to reduced mismatching loss becomes larger.

Now suppose instead of demanding 1 unit, in the second period consumers demand x units instead, so $x < 1$ means demand shrinks, and $x > 1$ means demand expands.

Proposition 18. *If $x > \frac{3}{2}$, then the efficiency increases when PD is allowed, and if $x < \frac{3}{2}$, then efficiency decreases when PD is allowed.*

3.6.4. Contrast with a poaching model

We will contrast it with a parallel proposition in the set-up of Fudenberg and Tirole (2000) with uniform distribution:

Proposition 19. *In a set-up where consumers' taste are fixed and they do not have switching cost, when sellers are allowed to charge history dependent prices in the second period, the first period profit is increased, the second period profit is decreased, the total equilibrium profit is decreased and the efficiency is decreased.*

We see that even though in both set-ups, price discrimination decreases seller's total profit, the two periods' respective profits go the opposite direction between the two models as a result of price discrimination. In Fundeberg and Tirole (2000)'s set-up, 1) first period demand elasticity is decreased when sellers can poach; 2) second period price competition is intensified when sellers can charge history dependent prices.

We see that even though the outside descriptive phenomenon might look undistinguishable: in both cases sellers charge a higher price to those who purchased from her in the previous period, the inner working is very different.

3.7. Concluding Remarks

Price discrimination can arise in a two period model either when the consumers' taste are fixed or when the consumers have switching cost. We examine the latter case and find that allowing price discrimination based on purchase history has different implications for equilibrium profits and efficiency from the former case.

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APPENDIX

Appendix for Chapter 1

1. Proofs for Benchmark 1: Full Commitment

Proof of Proposition 1.

Proof. We will first, in Step 1, solve for the outcome that gives the seller the highest expected profit among a set of potential equilibrium outcomes. Then we will show that such an outcome is an equilibrium outcome for the Full Commitment model.

Step 1. Find the profit maximizing outcome.

Some consumers must buy in the first period at the profit maximizing outcome because otherwise we can find another outcome where the timing is shifted earlier. There are two possibilities: H types buy in the first period (possibly with some L types) and only L types buy in the first period. If only L types buy in the first period, then the expected (at $t = 0$) future price of transactions with L types must be lower than L, which means that the seller's profit level is dominated by the outcome where all consumers buy in the first period. Similarly, if some H types do not buy in the first period, then the expected future price at transactions for H types at $t = 0$ must be lower, which implies that there is a more profitable outcome where all H types buy at $t = 0$. Therefore, we can restrict attention to outcomes where all H types buy in the first period.

Now we can characterize a potentially profit-maximizing outcome simply by the time that L types buy and the price at which H types buy. Here are some notations [1]: suppose

[1]For notational simplicity, it is understood that all selling and buying time belongs to the set $\{0, \Delta, 2\Delta, \ldots\}$

that L types buy at t_L^0 if demand never drops (and t_L^0 can be ∞, meaning that L types never buy on the outcome), and L types buy at $t_L^u \geq u$ if demand drops at $u \in \{\Delta, 2\Delta, \ldots\}$. Suppose H types accepts price P_0. Let $\mathbf{1}$ be an indicator function that returns value 1 if the statement that follows is true, and 0 otherwise. We solve the following problem, picking P_0 and $\{t_L^u\}_{\{u=0,\Delta,\ldots\}}$ to maximize seller's expected profit:

$$
\begin{aligned}
Max \quad & P_0 h + e^{-(r+s)t_L^0} L(1-h) \\
& + [\sum_{u=\Delta}^{t_L^0} (1-e^{-s\Delta}) e^{-s(u-\Delta)} e^{-rt_L^u}] L'(1-h)\mathbf{1}(t_L^0 > 0) \\
s.t. H - P_0 \geq & [\sum_{u=\Delta}^{t_L^0} (1-e^{-s\Delta}) e^{-s(u-\Delta)} e^{-rt_L^u}] (H'-L')\mathbf{1}(t_L^0 > 0) \\
& + e^{-(r+s)t_L^0}(H-L) \\
& L < P_0 \\
& t_L^u \geq u \quad \forall \quad u \in \{\Delta, 2\Delta, \ldots\} \\
& t_L^u \in \{0, \Delta, 2\Delta, \ldots\}.
\end{aligned}
$$

In the objective function, $(1-e^{-s\Delta})e^{-s(u-\Delta)}$ is the probability that the demand keeps high through $u-\Delta$ but is low at u. If $t_L^0 = 0$ then this term will disappear because the seller already sells to all the types at $t = 0$ and the game effectively ends there. Since the second constraint is implied if the first is binding, the first constraint on P_0 must bind, we have $P_0 = H - [\sum_{u=\Delta}^{t_L^0} (1-e^{-s\Delta}) e^{-s(u-\Delta)} e^{-rt_L^u}](H'-L')\mathbf{1}(t_L^0 \geq \Delta) - e^{-(r+s)t_L^0}(H-L)$ and thus we can rewrite the objective function as:

$$
\begin{aligned}
Max \quad & Hh + e^{-(r+s)t_L^0}(L - Hh) \\
& + [\sum_{u=\Delta}^{t_L^0}(1-e^{-s\Delta})e^{-s(u-\Delta)}e^{-rt_L^u}](L' - H'h)\mathbf{1}(t_L^0 \geq \Delta).
\end{aligned}
$$

The objective function is monotone in t_L^u for all $u > 0$ and whether it is increasing or decreasing solely depends on the sign of $L' - H'h$. Therefore, if $L' < H'h$, then $t_L^u = \infty$ for all $u > 0$; if $L' > H'h$, $t_L^u = u$ for all $u > 0$. [2]

Case 1. $L' < H'h$. Since $t_L^u = \infty$ for all $u > 0$, the objective function further reduces to:

$$
Max \quad Hh + e^{-(r+s)t_L^0}(L - Hh).
$$

Therefore, $t_L^0 = 0$ if $L > Hh$ and $t_L^0 = \infty$ if $L < Hh$.

Case 2. $L' > H'h$. Since $t_L^u = u$ for all $u > 0$, now the problem is reduced to, after some algebra:

$$
\begin{aligned}
Max_{t_L^0} \quad & Hh + \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(1 - e^{-(r+s)t_L^0})(L' - H'h)\mathbf{1}(t_L^0 \geq \Delta) \\
& + e^{-(r+s)t_L^0}(L - Hh).
\end{aligned} \tag{.1}
$$

Taking first order derivative with respect to t_L^0 gives us:

$$
(r+s)e^{-(r+s)t_L^0}[\frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) - (L - Hh)].
$$

[2]We will ignore the hair-line case of equality in the parameters.

Therefore, the solutions will be bang-bang depending on parameter cases:

Subcase 2a. If $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$, then optimal $t_L^0 = \infty$.

Subcase 2b. If $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) < L - Hh$, then optimal $t_L^0 = 0$.

So far we have shown that the outcome best for the seller among potential equilibrium outcomes is:

(1) Outcome-Contingent if $L' > H'h$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) > L - Hh$.

(2) Outcome-Only-H if $L' < H'h$ and $L < Hh$.

