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ABSTRACT

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Chapter 1 examines the question of how to sell a firm when potential buyers do not know how many other potential buyers there are. The seller can choose to sell the firm either through bilateral negotiations or through an auction. In equilibrium, if the seller observes the number of buyers before choosing the mechanism, the choice can signal information about the number of buyers and lower expected revenue. Broadly speaking, the seller chooses an auction if the expected number of buyers is high and negotiations otherwise. Empirical implications of the theory are (i) The revenue is higher if buyer valuations are less volatile; (ii) More risk-averse sellers choose auctions more often; (iii) If the seller risk-aversion is above (below) a threshold value, the average transaction price in auctions is greater (less) than that in negotiations. Committing to a mechanism before seeing the number of buyers increases the seller's revenue.

Chapter 2 considers the problem of optimal contracting with an M&A Advisor during the sale of a firm. Both the buyer and the seller are uninformed about the value of synergies, but they can hire an M&A Advisor. Suppose, though, that the seller and

buyer face a moral hazard problem. If the advisor's effort is not observable, he has the option of not exerting effort and reporting any of the possible values. Should the seller and buyer hire an advisor and what is the optimal contract that they should sign with him? We find that the probabilities with which the buyer and seller hire their advisors and the optimal contracts are determined simultaneously in equilibrium. Both contracts depend on two variables- whether the transaction succeeds or not and, if it does, the value of the transaction. The seller's optimal contract with his advisor is unique, but the buyer's optimal contract can take a variety of forms. The compensation of the seller's advisor is monotonically increasing in the transaction value. Neither advisor is paid if the transaction fails. In equilibrium, both advisors exert effort, report truthfully and do not extract any information rents. However, the first best is not obtained because the transaction can fail even though it is socially optimal.

Chapter 3 studies how merger decisions between public firms in the US are affected by the similarity between the product markets of the acquirer and the potential target. The relation between the likelihood of the merger and the product market similarity is non-monotonic, in the shape of an inverted U. We offer two reasons for this finding. First, when the product markets are very similar, there is a high chance that antitrust investigations will block the merger. We find that this effect is stronger in markets that are more concentrated and in years where antitrust regulatory intensity is high. Second, the synergies from the merger are less if the product markets are very related. Hence, firms are more likely to acquire targets with which they have a medium rather than a high level of product market similarity.

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CHAPTER 1

Auctions or Negotiations? A Theory of How Firms are Sold

1.1. Introduction

Consider a situation where a firm faces multiple buyers interested in purchasing it. There are two ways to sell the firm. The seller can either run an auction or conduct exclusive bilateral negotiations. Which of these would the seller prefer? This paper develops a theory that answers this question and provides empirical implications of the theory.

Recent studies show that neither auctions nor negotiations are universally chosen, with firm sales almost equally split between the two.[1] Another feature of firm sales is that the process is often completed before the merger is publicly announced.[2] Very often, buyers sign confidentiality agreements with the seller and are uncertain about the number

[1]For instance, among recent studies, Boone and Mulherin (2007) conclude that 51% of the firms in their sample were sold through auctions, while Aktas et al. (2010) find that 52% were auctions. They classify a transaction as an auction if multiple potential buyers are mentioned in the merger background section of the SEC filings, and as a negotiation when there is only one buyer mentioned.

[2]To understand the distinction between the public part of the sale which happens after the merger is announced and that which happens before, contrast the results obtained by Boone and Mulherin (2007) and Aktas et al. (2010) with those from prior studies. Earlier studies classified a sale an auction if there was more than one publicly announced bidder *after the takeover is publicly announced*. For example, Moeller et al. (2004) find that only 1.21% of this sample were public auctions with multiple bidders, while Betton et al. (2008) report that only around 3.4% were public auctions. Applying this criterion, one would conclude that few targets are sold in an auction. Hence, it is imperative to look at the private part of the takeover process before the public announcement of the merger to provide a better classification.

of other buyers taking part. The uncertainty is significant if the number of participating buyers is low, which is precisely the case in corporate takeovers.[3]

Motivated by these empirical observations, I modify the standard framework in mechanism design to one where the buyers take part in the sale process without knowing how many other buyers are participating. Each buyer's belief about the competition is captured by two parameters - the maximum number of buyers who may enter the sale process and the probability with which each enters. These are common knowledge to the buyers and the seller. In addition, the seller observes the actual number of buyers that participate in the process. For most of the analysis, I assume a common values setting where the value of synergies in the transaction is the same for all the buyers.

Hence, the setting I consider has two-sided information asymmetry. The buyers know their valuations, but the seller doesn't. However, the seller knows the number of buyers taking part in the sale process, which the buyers do not. The question is how far the information advantage of the seller helps him mitigate the usual information disadvantage about buyer valuations in any selling mechanism. In this aspect, the informed principal is similar to that in Maskin and Tirole (1990, 1992).

In my model, the key difference between an auction and a negotiation is as follows. In an auction, the outcome depends on the bids of all the buyers participating in the process. In contrast, in a negotiation between the seller and a buyer, the outcome of the negotiation is independent of any other buyer's actions. I model these mechanisms in this

[3]Boone and Mulherin (2007) report that in the auctions, the average number of bidders was 1.57. In many of the auctions, only one buyer submitted the final bid even though many buyers were contacted. The median number of buyers which submitted a final bid was in fact 1. It is worth mentioning that they only consider listed targets. The identity and number of the bidders are likely to be even more uncertain if the targets are unlisted.

manner for two reasons. First, this lets me capture the exclusivity inherent in negotiations where the competitive pressure is across the table between the seller and the buyer, in contrast to an auction where the competitive pressure is between buyers who are on the same side of the table among buyers. Second, it ties my model to empirical studies on the sale of firms where a takeover is classified as an auction if multiple potential buyers are mentioned in the filings and a negotiation when there is only a single buyer mentioned.

I assume that the auction is a first-price auction with a reserve price equal to the standalone value of the firm. Modelling negotiations also requires a few assumptions.[4] I assume that a negotiation involves a take-it-or-leave-it offer made by the seller. If a buyer rejects an offer during negotiations, the seller moves on to the next buyer and never goes back to it. This makes the take-it-or-leave-it offer credible. The result of the negotiation thus depends only on whether an agreement is reached between the seller and the buyer involved in the negotiation. Unlike in an auction, a competing buyer's strategy will not affect the result of a negotiation.

The main findings are as follows. The seller does not benefit from the ability to observe the number of buyers before choosing the mechanism. The buyers make inferences about the total number of buyers from the mechanism chosen by the seller. Because of the buyer inference, the seller cannot increase his revenue by making the choice between auctions and negotiations dependent on the number of buyers. There are multiple equilibria. In the best case for the seller, the seller does does not benefit from observing the number

[4]Gentry and Stroup (2017) point out that negotiations *"can take a variety of observationally indistinguishable forms... An observed single bidder sale could, for example, reflect a successful one-shot negotiation or a successful first stage in a sequential negotiation, among other possibilities."*

of buyers and in the worst case, his revenue decreases when he observes the number of buyers.

Thus, in equilibrium, the seller does not employ his knowledge to increase his payoff. But if this is the case, why should the seller not commit to choose the same mechanism irrespective of the actual number of buyers he observes? I show that if the seller can commit ex ante to choosing an auction or a negotiation irrespective of the number of buyers, the seller can increase the expected revenue. In the equilibria with commitment, the seller chooses negotiations up to a threshold probability of entry and chooses auctions above the threshold.

The threshold probability above which auctions are chosen decreases as the seller becomes more risk-averse or the variance of buyer valuations increases. The expected transaction price conditional on the sale being a negotiation is higher than that conditional on an auction for low levels of the relative risk aversion of the seller. If the seller's risk aversion is high, the transaction price in auctions is higher than in negotiations. The volatility of the price conditional on auctions is higher than that in negotiations.

I contribute to various strands of research. The central question in the paper is the choice between negotiations and auctions. I add two elements, which are very relevant in the context of the sale of firms, to previous studies examining the choice between auctions and negotiations. First, I assume that the buyers do not know the number of other buyers participating, a realistic assumption to model corporate takeovers. Second, I adapt the usual framework used to model negotiations to capture the exclusivity inherent in the negotiations during the sale of firms.[5]

[5]In many studies, the only difference between auctions and negotiations seem to be that auctions are simultaneous and negotiations sequential.