(3) Outcome-All if $L > Hh$ and $\frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(L' - H'h) < L - Hh$.

Step 2.

Case 1. Parameters are such that Outcome-Contingent is the optimal outcome for the seller.

Suppose the seller commits to the following strategy at the very beginning: price at $t = 0$ is $P_0 \equiv H - \frac{e^{-r\Delta}-e^{-(r+s)\Delta}}{1-e^{-(r+s)\Delta}}(H' - L')$, price at $t > 0$ equals L' if demand is low there and positive measure of consumers still have not bought and ∞ otherwise. It is easy to see that there exists a subgame equilibrium after such a commitment such that Outcome-Contingent is the outcome. L types will not buy at P_0 because $L - P_0 < L - (H - (H' - L')) = (H' - L') - (H - L) < 0$. L type does not want to deviate from buying at the first L' price offered because if he does, he will only face prices of ∞. H type does not want to deviate from buying at time 0, because he is exactly indifferent from buying now and or waiting for L' prices later. We go one step backward: since Outcome-Contingent is the best outcome for the seller, she will not deviate to other commitments. This shows that we have an equilibrium.

We further claim that this is the unique equilibrium *outcome*. We prove by contradiction. Suppose there is a different equilibrium outcome, then it must give the seller a lower profit because we have shown in Step 1 that the outcome is the unique one that maximizes the profit under the parameter case. Now we let the seller deviate to the following commitment: price at $t = 0$ is $P_0 \equiv H - \frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(H' - (L' - \epsilon))$, price at $t > 0$ equals $L' - \epsilon$ if demand is low there and positive measure of consumers still have not bought and ∞ otherwise, where $\epsilon > 0$. Such a commitment will give consumers strict incentive to buy where they are intended to buy on the Outcome-Contingent. When ϵ is small enough, this deviation profit beats the proposed equilibrium profit, which forms a contradiction.

Case 2. Parameters are such that Outcome-Only-H is the optimal outcome for the seller.

By the similar analysis as in Case 1, the unique equilibrium outcome is Outcome-Only-H and it can emerge after the seller has committed to the following strategy: price at $t = 0$ is $P_0 = H$, and prices thereafter is ∞.

Case 3. Parameters are such that Outcome-All is the optimal outcome for the seller.

By the similar analysis as in Case 1, the unique equilibrium outcome is Outcome-All and it can emerge after the seller has committed to the following strategy: price at $t = 0$ is $P_0 = L$, and prices thereafter is ∞.

□

2. Proof for Benchmark 2: No Commitment

Lemma 15. *If one type of consumers are alone in the market, then their equilibrium strategy must specify actions in the following form: they accept any price equal to or below their willingness-to-pay and reject all other prices.*

Proof. Let X be either H or L. First, we claim that when X types are alone in the market, their equilibrium strategy cannot randomize at two different prices. If they do then he must be indifferent between accepting and rejecting at two different prices, which implies that they expect the future price path to be different and to yield them different continuation payoffs depending on the two prices today. This means that the seller will get different continuation payoffs in the subgame following X types accepting the two different prices. This is impossible because by the definition of subgame-perfection, the seller must maximizes her payoff in the next period.

Now we know that X type's strategy when they are alone in the market must have at most one price at which X types will randomize. It is immediate that this price must be less than or equal to their willingness-to-pay. Let it be $\tilde{P}$ for the case of high demand and $\tilde{P'}$ for the case of low demand, then for all prices above $\tilde{P}\,(\tilde{P'})$, X types must reject, and for all price below $\tilde{P}\,(\tilde{P'})$, X types must accept. Then there cannot be any equilibrium because the seller wants to charge a price closer and closer to $\tilde{P}\,(\tilde{P'})$ from below. Therefore, X types' equilibrium strategy when they they are alone in the market must not randomize at any price, and also the interval of prices that they accept must be closed at the higher end. Therefore, there exists a cutoff price c such that H types accept any price $\in [0, c]$ X types accept and reject any price $\in (c, \infty)$ when they are alone in the market and demand is high, and there exists a cutoff c' for the case when demand is low. Now we claim that the cutoff $c\,(c')$ is equal to X types' willingness-to-pay. We prove by contradiction.

Case 1. Demand is low. Suppose $c' < X'$. Let the seller offer price $c' + \epsilon$ with $\epsilon > 0$. X types will reject it. Then in the next period, demand is low again, and the seller can offer price c and X types accepts. This forms a contradiction because when ϵ is small enough, X types are better off accepting earlier.

Case 2. Demand is high. Suppose $c < X$. Let the seller offer price $c + \epsilon$ with $\epsilon > 0$. X types will reject it. Then in the next period, if the demand is low, then by Case 1 we know that the seller can offer price X' and X types will accept it; if demand is high, then the seller can offer c for X types to accept it. This forms a contradiction because when ϵ is small enough, X types are better off accepting earlier.

□

Lemma 16. *On any NC equilibrium, for any demand realization, at the last transaction, there must be positive measure of L types buying.*

Proof. The claim is equivalent to saying that positive measure of H types are never alone in the market. We prove by contradiction. Suppose we at a time when all L types have already bought, and positive measure of H types are alone in the market. By Lemma 15, we know that the unique subgame equilibrium outcome is that the seller charges H types their willingness to pay and H types accept the price. This would imply that H types are better off buying at the same as L types, because they will get strictly positive utility. Therefore there will never be any positive measure of H types buying after L types have all bought. □

Remark: Notice that this is a weaker statement than the usual skimming property which claims that all H types buy before or at the same time as the *first* moment that L types buy. However, it is sufficient for our purpose.

Lemma 17. *After any history, a price that is equal to L types' willingness-to-pay will clear the whole market with probability one.*

Proof. Let $c' \leq L'$ be the supremum of the set of prices that all consumers accept with probability one as long as the demand is low and positive measure of both types are in the market. Let $c \leq L$ be the supremum of the set of prices that all consumers accept with probability one as long as demand is low and positive measure of both types are in the market.

Case 1. Fix any history after which the demand is low and positive measure of consumers are in the market.

First, we prove that $c' = L'$.

Suppose $c' < L'$. Let the seller offer price $c' + \epsilon$ with $\epsilon \in (0, L' - c')$. If only positive measure of one type of consumers reject it, then this type must be L (Lemma 16), then in the next period all the remaining L types will be charged their willingness-to-pay, which gives a contradiction because they should have accepted earlier as $P < L'$. If positive measure of both types of the consumers reject it, then the cutoff in the next period will be at least c' because there will be positive measures of both types in the market, which implies the price in the next period must be higher than or equal to c'. Because of discounting, when ϵ is small enough, all consumers will strictly prefer accepting the price to rejecting the price. Since we can start with any history after which demand is low and positive measure of consumers are in the market, we know that $c + \epsilon$ is in the set of prices that all consumers will accept with probability one if the demand is low and positive measure of both types are in the market. This contradicts the definition of a supremum for c'.

Now we know $c' = L'$. Suppose at least one type of consumers reject the price c' with positive probability after a history such that demand is low and positive measure of both types are in the market (i.e., the supremum is not in the set), then no optimal strategy exists for the seller after this history.