These two elements add to and modify some of the results obtained by previous theoretical models in the literature. For example, Bulow and Klemperer (1996) concludes that sellers almost always choose auctions since additional competition (i.e. the presence of an extra bidder in an auction) is preferable to a negotiation. However, since the "negotiation" in their analysis is actually an optimal auction and the "auction" a second price auction, their conclusion is actually a comparison of an optimal auction to a second price auction with an extra bidder. It is not surprising that in reality, empirical studies have repeatedly found that takeover auctions are much less prevalent than they suggest and negotiations much more common. Bulow and Klemperer (2009) study the choice between a simultaneous auction, and a sequential process in which potential buyers decide in turn whether to enter the bidding, which they call "negotiations". My model differs from theirs since I assume an exogenous participation probability independent of whether the seller chooses an auction or negotiations. This enables me to show that even if the probability of entry is the same in an auction and negotiations, sellers may prefer one mechanism to the other.

The second contribution is to the literature that characterises equilibria in games where the principal possesses information that the agent does not. The "principal" corresponds to the seller in my model and "agent" to the buyers. The seller has information on the number of buyers, which the buyers do not know. The informed principal is similar to that in Maskin and Tirole (1990, 1992). I give the principal all the bargaining power in choosing the mechanism as in Maskin and Tirole (1992). Also, like in their game, the agent's expected payoff depends on his interim beliefs about the principal's type, where the interim beliefs are obtained by updating the prior beliefs using the information conveyed by the prinicipal's contract proposal. My setting is more restrictive than theirs since the principal is just choosing between two specific mechanisms. In addition, the private information the principal has is of a very specific kind, about the number of agents. I find that in this setting, the information asymmetry can lead to equilibria where the seller is *in fact at a disadvantage* due to possessing information.

I also contribute to the research on auctions by studying the bidding strategies in a setting with an unknown number of bidders when the values are common. Previous studies which look at auctions with an unknown number of bidders, for example McAfee and McMillan (1987) and Harstad et al. (1990), derive bidding strategies and seller revenues if the bidders have independent values. I derive closed-form expressions for the bidding strategies and seller revenues in a case when the bidder valuations are not independent but (perfectly) correlated. In addition, unlike these studies which assume that the seller always chooses an auction, I show that the presence of unknown number of buyers can lead the seller to choose negotiations.

The empirical implications of my paper can be compared with the results found in empirical studies of takeovers, for instance Boone and Mulherin (2007) and Aktas et al. (2010). They find that sellers employ both auctions and negotiations frequently. This can be explained by the variation in the competition, risk aversion and number of possible buyers, both across industries and temporally, leading sellers to choose either mechanism depending on the parameter values. Mulherin and Womack (2015) find that private auctions exist even in the takeover market for REITs. The median number of bidders in their sample is one. Therefore, it is reasonable to suppose that the REIT takeover market is also characterized by uncertainty about the number of bidders. They report that larger firms choose to negotiate more often than smaller firms, possibly because the sellers of larger firms are less risk-averse. This is consistent with what my model predicts about the effect of seller's risk aversion on negotiations. In addition to being consistent with existing empirical research, I also provide directions for future empirical research.

I analyse the effect of the volatility of buyer valuations on negotiations, which goes some way towards explaining another puzzling phenomenon frequently reported in studies of takeovers. Many studies (Chang (1998), Moeller et al. (2004) and Faccio et al. (2006) to name a few) find that the acquisitions of private targets are associated with higher abnormal returns for the acquirer than acquisitions of public targets. Officer et al. (2008) also documents that the returns to acquirers are higher when a target is difficult to value irrespective of the target's public status. While public and private targets may differ on many dimensions (size, for example), one of the most significant differences between them is how volatile the target's value is. My analysis shows that more volatile (or private) targets can extract less of the surplus in negotiations. Hence, acquiring private targets through negotiations will benefit the acquirer.

The rest of the paper is organised as follows. Section 1.2 introduces the baseline model. I derive the optimal offers made by the seller in the sequential negotiations subgame, the buyers' bidding strategies in the auction subgame, and then use these results to describe the equilibria of the overall game. Section 1.3 suggests a way for the seller to increase the expected revenue by credibly committing ex ante to choose one of the two mechanisms. Section 1.4 considers extensions to the basic model where the buyer valuations are independent rather than correlated, and also considers the effect of the volatility of buyer valuations on the choice of the mechanism. Section 1.5 outlines empirical implications of the theory. Section 1.6 concludes.

1.2. The Baseline Model Setup

A risk-averse seller is trying to sell a firm of unknown value to a set of unknown buyers. The seller and buyer are both risk-averse, with CRRA utility function. The relative risk aversion of the seller is γ_s and that of the buyer γ_b with both γ_s and γ_b less than 1. The number of potential buyers participating in the sale of the firm is n. Here, n can be interpreted as a measure of asset-specificity of the firm being sold. The more specific the assets of the firm, the lesser is the number of potential buyers. Each buyer participates in the sale with probability p. Here, p measures how competitive the sales process is. I assume that p is exogenous. Both n and p are common knowledge. The seller also observes the number of buyers who actually participate, m. None of the buyers observe m. The seller chooses a mechanism after he observes m.

The value of the object to the i^{th} buyer is V_i ($i = 1, 2, \ldots m$ for $m > 0$).[6] Each buyer knows their value. The seller does not know any of the values, but knows the distribution of the values. I consider two cases:

(a) Common values - The values are the same. $V_i = V$ with $V \sim U[0, 1]$. This is the case I consider in the baseline model.

(b) Independent private values - The values are *iid*. $V_i \sim U[0, 1]$.

In negotiations, the seller proceeds sequentially. In each round of the negotiations, the seller picks one of the buyers randomly and makes a take-it-or-leave-it offer. If the offer is accepted, the negotiations end. If the offer is refused, the seller goes on to negotiate

[6] If $m = 0$, the firm is not sold.

with the next buyer, if there is one. If not, the negotiations end. The seller does not go back to a buyer who previously refused an offer.

In an auction, each buyer submits a sealed bid. It is a first- price auction with a reserve price equal to the standalone value of the firm.

I solve the game as follows. I first solve the negotiations subgame and the auctions subgame separately. Then, I solve the supergame where the seller makes the choice of mechanism by backward induction.

1.2.1. The Negotiations Subgame

With negotiations, the buyer's strategy is to accept the seller's offer if it is less than the value V_i. The buyer's strategy is independent of the beliefs about m, the number of other buyers participating.

Let $W(k, a)$ denote the expected utility for the seller from optimal sequential negotiations with k buyers when $V \sim U[0, a]$. Denote $W(k, 1)$ by W_k for the sake of simplicity.

Lemma 1. *The seller's expected utility from negotiations with k buyers when $V \sim U[0, a]$ is homogenous of degree $1 - \gamma_s$ in a, that is, $W(k, a) = W_k a^{1-\gamma_s}$*

Proof. First consider the case when $k = 1$. If a price $x \leq a$ is offered, the offer is accepted when $V \geq x$, which happens with probability $1 - \frac{x}{a}$. The seller's utility conditional on acceptance is $\frac{x^{1-\gamma_s}}{1-\gamma_s}$. So, the optimal offer price x_1^* maximises

$$W(1, a) = \underset{x \in [0,a]}{Max} \left(\frac{x^{1-\gamma_s}}{1 - \gamma_s} (1 - \frac{x}{a}) \right) \tag{1.1}$$

Differentiating and solving for x_1^* gives

$$x_1^* = \frac{1-\gamma_s}{2-\gamma_s}a \tag{1.2}$$

Substituting in equation 1.1 gives

$$W(1,a) = \frac{(1-\gamma_s)^{1-\gamma_s}}{(2-\gamma_s)^{2-\gamma_s}} \frac{a^{1-\gamma_s}}{1-\gamma_s} \tag{1.3}$$

This is indeed of the form $W_1 a^{1-\gamma_s}$ where $W_1 = \left(\frac{1-\gamma_s}{2-\gamma_s}\right)^{1-\gamma_s} \frac{1}{1-\gamma_s}$, so lemma 1 holds for $m = 1$. Now, I prove that if lemma 1 is true for $k = l-1$, it is also true for $k = l$. Consider the first stage of the negotiation when there are l buyers. An offer of x in the first stage is accepted when $V_i \geq x$, which happens with probability $1 - \frac{x}{a}$. The seller's utility conditional on acceptance is $\frac{x^{1-\gamma_s}}{1-\gamma_s}$. If the offer is rejected, which happens with probabililty $\frac{x}{a}$, there are $l-1$ buyers left to negotiate with. We also know that conditional on the offer being rejected, $V \sim U[0,x]$. So, the expected seller utility conditional on rejection is obtained by substituting $k = l-1$ and $a = x$ in $W(k,a)$, that is, $W(l-1,x)$. This gives us the recursive equation

$$W(l,a) = \underset{x\in[0,a]}{Max}\left(\frac{x^{1-\gamma_s}}{1-\gamma_s}(1-\frac{x}{a}) + W(l-1,x)\frac{x}{a}\right) \tag{1.4}$$

By assumption, lemma 1 is true for $k = l-1$. In other words,$W(l-1,x) = V_{l-1}x^{1-\gamma_s}$. Substituting in equation 1.4, we have

$$W(l,a) = \underset{x\in[0,a]}{Max}\left(\frac{x^{1-\gamma_s}}{1-\gamma_s}(1-\frac{x}{a}) + W_{l-1}x^{1-\gamma_s}\frac{x}{a}\right) \tag{1.5}$$

Differentiating equation 1.5 w.r.t. x and solving for the optimal offer, we have

$$x(l,a) = \frac{1-\gamma_s}{2-\gamma_s}\frac{a}{1-(1-\gamma_s)W_{l-1}} \tag{1.6}$$

Substituting back in equation 1.5, we get that

$$W(l,a) = \left(\frac{1-\gamma_s}{2-\gamma_s}\frac{1}{1-(1-\gamma_s)W_{l-1}}\right)^{1-\gamma_s}\left(\frac{1}{2-\gamma_s}\right)\frac{a^{1-\gamma_s}}{1-\gamma_s} = W_l a^{1-\gamma_s} \tag{1.7}$$

This is indeed of the form $W_l a^{1-\gamma_s}$ where $W_l = \frac{(1-\gamma_s)^{1-\gamma_s}}{(2-\gamma_s)^{2-\gamma_s}}\left(\frac{1}{1-(1-\gamma_s)V_{k-1}}\right)^{1-\gamma_s}\frac{1}{1-\gamma_s}$. In other words, lemma 1 is also true for $k = l$. Since lemma 1 is true for $k = 1$, this means that by induction, lemma 1 is true for all k. □

Theorem 1. *The expected utility for the seller from optimal sequential negotiations with k buyers when $V \sim U[0,1]$ is given by the recursion*

$$W_k = \frac{1}{(1-\gamma_s)^{\gamma_s}(2-\gamma_s)^{2-\gamma_s}}\left(\frac{1}{1-(1-\gamma_s)W_{k-1}}\right)^{1-\gamma_s} \tag{1.8}$$

with the boundary condition

$$W_0 = 0 \tag{1.9}$$

The offer in the i^{th} round of the negotiations is given by

$$x(k,i) = ((1-\gamma_s)(2-\gamma_s))^{\frac{i}{1-\gamma_s}}\prod_{j=0}^{i-1} W_{k-j}^{\frac{1}{1-\gamma_s}} \tag{1.10}$$

Proof. The proof follows from lemma 1, specifically equations 1.6 and 1.7, by substituting $a = 1$. □

Panel A of figure 1.1 plots the optimal offers in each round when the number of buyers is 4 as a function of the seller's coefficient of relative risk aversion. The graph shows that as the seller becomes more risk-averse, the offers in each round decrease. The intuition behind this is that as the risk-aversion increases, the seller increasingly prefers selling the firm even for a low price rather than being left with the firm as a result of the negotiations breaking down. So, the seller offers low prices to increase the probability of the transaction going through.

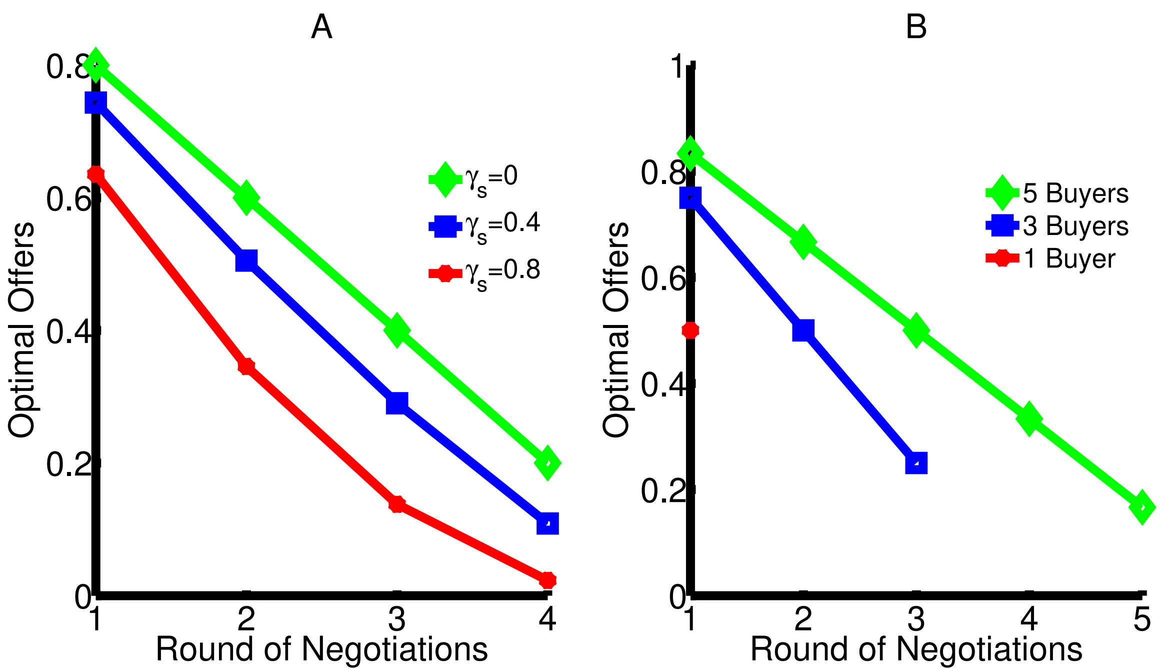

Figure 1.1. The optimal offers in each round of negotiations
Panel A shows the offers when the number of buyers is 4 for varying levels of the seller's coefficient of relative risk aversion. Panel B plots the offers made by a risk-neutral seller for varying number of buyers.

If the seller is risk-neutral, the expressions for the seller utility and the offers in each round simplify considerably.

Corollary 1. *If the seller is risk-neutral, the expected utility for the seller from optimal sequential negotiations with* m *buyers is given by*

$$W_m = \frac{1}{2}\frac{m}{m+1} \tag{1.11}$$

The offer in the i^{th} *round is given by*

$$x(m,i) = \frac{m+1-i}{m+1} \tag{1.12}$$

Proof. Risk-neutrality corresponds to the case where $\gamma_s = 0$. Substituting $\gamma_s = 0$ in equation 1.8, we obtain

$$V_m = \frac{1}{4(1-V_{m-1})} \tag{1.13}$$

with the boundary condition $V_0 = 0$. The last step consists in showing that this simplifies to $\frac{1}{2}\frac{m}{m+1}$. The proof, by induction, is provided in Appendix A.1. □

The i possible offers that the seller makes in the m rounds of the negotiations are equally spaced in the range of possible values $(0,1)$, with a maximum offer of $\frac{m}{m+1}$ in the first round and a minimum offer of $\frac{1}{m+1}$ in the m^{th} round.

Panel B of figure 1.1 plots the offers made by a risk-neutral seller in each round of the negotiations. Separate graphs are plotted for different values of k, the realised number of buyers in the sale. The seller starts by offering a high price in the initial rounds of the negotiation. In the later rounds, he offers lower prices since he learns that the value is less than the amount offer rejected by the buyers in the earlier rounds. The information revelation in negotiations is thus gradual in contrast to an auction where it is a simultaneous process.[7]

The range of offers made by the seller increases as the number of buyers increases because the maximum offer increases and the minimum offer decreases. As the number of buyers becomes very large, the maximum approaches 1 and the minimum 0. In the limit, the seller is able to offer prices ranging from 0 to 1, the entire range of possible values. Figure 1.1 shows that even in negotiations, as the competition increases, the seller can extract more and more surplus.

Figure 1.2 shows how the expected revenue of the transaction varies with the number of buyers m for a risk-neutral seller. As $m \to \infty$, there is perfect learning, and the seller is able to realize all the value of synergies, which equal 0.5 in expectation. Thus, as the number of buyers increases, the informational rent extracted by them decreases. Conditional on m, the seller's expected revenue does not depend on n and p, the maximum number of bidders and their chance of participation.

1.2.2. The Auctions Subgame

I construct a symmetric equilibrium where each bidder randomizes his bid x in the interval $[0, \bar{V}(n, p, \gamma_b)]$ for some maximum bid $\bar{V}(n, p, \gamma_b)$. The cumulative distribution function of x, $F(x)$, can have neither gaps nor atoms in this interval. A sketch of the reasoning is as follows.