Therefore, $c' = L'$ and all consumers accept this price with probability one after any history.

Case 2. Fix any history after which the demand is high and positive measure of consumers are in the market.

First we prove that $c = L$.

Suppose $c < L$. Let the seller offer price $P = c + \epsilon$ with $\epsilon \in (0, L - c)$. If some L types reject this price, then they will buy later. At the last transaction following they rejecting the price, if L types buy alone there, then by Lemma 15, they will get a payoff of zero. If they buy with H types, then the price must be at least c if the demand is high, and exactly L' by the analysis of *Case 1* if the demand is low. Then, when ϵ is small enough, L types will strictly prefer accepting the price P to rejecting the price. Since L types will accept P with probability one, H types must accept P with probability one too, because otherwise they will get zero payoff (Lemma 15). This means that P is in the set of prices accepted with probability one by both types after this history. Since we started with any history such that demand is high and positive measure of both types are in the market, we know that $P = c + \epsilon$ is in the set of the prices that all consumers accept with probability one as long as demand is high and positive measure of consumers are in the market. This contradicts the definition of a supremum for c.

Now we know $c = L$. Suppose at least one type of consumers reject the price c with positive probability after a history such that demand is high and positive measure of both types are in the market (i.e., the supremum is not in the set), then no optimal strategy exists for the seller after this history.

Therefore, $c = L$ and all consumers accept this price with probability one after any history. $\square$

Proof of Proposition 3.

Proof. Because of subgame perfection, there must be positive measure of consumers buy in each period until all consumers are gone regardless of demand realization. We know that at the last transaction for each demand realization, the price must be equal to L at the last transaction when the demand is high or equal to L' at the last transaction when demand is low by Lemma 17. Then by incentive compatibility of the consumers, for any $\epsilon > 0$ there exists a $\hat{\Delta} > 0$ small enough such that for any $\Delta < \hat{\Delta}$, the first transaction price (at $t = 0$) must be less than $L + \epsilon$. This is because when Δ is small enough, it costs almost nothing for the consumers at the beginning to deviate to get a price in the future that is at most L. This implies that the profit is also capped by an upper bound that converges to L as $\Delta \to 0$.

Lemma 17 also implies that the profit has an lower bound of L because the seller can charge price L and all consumers will buy with probability one at $t = 0$. □

3. Proofs for the BP model.

Proof of Lemma 2.

Proof. First we claim that if we are at a moment such that the remaining consumers are of the same type and the seller does not need to pay any refund to the previous consumers if she charges the willingness-to-pay of the remaining consumers, then the unique equilibrium outcome in the subgame starting from this moment has the seller charging a price equal to these consumers' willingness-to-pay and they accepting the price right away. The proof of this is the same as that of Lemma 15.

Now we show that after offering a price of L at $t = 0$, it is a equilibrium in the subgame that both H and L types accept. They do not have profitable deviations, because if one consumer rejects the price, he will eventually be charged his willingness-to-pay.

Next, we show that the uniqueness of such an outcome in the subgame is implied by its "optimality". As argued before, on the optimal equilibrium, some consumers must buy in the first period. Suppose only some of the consumers accept the price at $t = 0$, the prices where positive measure of consumers buy afterwards on the equilibrium path must all be less than or equal to L (otherwise the consumers would rather buy at $t = 0$ than waiting for a later price). However, the profit is strictly lower than that of the equilibrium that all consumers buy at $t = 0$, because the profits from any transaction after time $t = 0$ are discounted. This gives a contradiction.

□

Proof of Lemma 4.

Proof. Let $c \leq L$ be the supremum of the set of prices that L types accept with probability one after any history as long as H types have all bought and demand is still high. (This has to be a connected set because since they accept with probability one, there will be no consumers left after them and they cannot expect any future refunds. As a result, if they are indifferent between accepting and rejecting at one price, they strictly prefer accepting at all lower prices.) If $c = L$, but a positive measure of consumers still reject the price after some history (i.e., the supremum is not in the set), then no optimal strategy exists for the seller after that history. Suppose $c < L$. Let the seller offer price $c + \epsilon$ with $\epsilon \in (0, L - c)$. If a positive measure of consumers reject it, then the price in the next period must be higher than or equal to c. Because of discounting, and because of the possibility of a demand drop, when ϵ is small enough, all L types strictly prefer accepting the price. This contradicts the definition of a supremum. Therefore, $c = L$ and all L types still in the market accept this price with probability one after any history as long as H types have all bought and demand is high.

Let $c' \leq L'$ be the supremum of the set of the prices that L types accept with probability one after any history as long as H types have all bought and demand is already low. The analysis is the same as that for the case when demand is high. Therefore, $c' = L'$ and all L types accept this price with probability one after any history as long as H types have all bought and demand is already low. □

Proof of Proposition 7.

Proof. We define two actions. S means selling to L types at their willingness-to-pay and N means charging a price above P_H, which is a price that L types will not accept. By Lemma 4, we can now characterize the equilibrium outcome in this subgame starting at time Δ (where H types have all bought and all L types are in the market) by just a $\frac{T}{\Delta}$-element sequence: $\{(a_t, a'_t)\}_{\{t=\Delta,2\Delta,\ldots,T\}}$, where $a_t, a'_t \in \{S, N\}$ and prime means the case of low demand.

We use backward induction to solve the subgame starting at Δ.

Step 1. Consider the subgame starting at time T given that L types have not bought yet. We know that L types accept a price as high as their willingness-to-pay. The trade-off for the seller is between the discounting of profits from L types for one additional period, and the refunds she has to pay if she sells now. When demand is low, the trade-off is between $(L' - e^{-r\Delta}L')(1-h)$ and $z(P_H - L')h$. Therefore:

$$(L' - e^{-r\Delta}L')(1-h) > z(P_H - L')h \Rightarrow a'_T = S,$$

$$(L' - e^{-r\Delta}L')(1-h) < z(P_H - L')h \Rightarrow a'_T = N.$$

When $(L' - e^{-r\Delta}L')(1-h) = z(P_H - L')h$, the seller may randomize, so both outcomes are possible.