[7]In this model, gradual revelation of information does not make a difference since there is no discounting and nothing happens between two rounds of negotiation. But in a richer model where firms decide to enter based on the history of the previous round of negotiations, it does make a difference.

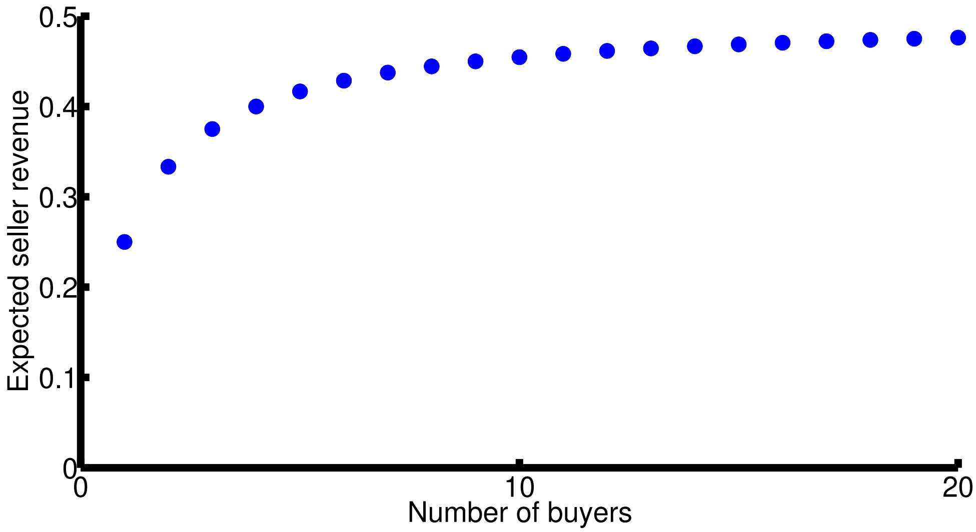

Figure 1.2. Expected seller revenue from negotiations

If there is a gap, it is not optimal to bid just above the gap. Decreasing the bid by ϵ keeps the probability of winning constant, but makes the payment on winning lesser. So, the gap would decrease till it shrinks to zero.

There cannot be an atom because in the presence of an atom in the other bidders' CDFs, a bidder would not bid in a suitably chosen small interval below the atom. Shifting his bid from this interval to another just above the atom would decrease his payoff only by an order less than it increases his probability of winning. So, he would prefer to bid above the atom. But this leads to a gap which we have already proved is impossible. So, the bidding is continuous in the interval $[0, \bar{V}(n, p, \gamma_b)]$.

Lemma 2. *$\bar{V}(n, p, \gamma_b)$, the maximum bid, is $(1-(1-p)^{\frac{n-1}{1-\gamma_b}})V$*

Proof. Bidding 0 gives the buyer an expected payoff of $(1-p)^{n-1}\frac{V^{1-\gamma_b}}{1-\gamma_b}$. So, no buyer would bid above $(1-(1-p)^{\frac{n-1}{1-\gamma_b}})V$ since even if he wins the auction with certainty, his payoff will not be greater than $(1-p)^{n-1}\frac{V^{1-\gamma_b}}{1-\gamma_b}$ which he could have got by bidding 0.

Hence, $\bar{V}(n,p,\gamma_b) \leq (1-(1-p)^{\frac{n-1}{1-\gamma_b}})V$. If $\bar{V}(n,p,\gamma_b)$ were less than $(1-(1-p)^{\frac{n-1}{1-\gamma_b}})V$, there is a profitable deviation for any buyer to bid at $\bar{V}(n,p,\gamma_b)+\epsilon$ with probability 1. $\square$

Lemma 3. *The bidding is continuous in the interval* $[0,\bar{V}(n,p,\gamma_b)]$. *The cumulative distribution function of each buyer's bid* x, $F(x)$, *is given by*

$$F(x) = \frac{1-p}{p}\left(\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}} - 1\right) \tag{1.14}$$

Proof. To derive the functional form of $F(x)$, equate the expected utility from bidding any x in the interval to the expected utility from bidding 0. (These have to be equal to keep the buyer indifferent throughout the interval over which he mixes).

If a bidder bids x, he wins only if all other $n-1$ bidders bid less than x. For any bidder to bid less than x, he either does not enter, which happens with a probability $1-p$ or, if he enters, bid less than x, which happens with probability $pF(x)$. The probability that any one bidder bids less than x is $(1-p)+pF(x)$. The probability that all $n-1$ bidders bid less than x is $((1-p)+pF(x))^{n-1}$. So, if the bidder bids x, he wins with probability $((1-p)+pF(x))^{n-1}$ and gets utility $\frac{(V-x)^{1-\gamma_b}}{1-\gamma_b}$. The expected utility has to be equal to $(1-p)^{n-1}\frac{V^{1-\gamma_b}}{1-\gamma_b}$.

$$((1-p)+pF(x))^{n-1}\frac{(V-x)^{1-\gamma_b}}{1-\gamma_b} = (1-p)^{n-1}\frac{V^{1-\gamma_b}}{1-\gamma_b} \tag{1.15}$$

Simplifying this equation leads to the expression for $F(x)$ in equation 1.14. $\square$

Theorem 2. *The distribution of the bids for a given value of the maximum number of buyers* n, *probability of entry* p *and the buyers' coefficient of risk aversion* γ_b *dominates*

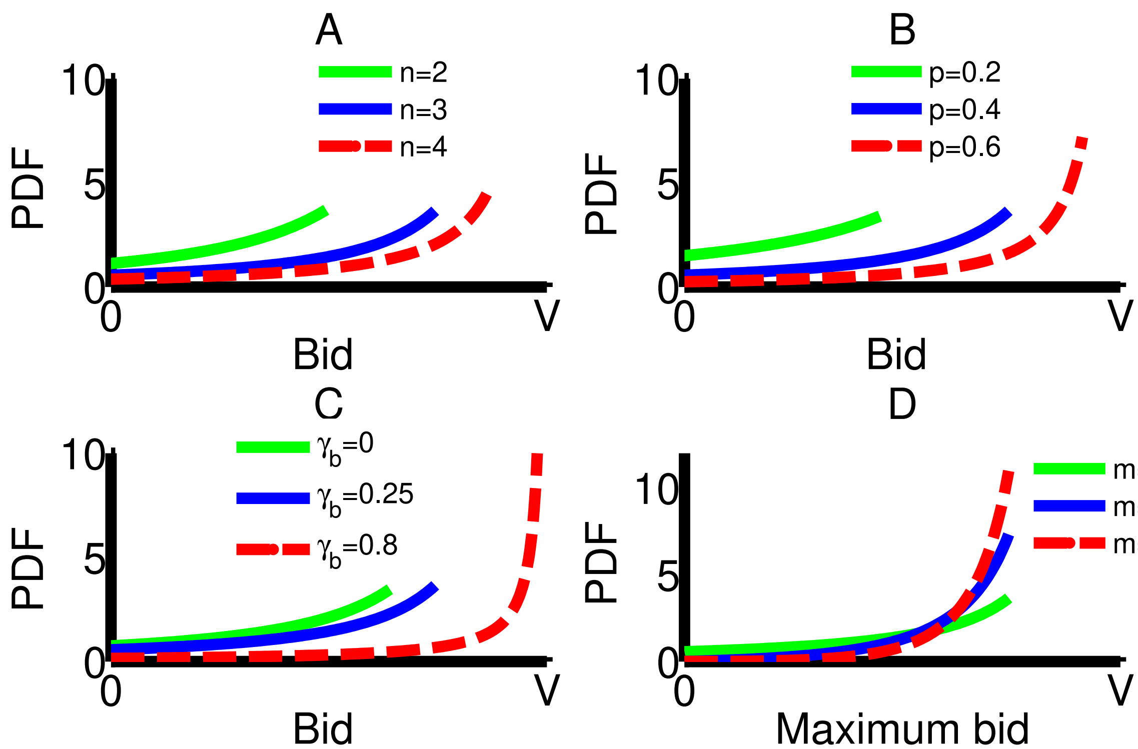

Figure 1.3. PDF of the bid for different values of n, p, γ_b and m
Panels A, B and C plot the the probability density function of each buyer's bid for different values of the maximum number of buyers n, probability of entry p, and buyer's relative risk-aversion coefficient γ_b. The default values are $n = 3$, $p = 0.4$ and $\gamma_b = 0.25$. In each of the panels A, B and C, a parameter is varied keeping the other 2 parameters at default levels. Panel D plots the PDF of the maximum bid when the actual number of buyers m varies from 1 to 3.

the distribution of bids for a lower value of n, p or γ_b in the sense of first-order stochastic domiance

Proof. When n, p or γ_b increase, the value of $F(x)$, given in equation 1.14, weakly decreases for all values of x. This is because the ratio $\frac{V}{V-x} \geq 1$ and the exponent $\frac{1-\gamma_b}{n-1}$ decreases when γ_b or n increases. So, the new distribution of bids dominates the old one in the sense of first- order stochastic dominance $\square$

Panels A, B and C of figure 1.3 plot the probability density function of the bid for different values of n, p and γ_b. The default values are assumed to be $n = 3$, $p = 0.4$ and $\gamma_b = 0.25$. In each figure, two parameters are held fixed at the default values and the third varied. The graphs of the PDF clearly show the shift in the distribution as n, p and γ_b increase. Not only does the probability mass of the distribution shift to the right as first-order stochastic dominance implies, the interval of bidding extends to the right too. In other words, not only are higher bids more likely, but also amounts which were not bid earlier are now being bid.

The intuition for the shift in the distribution of bids is as follows. As n increases, each buyer considers it more possible that there are competing buyers present in the auction and shifts his bid higher to increase the probability of being the maximum bidder. The effect of an increase in p is very similar. In both cases, the competition increases, due to more number of potential entrants or higher entry probability of each.

The effect of an increase in the relative risk-aversion coefficient of the buyers is not due to an increase in perceived competition since the competition is held constant in each of the graphs in panel C of figure 1.3. The increase in γ_b shifts the distribution of bids because the buyers' incentives to take a chance by bidding low decreases as the buyers become more risk-averse, their utility from bidding high and winning the auction with a high probability becomes higher than that of bidding low and winning the auction with a low probability.

Lemma 4. *If the actual number of buyers is m, the cumulative distribution function of the highest bid is $(F(x))^m$ where $F(x)$ is the cumulative distribution function of each buyer's bid.*

Proof. The m bids are all independent random variables and are identically distributed. The probability that all m bids are less than x is the product of the individual cumulative probability distribution functions, $(F(x))^m$. □

Corollary 2. *The distribution of the highest bid for a given value of the maximum number of buyers n, probability of entry p, actual number of buyers participating m and the buyers' coefficient of risk aversion γ_b dominates the distribution of highest bid for a lower value of n, p or γ_b in the sense of first-order stochastic dominance.*

Proof. From lemma 4, the CDF of the maximum bid is $F(x)^m$. Since $F(x) \leq 1$, $F(x)^m$ decreases as m increases. In addition, $F(x)$ decreases as n, p or γ_b increase from theorem 2. So, $F(x)^m$ decreases as any of n, p , m or γ_b increase. □

The pdf of the maximum bid is plotted in panel D of figure 1.3. Notice the difference between an auction and a negotiations in this aspect. In negotiations, we have seen that the actual number of buyers m is a sufficient statistic for the seller's expected utility since, conditional on m, the seller's expected utility does not depend on n and p. In an auction, even conditional on knowing m, the expected utility of the seller depends on n and p. The m buyers participating bid based on their beliefs about the competition they face, which depends on n and p. I now derive the expected utility for the seller in an auction as a function of m, n and p.

Theorem 3. $\Pi_V(m,n,p,\gamma_s,\gamma_b)$, *the expected utility for the seller for a realisation of the value V when m buyers participate is given by*

$$\Pi_V(m,n,p,\gamma_s,\gamma_b) = (1-\gamma_b)m\left(\frac{1-p}{p}\right)^m \frac{V^{1-\gamma_s}}{1-\gamma_s}$$

$$\left(\int_1^{\frac{1}{(1-p)^{\frac{1}{1-\gamma_b}}}} (y^{1-\gamma_b}-1)^{m-1}y^{-\gamma_b}(1-\frac{1}{y^{n-1}})^{1-\gamma_s}\mathrm{d}y\right) \tag{1.16}$$

Proof. The expected utility for the seller is the expected utility from the maximum bid. Since we know the probability density function of the maximum bid, the expected utility can be calculated. The proof is provided in Appendix A.5. □

Corollary 3. *The expected utility of the seller* $\Pi_V(m,n,p,\gamma_s,\gamma_b)$ *is increasing in* m,n,p *and* γ_b

Proof. This follows from corollary 2, which proves that the cumulative distribution function of the maximum bid undergoes a shift in the sense of first-order stochastic dominance as m,n,p or γ_b increase. Because of the shift, any decision maker with an increasing utility function would prefer the new distribution. So, the seller's expected utility is higher under the new distribution. □

It is useful to compare the expression in equation 1.16 to the maximum expected utility that the seller can extract from the sale. This corresponds to the case where the seller has perfect information and complete bargaining power. In that case, the seller would know V and would be able to extract the entire surplus due to the bargaining power. The seller's utility conditional on V would be $\frac{V^{1-\gamma_s}}{1-\gamma_s}$.

Corollary 4. *The expected utility of the seller $\Pi_V(m, n, p, \gamma_s, \gamma_b)$ is a fraction ϕ ofthe benchmark case where the seller has perfect information and complete bargaining power. The fraction ϕ is independent of the realised value V.*

Proof. The expected utility of the seller $\Pi_V(m, n, p, \gamma_s, \gamma_b)$ is is the benchmark $\frac{V^{1-\gamma_s}}{1-\gamma_s}$ multiplied by the term

$$\phi = (1-\gamma_b)m\left(\frac{1-p}{p}\right)^m \left(\int_1^{\frac{1}{(1-p)^{\frac{1}{1-\gamma_b}}}} (y^{1-\gamma_b}-1)^{m-1}y^{-\gamma_b}(1-\frac{1}{y^{n-1}})^{1-\gamma_s}\mathrm{d}y \right)$$

This term is independent of V □

Corollary 5. *The unconditional expected utility of the seller $\Pi(m, n, p, \gamma_s, \gamma_b)$ is increasing in m, n, p and γ_b*

Proof. Since $\Pi_V(m, n, p, \gamma_s, \gamma_b)$ is increasing in m, n, p and γ_b for any value of V, the unconditional expectation is also increasing in m, n, p and γ_b

□

Theorem 4. $\Pi(m, n, p, \gamma_s, \gamma_b)$*, the expected utility for the seller when m buyers participate is given by*

$$\Pi(m, n, p, \gamma_s, \gamma_b) = \frac{1}{(2-\gamma_s)(1-\gamma_s)}\phi \tag{1.17}$$

Proof. This follows from $V \sim U[0, 1]$, so the

$$\mathbb{E}_V\left[\frac{V^{1-\gamma_s}}{1-\gamma_s}\right] = \int_0^1 \frac{\mathrm{V}^{1-\gamma_\mathrm{s}}}{1-\gamma_\mathrm{s}}\mathrm{d}V = \frac{1}{(2-\gamma_s)(1-\gamma_s)}$$

□

Theorem 5. $\Pi(m, n, p, \gamma_s, \gamma_b)$ *is dependent on the distribution of the value of synergies only through the certainty equivalent of the distribution. Consequently, any two distributions which have the same certainty equivalent give the seller the same expected utility.*

Proof. The only dependence of $\Pi(m, n, p, \gamma_s, \gamma_b)$ on the distribution of V is through the expression $\mathbb{E}_V\left[\frac{V^{1-\gamma_s}}{1-\gamma_s}\right]$ The certainty equivalent of the distribution is defined as the constant value c which satisfies

$$\frac{c^{1-\gamma_s}}{1-\gamma_s} = \mathbb{E}_V\left[\frac{V^{1-\gamma_s}}{1-\gamma_s}\right]$$

So, $\Pi(m, n, p, \gamma_s, \gamma_b)$ can be rewritten as

$$\Pi(m, n, p, \gamma_s, \gamma_b) = \phi\frac{c^{1-\gamma_s}}{1-\gamma_s}$$

which is the same for any distributions of the value of synergies which have the same certainty equivalent □

Corollary 6. *If the seller is risk-neutral, any two distributions of the value of synergies which have the same mean give the seller the same expected utility.*

Proof. If the seller is risk-neutral, the certainty equivalent is the same as the expected value. The proof then follows from theorem 5. □

Corollary 7. *The expected revenue from the auction* $R(m,n,p,\gamma_b)$ *is given by*

(1.18)

$$R(m,n,p,\gamma_b) = (1-\gamma_b)m\left(\frac{1-p}{p}\right)^m \frac{1}{2}\left(\int_1^{\frac{1}{(1-p)^{\frac{1}{1-\gamma_b}}}} (y^{1-\gamma_b}-1)^{m-1}y^{-\gamma_b}(1-\frac{1}{y^{n-1}})\mathrm{d}y\right)$$

Proof. The expected revenue is obtained by setting $\gamma_s = 0$ in equation 1.17 □

How does the expected seller utility change with n, the maximum number of buyers, m, the number of buyers in play and p, the belief each buyer has about the latent competition? Figure 1.4 plots the expected seller utility for different values of n, m and p when the buyers and the seller are risk-neutral. Panel A plots the expected revenue against p for $n = 2$ and $m = 1, 2$ and Panel B for $n = 3$ and $m = 1, 2, 3$. (I derive closed form expressions for these cases in Appendix A.3).