When demand is high, the trade-off is instead between $e^{-s\Delta}(L - e^{-r\Delta}L)(1-h) + (1 - e^{-s\Delta})(L - e^{-r\Delta}L')(1-h)$ and $z(P_H - L)h$. Therefore:

$$[e^{-s\Delta}(L - e^{-r\Delta}L) + (1 - e^{-s\Delta})(L - e^{-r\Delta}L')](1-h) > z(P_H - L)h$$
$$\Rightarrow a_T = S,$$

$$[e^{-s\Delta}(L - e^{-r\Delta}L) + (1 - e^{-s\Delta})(L - e^{-r\Delta}L')](1-h) < z(P_H - L)h$$
$$\Rightarrow a_T = N.$$

Since $[e^{-s\Delta}(L - e^{-r\Delta}L) + (1 - e^{-s\Delta})(L - e^{-r\Delta}L')](1-h) > (L' - e^{-r\Delta}L')(1-h)$ and $z(P_H - L)h < z(P_H - L')h$, we have:

$$a'_T = S \Rightarrow a_T = S.$$

In other words, the seller is more inclined to sell now when the demand is high, because compared to the case of low demand she can pay a lower refund if she sells to L types while the cost of waiting is bigger. Therefore there are three possible outcomes in the subgame starting at $t = T$: $(a_T, a'_T) \in \{(S,S),(S,N),(N,N)\}$. Rearranging the inequalities, we get the sufficient conditions for the three possible outcomes:

(1) If $P_H > \overline{P_H}(\Delta, z)$, then the unique equilibrium outcome is that the seller sells to L types at $t = T + \Delta$ regardless of demand, i.e., $\{(N,N)\}$

(2) If $\underline{P_H}(\Delta, z) < P_H < \overline{P_H}(\Delta, z)$, then the unique equilibrium outcome is that the seller sells to L types at $t = T$ is demand is high at $t = T$, and to L types at $t = T + \Delta$ is demand is low at $t = T$, i.e., $\{(S,N)\}$.

(3) If $L < P_H < \underline{P_H}(\Delta, z)$, then the unique equilibrium outcome is that the seller sells to L types at $t = T$ regardless of demand, i.e., $\{(S,S)\}$.

Obviously, the necessary conditions for these outcomes are just the weak inequality versions of the sufficient conditions.

Step 2. Consider the game at $T - \Delta$. We break the analysis into three cases depending on the outcomes a period later.

Case 1. $(a_T, a'_T) = (N,N)$.

The only difference from the analysis in *Step 1* is that the trade-off the seller faces here is between discounting for *two* periods (instead of just one period) and the refunds she has to pay now. By the same logic, we know that (N,S) cannot happen at $T - \Delta$, so $\{(a_{T-\Delta}, a'_{T-\Delta}), (a_T, a'_T)\} \in \{\{(N,N),(N,N)\},\{(S,N),(N,N)\},\{(S,S),(N,N)\}\}$.

Case 2. $(a_T, a'_T) = (S,N)$.

We first claim that when the demand is high at $T - \Delta$, the seller wants to sell right now. A period later, the seller will sell if the demand is high which implies that $P_H \leq \overline{P_H}(\Delta, z) \Rightarrow L(1-h) - z(P_H - L)h \geq e^{-s\Delta}e^{-r\Delta}L(1-h) + (1-e^{-s\Delta})e^{-r\Delta}L'(1-h) \Rightarrow L(1-h) - z(P_H - L)h > e^{-s\Delta}e^{-r\Delta}(L(1-h) - z(P_H - L)h) + (1-e^{-s\Delta})e^{-2r\Delta}L'(1-h)$.

The left-hand-side of the last inequality is exactly the profit from selling now and the right-hand-side is the profit from not selling now. Thus the seller wants to sell right away, i.e., $a_{T-\Delta} = S$.

If the demand is low at $t = T - \Delta$, depending on the parameters the seller may or may not sell.

Therefore, $\{(a_{T-\Delta}, a'_{T-\Delta}), (a_T, a'_T)\} \in \{\{(S,S),(S,N)\},\{(S,N),(S,N)\}\}$.

Case 3. $(a_T, a'_T) = (S,S)$.

The seller must sell at time $t = T - \Delta$. If not, she will sell a period later for sure. By selling earlier, she gets the profit earlier and if the demand is high she can also avoid the less profitable case of a demand drop. (There must be a positive profit from selling to L types because otherwise the seller will wait until the policy expires.) Therefore, $\{(a_{T-\Delta}, a'_{T-\Delta}), (a_T, a'_T)\} = \{(S,S),(S,S)\}$.

Step 3. We go further back to $t = T - 2\Delta$, and so on. Similar analysis applies and shows the following:

From the reasoning in *Case 1 of Step 2*, we know that if the seller sells now when facing a low demand, then she will also sell now when facing a high demand:

$$a'_t = S \Rightarrow a_t = S \text{ for } t = \Delta, 2\Delta, \ldots$$

From the reasoning in *Case 2 of Step 2*, we know that if in the next period the seller will sell if demand is high and there are still L types in the market, then the seller will sell now if the demand is high:

$$a_{t+\Delta} = S \Rightarrow a_t = S \text{ for } t = \Delta, 2\Delta, \ldots$$

From the reasoning in *Case 3 of Step 2*, we know that if the demand has already dropped and in the next period the seller will sell when there are still L types in the market, then the seller will sell now:

$$a'_{t+\Delta} = S \Rightarrow a'_t = S \text{ for } t = \Delta, 2\Delta, \ldots$$

Step 4. By definition, Subgame-Outcome-Wait corresponds to the sequence $\{(a_t, a'_t)\}_{\{t=\Delta,2\Delta,\ldots,T\}} = \{(N,N),\ldots,(N,N)\}$. From the analysis of *Step 3.*, we know that the first element of this sequence being (N,N) is equivalent to the whole sequence of being composed of only (N,N). It then follows that since $P_H > \overline{P_H}(T,z)$ is the sufficient condition for the first element to be (N,N), it is the sufficient for the whole sequence to be composed of (N,N) and the necessary condition is the version with a weak inequality.

By definition, Subgame-Outcome-Semi-Wait corresponds to a sequence starting with the element $(a_\Delta, a'_\Delta) = (S,N)$, i.e., there exists at least one period $u \in \{\Delta, 2\Delta, \ldots\}$ such that $a_u = S$ and for all $t = \Delta, 2\Delta, \ldots$, $a'_t = N$. In other words, if demand is high, the seller does not want to wait for T periods to sell (but she may want to wait for $T - \Delta$ periods), and if demand is low the seller wants to wait for at least T periods. So the sufficient condition is $P_H < \overline{P_H}(T,z)$ and $P_H > \underline{P_H}(T,z)$.

By definition, Subgame-Outcome-No-Wait corresponds to a sequence starting with the element $(a_\Delta, a'_\Delta) = (S,S)$. I.e., there exists at least one period $t \in \{\Delta, 2\Delta, \ldots\}$ such that $a_t = S$ and at least one period $u \in \{\Delta, 2\Delta, \ldots\}$ such that $a'_u = S$. In other words, the seller does not want to wait for T periods regardless of demand. Since the condition under low demand is more binding, the sufficient condition is $P_H < \underline{P_H}(T,z)$.

$\square$

Proof of Lemma 7.

Proof. *Step 1.* We first define an auxiliary problem where the objective function is the same, but the constraint set is a continuous set $[0, \infty)$ instead of the discrete set of the original problem. Let's call it the *Continuous Constraint Problem*:

$$Max_T \quad \Pi_1^*(\Delta, T) \quad s.t. \quad T \in [0, \infty).$$

We will show that the problem has an interior solution.