Figure 1.4 shows that the expected revenue is increasing in the three parameters n, m and p when the other two are held constant. As one would expect, the seller revenue increases when the perception of competition among the buyers is high (n and p) and also when the number of buyers that participate (m) is high. The expected revenue can be concave, linear or convex in p for different combinations of m and n.

However, for all values of m and n, as p approaches 1, the expected revenue approaches $\frac{1}{2}$, the expected value of synergies. This is because when $p \to 1$, each buyer is virtually certain that the other will enter. So, both bid V and the seller is able to extract all the surplus.

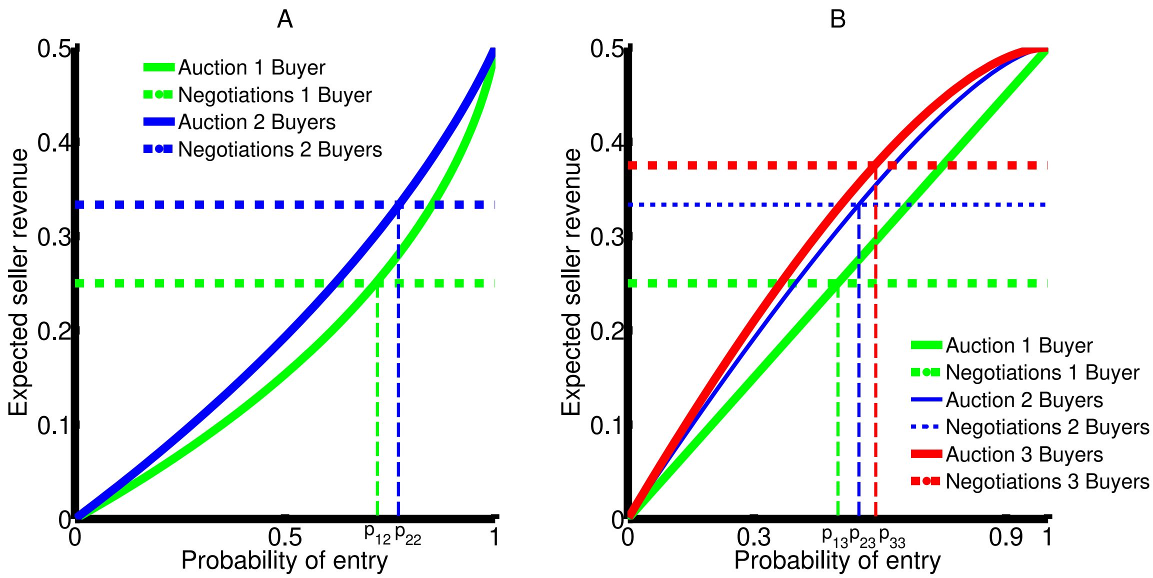

Figure 1.4. Expected seller revenue from an auction and negotiations for $n = 2$ and $n = 3$

The figure shows the expected revenue from an auction and negotiations as a function of the probability of entry p and the actual number of buyers participating m. Buyers are assumed to be risk-neutral. Panel A is for a maximum of 2 buyers, that is $n = 2$, and Panel B for a maximum of 3 buyers, that is $n = 3$. The dotted lines show the revenue from negotiations, the solid curves those from auctions. The probability at which the expected revenue from the auction equals that from negotiations is labelled p_{mn}.

On the other extreme, as p approaches 0, each buyer is almost sure that he is the only buyer and bids 0. The seller suffers from the perceived lack of competition and his expected payoff approaches 0.

1.2.3. Seller's Choice of Mechanism in the Supergame

So far, I have derived expressions for the expected seller utility in the negotiations and auction subgames. I now consider the question of whether the seller would choose auctions or negotiations in the supergame, which would determine which of these subgames occurs.

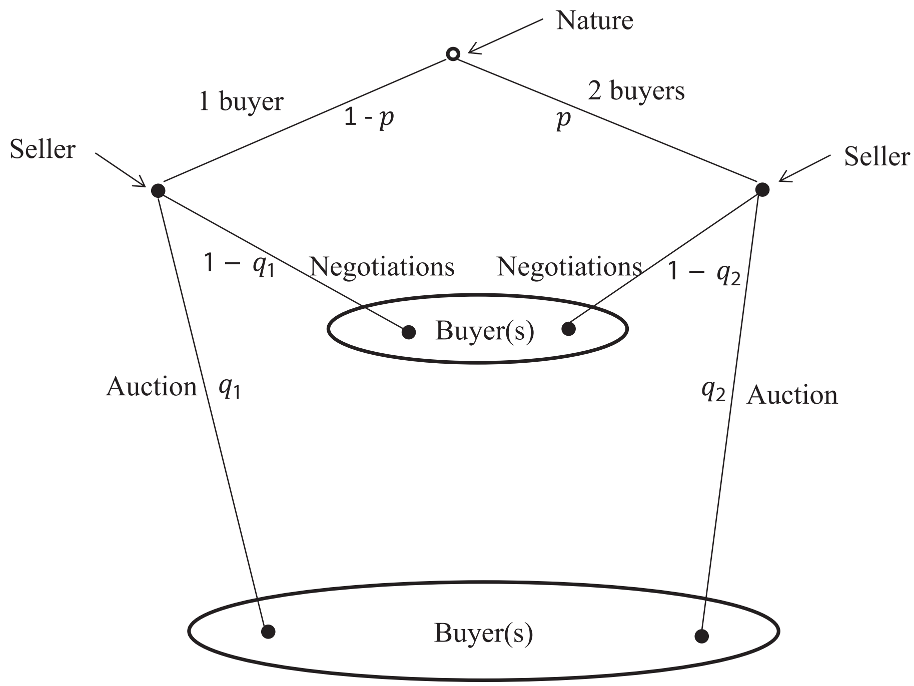

Figure 1.5. The evolution of the supergame
Nature moves first and reveals the number of buyers. The seller moves next after seeing the number of buyers. The seller randomizes between an auction and negotiations, choosing an auction with probability q_1 when there is 1 buyer and with probability q_2 when there are two buyers. The information sets of the buyers are not singleton sets since the buyers do not see whether there is another buyer or not.

Since the seller observes m before he chooses the mechanism, the choice of the mechanism can depend on n, m and p. The seller can also employ a mixed strategy, which involves randomising between an auction or negotiations with different probabilities based on the number of buyers. Figure 1.5 depicts the relevant part of the game tree for $n = 2$ and m at least 1.

I begin the analysis by assuming that the seller and the buyers are risk-neutral. I relax this assumption later, and show that the conclusions remain broadly the same even when the buyers and seller are risk-averse.

The solution concept I use is perfect Bayesian equilibrium. In any perfect Bayesian equilibrium, the following twin requirements have to be satisfied-

- The strategies have to be sequentially rational given the beliefs of the players and
- The beliefs in any information set which is on the equilibrium path have to be consistent with the strategies chosen by the players.

In other words, the buyers update their prior belief about there being another buyer based on the strategy of the seller. It is useful to illustrate this with an example.

Say there are 2 possible buyers, and each buyer's prior probability that there is another buyer participating is p. Say the seller's strategy is to choose negotiations when there is one buyer ($m = 1$) and auctions when there are two buyers ($m = 2$). If a buyer sees an auction, he will update the probability that there is another buyer from the prior of p to the posterior of 1. Similarly, conditional on seeing a negotiation, a buyer's posterior belief that there is another buyer is 0 and not p.

1.2.3.1. A Maximum of 2 Buyers ($n = 2$). I start with the case of $n = 2$. Panel A of figure 1.4 shows the revenues from auction and negotiation for $n = 2$ and $m = 1, 2$ as a function of p in the same graph for convenience.

The expected revenue from negotiations is independent of p while that with auctions increases with p. Define p_{12} as the prior probability at which the expected revenue from the auction equals that from negotiations for $m = 1$ and p_{22} as the prior probability at which the expected revenue from the auction equals that from negotiations for $m = 2$. As can be seen from panel A of figure 1.4, $p_{12} < p_{22}$.

If the seller chooses to negotiate, the posterior beliefs of the buyer are not relevant to the seller revenue since it is a take-it-or-leave-it offer. So, the beliefs play no part in the sequential rationality of the seller.

However, if the seller chooses an auction, the expected revenue will depend on the prevailing belief that there are two buyers given that he has chosen an auction. Denote this belief by p^*. Let the probability of the seller choosing an auction when he sees one buyer be q_1 and when he sees two buyers be q_2. The belief p^* is given by a Bayesian updating of the prior belief p as per the formula

$$p^* = \frac{pq_2}{(1-p)q_1 + pq_2} \text{ if at least one of } q_1 \text{ or } q_2 \text{ is not zero.} \tag{1.19}$$

$$p^* \in [0, 1] \text{ if both } q_1 \text{ and } q_2 \text{ are zero.} \tag{1.20}$$

where equation 1.20 follows from the fact that if both q_1 and q_2 are zero, the information set corresponding to auctions is reached only with zero probability. So, we are free to assign any belief to it since it is off the equilibrium path, *provided that given* p^*, $q_1 = q_2 = 0$ *is sequentially rational for the seller.*

Theorem 6. ***The equilibria for*** $\boldsymbol{n = 2}$

If $p < p_{22}$*, that is, if the probability of each buyer participating is low, the unique equilibrium is that the seller chooses negotiations independent of the actual number of* *buyers in play. If the buyers see an auction, which happens off the equilibrium path, they believe that there is only one buyer.*

If $p \geq p_{22}$, that is, if the probability of each buyer participating is high, there are multiple equilibria which fall into one of three categories

(1) *The seller chooses negotiations independent of the actual number of buyers in play. If the buyers see an auction, which happens off the equilibrium path, they believe that there is only one buyer.*

(2) *The seller chooses an auction independent of the actual number of buyers in play.*

(3) *The seller chooses an auction if there is one buyer in play and mixes between an auction and negotiations if there are two buyers in play. The probability of choosing an auction when there are two buyers, q_2, is given by the solution to $p_{22} = \frac{pq_2}{1-p+pq_2}$.*

In equilibria 1 and 2, the buyer's posterior belief that there is another buyer conditional on seeing an auction, p^*, is the same as p, the prior belief, since the buyers learn nothing about m from the seller's choice. In equilibrium 3, $p^* = p_{22}$, since the buyers update their prior probability p to reflect the seller's choice. In equilibrium 3, the posterior belief that there is another buyer on seeing a negotiation is 1.

Proof. See Appendix A.4 □

Theorem 7. *When $p \geq p_{22}$, the expected revenue of the seller is highest in the "always-auction" pure strategy equilibria.*

Proof. The ex-ante belief of there being another buyer is p. If auctions are always chosen irrespective of the actual number of buyers m, the posterior belief of there being another buyer, p^* is also p, since there is no Bayesian updating. If $p^* > p_{22}$, from figure 1.4, the seller revenue on choosing auctions is higher than from choosing negotiations even when $m = 2$.

In the mixing equilibria, the seller is mixing between auctions and negotiations when $m = 2$. So, the seller is indifferent between the payoffs from choosing either. So, the two payoffs must be equal, which implies that the seller revenue from choosing auctions is the same as that from negotiations. Hence, the payoff from the mixing equilibria must be less than that from the "always-auction" pure strategy equilibria when $m = 2$.

When $m = 1$, the seller chooses auctions in both equilibria and gets the same payoff.

Hence, the overall payoff must be greater in the "always-auction" pure strategy equilibria $\square$

Theorem 8. *In any pure strategy equilibria, the seller's choice of an auction or negotiations depends only on the ex-ante probability of entry and is independent of actual number of buyers.*

Proof. This follows from the fact that in one pure strategy equilibrium, the seller always chooses auctions and in the other, the seller always chooses negotiations. $\square$

1.2.3.2. A Maximum of 3 Buyers ($n = 3$). The expected revenue from negotiations is independent of p while that with auctions increases with p. Define p_{13} as the prior probability at which the expected revenue from the auction equals that from negotiations for $m = 1$, p_{23} as the prior probability at which the expected revenue from the auction equals that from negotiations for $m = 2$ and p_{33} as the prior probability at which the expected revenue from the auction equals that from negotiations for $m = 3$. As can be seen from Panel B of figure 1.4, $p_{13} < p_{23} < p_{33}$.

The expected revenue in any equilibrium can depend on the prevailing equilibrium beliefs that there are 1, 2 or 3 buyers. While the ex-ante probabilities each buyer assigns to there being 0, 1 or 2 other buyers can be characterised in terms of the entry probability p,[8] the posterior probabilities depend on the probability that the seller uses to randomise between an auction and a negotiation. They cannot be represented by a single parameter p.

Theorem 9 gives the expected seller utility when the maximum number of buyers is n for a general probability vector $(p_1, p_2, \ldots, p_n)$, $\sum_{i=1}^{n} p_i = 1$ and $p_i \geq 0$, where p_i denotes the equilibrium probability each of the m participating buyers assigns to there being a total of i buyers in the auction.

Theorem 9. *Let n be the maximum number of buyers. The expected utility of the seller for a given V when there are m actual buyers is given by*

$$\Pi_V(m, n, p, \gamma_s, \gamma_b) = m\frac{V^{1-\gamma_s}}{1-\gamma_s}\int_0^1 \left[1 - \left(\frac{p_1}{p_1 + p_2 y + p_3 y^2 + \cdots + p_n y^n}\right)^{\frac{1}{1-\gamma_b}}\right]^{1-\gamma_s} y^{m-1}\mathrm{d}y$$

Proof. See Appendix A.5. □

Corollary 8. *When the seller and buyers are risk-neutral, the expected utility for the seller (which is also the expected revenue) is given by*

$$R(m, n, p) = \frac{1}{2}\left[1 - mp_1\int_0^1 y^{m-1}\left(\frac{1}{p_1 + p_2 y + p_3 y^2 + \cdots + p_n y^n}\right)\mathrm{d}y\right]$$

[8]These are the binomial probabilities $(1-p)^2$, $2p(1-p)$ and p^2 respectively.

Proof. The result follows from setting $\gamma_s = \gamma_b = 0$ in theorem 9. □

Now that we have the expression for seller revenues for negotiations and auctions including in equilibria that feature mixing, we can characterise the equilibria for $n = 3$.

Theorem 10. ***The equilibria for $n = 3$***

If $p < p_{33}$, that is, if the probability of each buyer participating is low, the unique equilibrium is that the seller chooses to negotiate independent of the actual number of buyers in play. If the buyers see an auction, which happens off the equilibrium path, they believe that there is only one buyer.

If $p \geq p_{33}$, that is, if the probability of each buyer participating is high, there are multiple equilibria which fall into one of three categories

(1) *The seller chooses negotiations independent of the actual number of buyers in play. If the buyers see an auction, which happens off the equilibrium path, they believe that there is only one buyer.*
(2) *The seller chooses an auction independent of the actual number of buyers in play.*
(3) *The seller chooses an auction if there is one or two buyers in play and mixes between an auction and negotiations if there are three buyers in play. The probability of choosing an auction when there are three buyers, q_3, is unique for any given p.*

Proof. The proof consists of numerically checking for equilbria which obey both sequential rationality and the consistency conditions for belief in a PBE for various values of p.

A sketch of the proof is as follows. I divide p into the intervals $[0, p_{13}]$, $[p_{13}, p_{23}]$, $[p_{23}, p_{33}]$ and $[p_{33}, 1]$. For each interval, I rule out candidate equilibria which violate either sequential rationality or consistency of beliefs. For example, if p is in the range $[0, p_{13}]$ and the seller chooses negotiations when $m = 1$, he must choose negotiations for $m = 2$ and $m = 3$ as well due to sequential rationality. In other words, all equilibria where he chooses negotiations for $m = 1$ and an auction when $m = 2$ or $m = 3$ can be ruled out. I repeat this process for all the sub-intervals of p till I am left with the equilibria listed. $\square$

When $p \geq p_{33}$, the expected revenue of the seller is highest in the "always-auction" pure strategy equilibria. Also, in any pure strategy equilibria, the seller's choice of an auction or negotiations depends only on the ex-ante probability of entry and is independent of actual number of buyers.