First order derivative of the objective function wrt. T is:

$$f^1(T) \equiv$$
$$(-re^{-r(T+\Delta)} + (r+s)e^{-(r+s)(T+\Delta)})(L' - H'h) - (r+s)e^{-(r+s)(T+\Delta)}(L - Hh).$$

First order derivative evaluated at $T = 0$ can be rewritten as:

$$\begin{aligned} f^1(0) = & se^{-(r+s)\Delta}\frac{1 - e^{-r\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) \\ & + (r+s)e^{-(r+s)\Delta}[\frac{e^{-r\Delta} - e^{-(r+s)\Delta}}{1 - e^{-(r+s)\Delta}}(L' - H'h) - (L - Hh)] > 0 \end{aligned}$$

$f^1(T)$ goes to 0 as T goes to ∞ and the second order derivative $f^2(T)$ is strict positive when T is large enough.

To see this, define $a \in \Re$ by $a \equiv -\frac{L-Hh}{L'-H'h}$.

The second order derivative wrt. T is:

$$
\begin{aligned}
f^2(T) &= (r^2 e^{-r(T+\Delta)} - (r+s)^2 e^{-(r+s)(T+\Delta)})(L' - H'h) \\
&\quad + (r+s)^2 e^{-(r+s)(T+\Delta)}(L - Hh) \\
&= r^2 e^{-r(T+\Delta)}[1 - (1+a)\frac{(r+s)^2}{r^2} e^{-s(T+\Delta)}](L' - H'h). \qquad (.2)
\end{aligned}
$$

When T is large enough, the expression in the square bracket is strictly positive, therefore, the whole expression is strictly positive.

This implies that the first order derivative is negative when T is large enough.

Since the objective function is continuous, increasing in T for T small enough, and decreasing in T for T large enough, the maximizer(s) must exist and are interior (i.e., finite and strictly positive).

Step 2. We show that the objective function is quasi-concave in T. We only need to show $f^1(T) = 0 \Rightarrow f^2(T) \leq 0$:

$$
f^1(T) = 0 \Rightarrow f^2(T) = -rse^{-r(T+\Delta)}(L' - H'h) < 0.
$$

This implies that the set of maximizers of the Continuous Constraint Problem is a connected interval. For this interval not to be a singleton, it must be the case that the objective function is constant over an interval, which means that $f^2(T) = 0$ over an interval, which is impossible by expression (.2). Therefore, the set of maximizers must be a singleton for the Continuous Constraint Problem. Denote the unique maximizer by $\tilde{T}_1(\Delta) \in [0, \infty)$.

Then the Original Problem with the discrete constraint set has at most two maximizers, which are within Δ distance from $\tilde{T}_1(\Delta)$, i.e., $|T_1^*(\Delta) - \tilde{T}_1(\Delta)| \leq \Delta$ for any $T_1^*(\Delta) \in \Gamma_1(\Delta)$. Since $\tilde{T}_1(\Delta)$ is finite, this implies that $T_1^*(\Delta)$ exist(s) and is/are also finite.

Step 3. Now we define another auxiliary problem for parameters such that $L' > H'h$ and $\frac{s}{r+s}(L' - H'h) > L - Hh$:

$$Max_{T \in [0,\infty)} \quad \Pi_{lim}(T) \equiv L + (e^{-rT} - e^{-(r+s)T})(L' - H'h) - (1 - e^{-(r+s)T})(L - Hh).$$

We call it the *Limit Problem.* We denote it's maximizer by T^*, so the maximized value is $\Pi_{lim}(T^*)$. It is easy to see that T^* is unique and satisfies FOC:

$$e^{sT^*} = \frac{r+s}{r}(1 - \frac{L - Hh}{L' - H'h}) \Rightarrow T^* = \frac{1}{s}\ln(\frac{r+s}{r}(1 - \frac{L - Hh}{L' - H'h})) > 0.$$

Since $T^* > 0$ and the maximizer is unique, we have $\Pi_{lim}(T^*) > \Pi_{lim}(0) = \Pi_4$. Since $\Pi_3^*(\Delta) \to \Pi_4$ as $\Delta \to 0$, we have that when Δ is small enough, $\Pi_{lim}(T^*) > \Pi_3^*(\Delta)$.

Step 4. We now use Theorem of Maximum on the Continuous Constraint Problem, so $\tilde{T}_1(\Delta) \to T^*$ as $\Delta \to 0$ and the maximized value of the Continuous Constraint Problem converges to the maximized value of the Limit Problem, i.e., $\Pi_1^*(\Delta, \tilde{T}_1(\Delta)) \to \Pi_{lim}(T^*)$.

As $\Delta \to 0$, $|T_1^*(\Delta) - \tilde{T}_1(\Delta)| \leq \Delta$ and $\tilde{T}_1(\Delta) \to T^*$ imply $T_1^*(\Delta) \to T^*$ and the maximized value of the Original Problem converges to the maximized value of the Limit Problem $\Pi_{lim}(T^*) = \Pi_1^*(0, T^*)$. From Step 3, we conclude that $\Pi_1^*(\Delta, T_1^*(\Delta))$ is strictly greater than $\Pi_3^*(\Delta)$ and Π_4 when Δ is small enough. By Lemma 6, we also know that $\Pi_3^*(\Delta) > \Pi_2^*(\Delta, T_1^*(\Delta))$.

$\square$

Proof of Lemma 9.

Proof. Given a policy length with length T, we claim, when Δ is small enough, that if there is an equilibrium outcome for which on the equilibrium path H types buy at different moments given some demand realization, then it gives the seller strictly less profit than another outcome which is Outcome 1* for a different BP policy.

We only need to consider outcomes where the transaction timing is contingent on demand realization, because if it is not, then there exits another outcome, where H types buy together at the first moment that some H types buy, that is better for the seller, which can be sustained by a policy with a large enough refund scale.

Let's first consider the following outcome with transaction timing contingent on the realization of demand and with H types buy at potentially two different moments. It is never optimal to pay refunds to some H types while selling to some other H types, so we will look at outcomes where H types buy at moments that are out of the policy length from the previous H types.

The seller sells to some H types at $t = 0$, then at $t = T + \Delta$, if the demand is high there, she sells to the remaining H types there and then sells to all L types at $t = 2T + 2\Delta$ at their willingness-to-pay; if the demand is low at $t = T + \Delta$, she sells to all the remaining consumers (H and L types) at price L'. (The parameter case $L' > H'h$ implies that if demand drops, all the remaining consumers buy together at L'.)

Such an outcome is at its best if the mass of H types at $t = T + \Delta$ is just enough to make the seller indifferent between selling to all the remaining consumers together at price L and selling to H types first and delaying selling to L types until $T + \Delta$ time later. This is the best because it causes the least amount of discounting while delays the transaction with L types if demand keeps high. Let the mass of H types who buy at $t = 0$ be $\eta(\Delta, T)$, then it is defined by the following equality:

$$(H - e^{-(r+s)(T+\Delta)}H - (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})H')(h - \eta(\Delta, T)) =$$
$$(L - e^{-(r+s)(T+\Delta)}L - (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})L')(1 - \eta(\Delta, T)).$$

which gives us:

$$\eta(\Delta, T) = \frac{h - e(\Delta, T)}{1 - e(\Delta, T)}.$$

where

$$e(\Delta, T) \equiv \frac{L - e^{-(r+s)(T+\Delta)}L - (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})L'}{H - e^{-(r+s)(T+\Delta)}H - (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})H'} < h.$$

Since on the equilibrium path, each consumers is charged a price such that they are indifferent from accepting and rejecting, the profit of such an outcome is at most: [3]

$$\Pi_r(\Delta, T) \equiv$$
$$[H - e^{-(r+s)(T+\Delta)}H - (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})H'](\eta(\Delta, T) + e^{-(r+s)(T+\Delta)}h)$$
$$+ e^{-2(r+s)(T+\Delta)}L + (e^{-(2r+s)(T+\Delta)} - e^{-2(r+s)(T+\Delta)})L' + (e^{-r(T+\Delta)} - e^{-(r+s)(T+\Delta)})L'.$$

[3] Subscript “r” stands for randomization by H types, and subscript “p” stands for “pure”.

Now consider a different policy length $T' = 2T$, and consider refund scale large enough such that the following outcome is the Outcome 1* for such a BP policy: all H types buy together at $t = 0$, and L types buy at their willingness-to-pay at $t = 2T + \Delta$. It gives the following profit:

$$\Pi_p(\Delta, T) \equiv [H - e^{-(r+s)(2T+\Delta)}H - (e^{-r(2T+\Delta)} - e^{-(r+s)(2T+\Delta)})H']h +$$
$$e^{-(r+s)(2T+\Delta)}L(e^{-r(2T+\Delta)} - e^{-(r+s)(2T+\Delta)})L'.$$

Since we will compare profit levels when $\Delta \to 0$, we will directly work with the limit of profit $\Pi_r(\Delta, T)$ and $\Pi_p(\Delta, T)$ which we denote by $\Pi_r(T)$ and $\Pi_p(T)$ respectively, and let $\eta(T)$ be the limit of $\eta(\Delta, T)$:

$$\begin{aligned}\Pi_r(T) =&[H - e^{-(r+s)T}H - (e^{-rT} - e^{-(r+s)T})H'](\eta(T) + e^{-(r+s)T}h)+\\ &e^{-2(r+s)T}L + (e^{-(2r+s)T} - e^{-2(r+s)T})L' + (e^{-rT} - e^{-(r+s)T})L'\\ \Pi_p(T) =&[H - e^{-2(r+s)T}H - (e^{-2rT} - e^{-2(r+s)T})H']h+\\ &e^{-2(r+s)T}L(e^{-2rT} - e^{-2(r+s)T})L'.\end{aligned}$$

Since $\eta(T) < he^{-rT}$, after some algebra we get, for any $T > 0$:

$$\Pi_p(T) - \Pi_r(T) > [(1 - e^{-(r+s)T})H - (e^{-rT} - e^{-(r+s)T})L'](1 - e^{-rT}) > 0.$$

Therefore, $\Pi_p(T) - \Pi_r(T) > 0$ for any positive T.

Once we have established the result for the case where H types buy at two different moments on the equilibrium path, we can prove it for the case where H types buy at more than two different moments.

Suppose H types buy at three moments if demand keeps high: $t = 0$, $t = T + \Delta$ and $t = 2T + 2\Delta$. We can first show that such an outcome is worse than (when Δ is small enough) an alternative outcome which is the same as the original outcome except that at $t = T + \Delta$, if the demand is high, then all the remaining H types buy and L types do not buy until time $t = 2T + \Delta$. Then we can further show that this outcome is still worse than (when Δ is small enough) a third outcome where all H types buy at $t = 0$ and all L types buy at $t = 3T + \Delta$. Therefore, eventually, we have found an Outcome 1* given policy length $3T$ that performs better for the seller. This method works for any outcome where H types buy at multiple moments on the equilibrium path.

□

Proof of the existence of equilibrium in Lemma 10.

Proof. For $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$, let $x : (P, \Delta) \mapsto (0, 1)$ be defined by the following equality :

$$
\begin{aligned}
&H - P + x(P,\Delta)(e^{-(r+s)\Delta}(P - Q_1(\Delta, T_1^*(\Delta))) \\
&\qquad = x(P,\Delta)[e^{-(r+s)\Delta}(H - Q_1(\Delta, T_1^*(\Delta))) + e^{-rT_1^*(\Delta)}(1 - e^{-s\Delta})(H' - L')] \\
&\qquad\qquad + (1 - x(P,\Delta))[e^{-(r+s)T_1^*(\Delta)}(H - L) + e^{-rT_1^*(\Delta)}(1 - e^{-sT_1^*(\Delta)})(H' - L')].
\end{aligned}
$$

Notice that $x(P, \Delta) \in (0, 1)$ for any $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$. This is because $P < \bar{P}(\Delta)$ implies that the RHS is larger if $x(P, \Delta) = 1$, and $P > Q_1(\Delta, T_1^*(\Delta))$ implies that the RHS is smaller if $x(P, \Delta) = 0$.

We also define $\eta : (P, \Delta) \mapsto (0, h)$ for $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$ to be the mass of H types such that if they accept the price P alone at $t = 0$, then in the next period if the demand is high, the seller is indifferent between two outcomes in the subgame starting at time Δ given that demand is high at time Δ: i) selling at $Q_1(\Delta, T_1^*(\Delta))$ to the rest of the H types (while paying refunds to the earlier buyers) and then later to L types at $T_1^*(\Delta)+2\Delta$ at their willingness-to-pay, and ii) waiting until $T_1^*(\Delta)+\Delta$ to sell to the remaining consumers at L types' willingness to pay:

$$
\begin{aligned}
Q_1(\Delta, T_1^*(\Delta))h - P\eta(P, \Delta) + e^{-(r+s)(T_1^*(\Delta)+\Delta)}L(1-h) \\
+ e^{-r(T_1^*(\Delta)+\Delta)}(1 - e^{-s(T_1^*(\Delta)+\Delta)})L'(1-h) \\
= e^{-(r+s)T_1^*(\Delta)}L(1 - \eta(P, \Delta)) + e^{-rT_1^*(\Delta)}(1 - e^{-sT_1^*(\Delta)})L'(1 - \eta(P, \Delta)).
\end{aligned}
$$

Notice that $\eta(P, \Delta) \in (0, h)$ for any $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$. This is because the seller prefers the outcome of selling to the rest of H types at time Δ if $\eta(P, \Delta) = 0$, and prefers the outcome of waiting if $\eta(P, \Delta) = h$ for small enough Δ.

When $\Delta \to 0$, we have $[\bar{P}(\Delta) - Q_1(\Delta, T_1^*(\Delta))] \to 0$, so $\eta(P, \Delta) \to h$ from below for all $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$. This implies that when Δ is small enough, it is sequential rational for the seller to sell to all the remaining consumers at L types' willingness-to-pay at $T_1^*(\Delta) + \Delta$, instead of selling only to the remaining high types at $T_1^*(\Delta) + \Delta$.

Consider the following strategy profile:

H types' strategy: H types accept with probability one if the price offered P satisfies $P \leq Q_1(\Delta, T_1^*(\Delta))$ and if L types have not bought, and reject with probability one if $P \geq \bar{P}(\Delta)$, and accept with probability equal to $\eta(P,\Delta)$ if $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$, and accept price equal to or lower than their willingness-to-pay with probability one if L types have already bought.

L types' strategy: L types accept price equal to or lower than their willingness-to-pay with probability one.

The seller's strategy: The seller offers price $Q_1(\Delta, T_1^*(\Delta))$ if there is no previous transaction and demand is high. The seller offers price L' if there is no previous transaction and demand is low. The seller offers price equal to H for T/Δ consecutive periods regardless of demand realization after all H types have accepted. The seller offers L types willingness-to-pay if L types are alone in the market and the last transaction was at least T/Δ periods earlier. The seller offers $Q_1(\Delta, T_1^*(\Delta))$ with probability $x(P,\Delta)$ and offers an infinite price with the complementary probability if only part of H types bought in the last period and the last period price is P.

Consumers' incentives are easy to check. Notice that if one infinitesimally small H types deviates at $t = 0$ to not buy there, he will subsequently buy when demand is high before $T^*(\Delta) + \Delta$, because that entitles him to receive a refund later, but does not cause him any negative payoff for that period, but he will not buy if the demand drops. He gets an expected payoff equal to $e^{-(r+s)(T^*(\Delta)+\Delta)}(H - L) + e^{-r(T_1^*(\Delta)+\Delta)}(1 - e^{-s(T_1^*(\Delta)+\Delta)})(H' - L')$, where the first part comes from refunds and the second part comes from consumption. By the definition of $Q_1(\Delta, T_1^*(\Delta))$, this deviation profit is equal to (so not strictly better than) the equilibrium payoff.

Next we show that at $t = 0$, the seller does not want to deviate to a price above $Q_1(\Delta, T_1^*(\Delta))$ precisely due to $Q_1(\Delta, T_1^*(\Delta))h > P\eta(P, \Delta)$.

If the seller deviates to a price $P \in (Q_1(\Delta, T_1^*(\Delta)), \bar{P}(\Delta))$, then she gets profit:

$$P\eta(P,\Delta) + e^{-(r+s)(T_1^*(\Delta)+\Delta)}L(1-\eta(P,\Delta)) + e^{-r(T_1^*(\Delta)+\Delta)}(1-e^{-s(T_1^*(\Delta)+\Delta)})L'(1-\eta(P,\Delta))$$

If the seller charges $Q_1(\Delta, T_1^*(\Delta))$, then her profit is:

$$Q_1(\Delta, T_1^*(\Delta))h + e^{-(r+s)(T_1^*(\Delta)+\Delta)}L(1-h) + e^{-r(T_1^*(\Delta)+\Delta)}(1-e^{-s(T_1^*(\Delta)+\Delta)})L'(1-h)$$

$\square$

The difference is, by the definition of $\eta(P, \Delta)$:

$$[e^{-(r+s)T_1^*(\Delta)}L(1-\eta(P,\Delta)) + e^{-rT_1^*(\Delta)}(1-e^{-sT_1^*(\Delta)})L'(1-\eta(P,\Delta))] -$$
$$[e^{-(r+s)(T_1^*(\Delta)+\Delta)}L(1-\eta(P,\Delta)) + e^{-r(T_1^*(\Delta)+\Delta)}(1-e^{-s(T_1^*(\Delta)+\Delta)})L'(1-\eta(P,\Delta))] > 0.$$

There shows there is no incentive to raise price for the seller at $t = 0$.

Proof of Proposition 6.

Proof. Let $x \equiv \frac{r}{r+s}$ and $k \equiv 1 - \frac{L-Hh}{L'-H'h}$. Our interesting parameter case implies that $x < k$.

(1) First, we prove the result on the optimal policy length.

By an abuse of notation, we let T^* denote a mapping from s to the optimal policy length.

Taking derivatives with respect to s gives us:

$$\frac{dT^*}{ds} = -\frac{1}{s^2}\ln\left(\frac{r+s}{r}k\right) + \frac{1}{(r+s)s} = \frac{1}{s^2}[\ln(x) - \ln(k)] + \frac{1}{s^2}[1-x]$$

Notice that the first term is negative. It is the effect that lower expected dropping time leads to lower optimal delay. Algebra gives us that when $L' - H'h \in (\frac{r+s}{s}, \frac{1}{1-\frac{r}{r+s}e^{\frac{s}{r+s}}}(L - Hh))$ and $L - Hh > 0$, the sign of the derivative is positive, i.e., the second effect is dominating and the optimal policy length increases with s; when $L' - H'h \in (\frac{1}{1-\frac{r}{r+s}e^{\frac{s}{r+s}}}(L - Hh), \infty)$, the sign of the derivative is negative, i.e., the first effect is dominating and the optimal policy length decreases with s, which includes the whole region I of our interesting parameter case.

(2) Second, we prove the result on price at the beginning.

The expression for the price at the beginning is thus:

$$\begin{aligned}
\lim_{\Delta\to 0}\frac{dQ_1(\Delta, T^*(\Delta))}{dx} &= \lim_{\Delta\to 0}\frac{d}{dx}(H - e^{-(r+s)(T^*(\Delta)+\Delta)}(H-L)) \\
&= -\frac{1}{(1-x)^2 x}\frac{k}{x}^{\frac{1}{1-x}}(\ln\left(\frac{k}{x}\right)x - 1 + x)[(H-L) - (H'-L')] \\
&- \frac{1}{(1-x)^2}\frac{k}{x}^{\frac{x}{1-x}}(\ln\left(\frac{k}{x}\right)x - 1 + x)[H' - L']
\end{aligned}$$

Therefore, when k is big enough (or in other words, when h is big enough keeping all else equal), the price is decreasing in x, or in other words, increasing in s.

(3) Finally, we prove the result on the equilibrium profit.

Directly from the optimization problem with respect to $T(\Delta)$, we get:

$$\frac{d}{dx}\Pi_1^*(\Delta, T^*(\Delta)) = (T^*(\Delta)+\Delta)e^{-(r+s)(T^*(\Delta)+\Delta)}[(L'-H'h)-(L-Hh)] > 0$$

$\square$

Proof of Proposition 8.

Proof. Recall that all consumers' equilibrium strategy must specify accepting a price equal to or low than L with probability one regardless of history as long as demand is high.

Suppose that the consumers have the following off (potential)-equilibrium behavior at time $t = 0$: they accept with probability one if the deviation price belongs to $[0, L)\bigcup(L,\ L+(1-e^{-(r+s)\Delta})(H-L)+e^{-r\Delta}(1-e^{-s\Delta})(H'-L'))$ and they reject with probability one if the deviation price belongs to $[L+(1-e^{-(r+s)\Delta})(H-L)+e^{-r\Delta}(1-e^{-s\Delta})(H'-L'), \infty)$. The threshold price between accepting and rejecting is such that it makes H types indifferent between accepting and rejecting if they expect the next period price equals to L types' willingness-to-pay. Given such consumer strategy, if the seller has no incentive to deviate, then we have shown that pricing at L and selling to all is an equilibrium outcome.

Let's consider the possible pure strategy deviations for the seller. If there is no strictly positive pure deviations, then there is no profitable mixed deviations as well.

Deviating to a price below L is clearly not profitable, because all will accept that price and the profit is lower.

Now suppose the seller deviates to a price in $(L,\ L+(1-e^{-(r+s)\Delta})(H-L)+e^{-r\Delta}(1-e^{-s\Delta})(H'-L'))$, where H types accept with probability one, but all low types reject with probability one. Notice that when $\Delta \to 0$, $L + (1 - e^{-(r+s)\Delta})(H - L) + e^{-r\Delta}(1 - e^{-s\Delta})(H' - L') \to L$, while $T > 0$ implies that $\lim_{\Delta \to 0} \overline{P_H}(T, 1) > L$. In other words, when Δ is small enough, the deviation price is strictly below $\overline{P_H}(T, 1)$. Therefore, when H types accept this price with probability one, there are two subgame possibilities: Subgame-Outcome-Semi-Wait and Subgame-Outcome-No-Wait (defined in Proposition 7). Therefore the deviation profits are bounded by Outcome 2^* and Outcome 3^*. Next, we will show that these profit bounds are even lower than L when Δ is small enough, i.e., they are not profitable deviations:

$$\begin{aligned} &\overline{P_H}(T,1)h + e^{-(r+s)\Delta}(L - \overline{P_H}(T,1)h) + e^{-r(T+\Delta)}(1 - e^{-s\Delta})L'(1-h) \\ &= L + e^{-r(T+\Delta)}(1-e^{-s\Delta})L'(1-h) - e^{-r(T+\Delta)}(1-e^{-(r+s)\Delta})[(1-e^{-s\Delta})L' + e^{-s\Delta}L](1-h) < L \end{aligned}$$

$$\begin{aligned} &\underline{P_H}(T,1)h + e^{-(r+s)\Delta}(L - \underline{P_H}(T,1)h) + e^{-r\Delta}(1 - e^{-s\Delta})(L' - \underline{P_H}(T,1)h) \\ &= (1 - e^{-(r+s)\Delta} - e^{-r\Delta}(1 - e^{-s\Delta}))[L' - e^{-rT}L'(1-h)] + e^{-(r+s)\Delta}L + e^{-r\Delta}(1 - e^{-s\Delta})L' < L \end{aligned}$$

$\square$

APPENDIX

Appendix for Chapter 2

Proof of Lemma 12

Proof. We prove by contraction. There are three cases we need to contradict: 1) a symmetric equilibrium where AU and BU do not buy in the first period, but AC buy from A and BC buy from B; 2) a symmetric equilibrium where AC and BC do not buy in the first period, but AU buy from A and BU buy from B. 3) a symmetric equilibrium where no consumer buys in the first period.

Case 1. Suppose AU and BU do not buy in the first period, but but AC buy from A and BC buy from B. For AU and BU to not buy in the first period, it must be that they want to take advantage of the lower price offered to new customers by their preferred seller. However, that implies AC and BC will not buy either, because they can benefit more from lower price offered to new customers by their preferred seller. This forms a contradiction.

Case 2. Suppose AC and BC do not buy in the first period, but AU buy from A and BU buy from B. This means that in the second period, turf A, turf B and turf N are all symmetric, so there will be no difference between the price offered to new and old customers, so for AC consumers to be willing to not buy and AU consumers to be willing to buy in the first period, it must be the case that first period P_A^S must exactly extract all the first period surplus, i.e. $P_A^S = 1 + \frac{t}{4}$. Then the seller can deviate to charge a price slightly below $1 + \frac{t}{4} - \frac{t}{2}$. Following such a deviation, the unique subgame equilibrium is such that all consumers buy from seller A, resulting in a profit of $(1-\frac{t}{4})+\frac{t}{2}$, which is a profitable deviation.

Case 3. Suppose no consumer buy in the first period. The second period prices will all be t and all consumers' second period surplus is $1-\frac{3}{4}t$. We claim that if A deviates to a price less than $\frac{t}{2}$, then the unique subgame equilibrium is that all consumers buy from her, and this improves her profit because it does not depress second period prices, but also allows her to get positive first period profit. The hardest to get consumers for A in the first period is BC. Their worst fear of buying A's ST contract is to lose all future surplus. But $P_A^S<\frac{t}{2}$ implies that $(1-\frac{t}{4})-P_A^S>1-\frac{3}{4}t$ Therefore, this is a profitable deviation, which forms a contradiction.

□

Proof of Lemma 13

Proof. We need to rule out 8 outcomes as impossible to appear on equilibrium: 1) No one buys in the first period. 2) AU and BU buy ST contracts in the first period, but AC and BC do not buy. 3) AC and BC buy ST contracts in the first period, but AU and BU do not buy. 4) All buy ST contracts in the first period. 5) AU and BU buy LT contracts in the first period, but AC and BC do not buy. 6) AC and BC buy LT contracts in the first period, but AU and BU do not buy. 7) all buy LT contracts 8) AU and BU buy LT contracts, while AC and BC buy ST contracts in the first period. The first three cases can be contradicted by the same logics as those in the proof of Lemma 12.

Case 4. AU and AC buy from A's ST contract and BU and BC buy from B's ST contract. Suppose the contracts are priced at Q^S and Q^L. The fact that no one buys LT contract implies that $(1+\frac{t}{4})+U_{AC}(\frac{1+e}{3(1-e)}t,\frac{2-e}{3(1-e)}t)-Q^S\geq 1+\frac{t}{4}+U_{AC}(t,t)-Q^L$.

Suppose seller A deviates by charging the same price for her ST contract, but reduces the LT contract price such that $Q_A'^L$ is just slightly below the threshold that will make the above inequality an equality. After such a deviation, the unique subgame equilibrium is that BU and BC keep on buying B's ST contract, AU buy A's ST contract, but AC buy A's LT contract. This is a profitable deviation for the seller, because it makes turf A more symmetric and increases prices on turf A.

Case 5. AU and BU buy LT contracts in the first period, but AC and BC do not buy. This is impossible because AC and BC will deviate to buy the LT contracts given that LT contracts meet the rationality condition of AU and BU consumers.

Case 6. AC and BC buy LT contracts in the first period, but AU and BU do not buy. Seller A can deviate by reducing price of her ST contract so that AU and BU consumer buy her ST contract instead of not buying, which is a profitable deviation.

Case 7. All consumers buy from LT contracts. Suppose such an equilibrium exists. Since if A deviates to a long-term contract price of $Q_A^L - t$ and a short-term contract of infinity, then there is unique subgame equilibrium that all consumers buy from A. Since this is not a profitable deviation, we must have $Q_A^L - t \leq \frac{1}{2}Q_A^L \Rightarrow Q_A^L \leq 2t$. However, if $Q_A^L \leq t$, A can deviate to a LT price slightly less than $Q_A^L + \frac{t}{2}$ and makes ST price infinity. Then after such a deviation, AU and AC consumers will still buy from A, so this is a profitable deviation. This forms a contradiction.

Case 8. AU and BU buy LT contracts, while AC and BC buy ST contracts in the first period. This is impossible because given any two LT and ST contracts of seller A, if AU consumers prefer LT to ST contract, then AC consumers must strictly prefer LT to ST contract.

$\square$