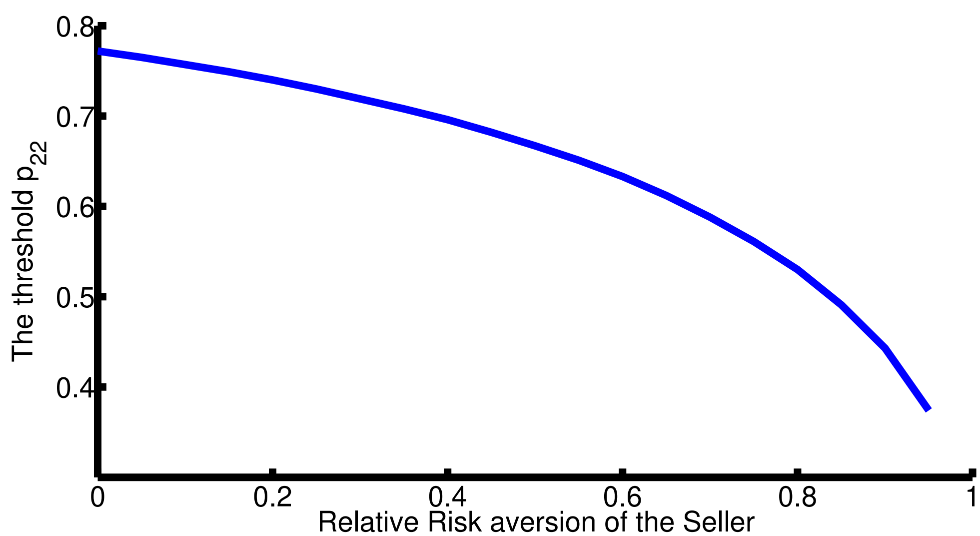

Figure 1.6. The threshold probability for auctions, p_{22} as a function of the seller's relative risk aversion γ_s

Figure 1.6 shows how the threshold probability p_{22} changes as the seller's risk-aversion increases.[9] As the seller becomes more risk-averse, the threshold probability above which he chooses auctions decreases. The intuition behind this is that the offers made by the seller during negotiations decreases as the risk-aversion increases. In auctions, the buyers' bid depend on their risk aversion, not the seller's, since they are competing with each other and not the seller. Hence, an increase in the seller's risk aversion has no effect on the bidding behaviour of the buyers in the auction. So, the effect of the risk-aversion on the seller's utility is greater in negotiations than in auctions.

1.3. Committing Ex Ante to a Particular Mechanism Irrespective of the Actual Number of Buyers

The analysis till now shows that the seller does not benefit from being able to see the actual number of buyers m before he chooses the mechanism. In the revenue-maximising equilibria, the seller's choice depends only on the buyers' belief about the competition (n and p), not the actual competition (m) that he is able to observe. In fact, the ability to observe the number of buyers *decreases* the seller's revenue in the mixed-strategy equilibria. Thus, the buyer inference about the actual competition from the seller's choice makes the information advantage of observing m useless and in fact worsens the seller's situation.

I next examine whether the seller can benefit from committing ex ante to choosing the same mechanism irrespective of how many buyers the seller sees. If the seller does so, his choice of the mechanism will reveal no extra information to the buyers about their competition since the choice is made before the seller observes the number of buyers. In

[9]The buyer is assumed to be risk-neutral.

this case, the expected revenue from auctions and negotiations depend only on n and p but is independent of m.[10]

1.3.1. Expected Revenue from Ex Ante Commitment

Auction

We know the expression for $R(m,n)$ the expected revenue for each value of m when the maximum number of buyers is n. To calculate the expected revenue ex-ante, we need to multiply this by the probability that m buyers participate, which is just the binomial probability ${}^nC_m p^m(1-p)^{n-m}$.

$$\text{Expected Revenue from an auction} = \sum_{m=1}^{n} ({}^nC_m)p^m(1-p)^{n-m}R(m,n)$$

Since this is independent of m, we drop the suffix m and denote this by R_n.

Theorem 11. *R_n, the expected revenue from committing to an auction ex ante when there is a maximum of n buyers, is given by*

$$R_n = \frac{1}{2}\left(1-(1+(n-1)p)(1-p)^{n-1}\right) \tag{1.21}$$

Proof. See Appendix A.6 for the proof. □

Negotiations

$$\text{Expected Revenue from negotiations} = \frac{1}{2}\sum_{m=0}^{n} ({}^nC_m)p^m(1-p)^{n-m}\frac{m}{m+1} \tag{1.22}$$

[10] In this section, I assume that the sellers and buyers are risk-neutral, but the results for risk-averse sellers and buyers are similar.

1.3.2. The Benefits of Commitment for $n = 2$ and $n = 3$

I illustrate the benefits of commitment for $n = 2$ and $n = 3$.

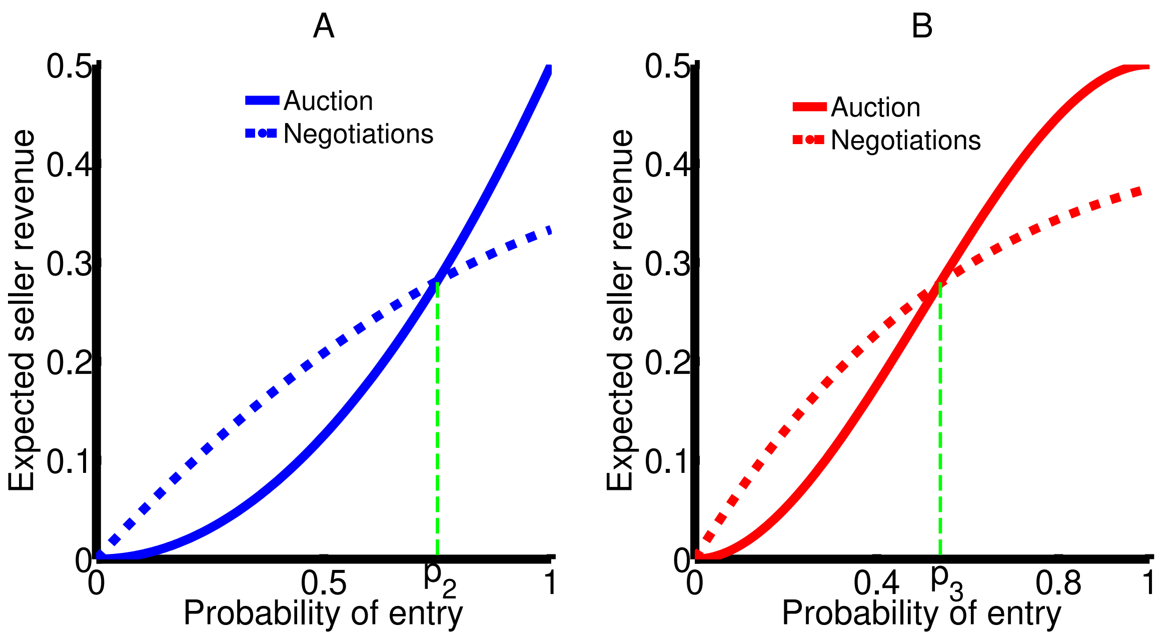

Figure 1.7. Expected revenue with commitment
The figure plots the expected seller revenue from an auction or from negotiations if the seller can credibly commit ex ante that he will choose the same mechanism irrespective of m. Panel A is for $n = 2$ and Panel B for $n = 3$. The probability at which the revenues from the auction and negotiations are the same is labelled p_n.

First, consider $n = 2$. Substituting in equations 1.21 and 1.22 above gives

$$\begin{aligned}
\text{Expected revenue from an auction} &= \frac{1}{2}\left(1 - (1+p)(1-p)\right) \\
&= \frac{1}{2}p^2 \\
\text{Expected revenue from negotiations} &= \frac{1}{2}\left(2p(1-p)\frac{1}{2} + p^2\frac{2}{3}\right) \\
&= \frac{1}{2}p(1 - \frac{p}{3})
\end{aligned}$$

The revenue from the two mechanisms is plotted against p in Panel A of Figure 1.7. Though the revenue from both auctions and negotiations increases as p increases, the graph exhibits single crossing. Hence, the seller chooses an auction if p is above a threshold p_2, and a negotiation otherwise. The point of indifference p_2 is the solution to the equation

$$\frac{1}{2}p_2^2 = \frac{1}{2}p_2(1 - \frac{p_2}{3}) \implies p_2 = \frac{3}{4}$$

The seller benefits from the ability to commit since the threshold probability of entry above which he chooses an auction with commitment, p_2, is less than the threshold above which he chooses auctions without commitment, p_{22}. The seller is now able to employ auctions even at a lower level of competition, which the buyer inference prevented it from doing without commitment. Thus, committing to a mechanism ex-ante increases the seller's revenue since he can choose auctions in a greater range.

Next, consider $n = 3$. Substituting in equations 1.21 and 1.22 above gives

$$\begin{aligned}
\text{Expected Revenue from an auction} &= \frac{1}{2}\left(1 - (1+2p)(1-p)^2\right) \\
&= \frac{1}{2}p^2\left(3 - 2p\right) \\
\text{Expected Revenue from negotiations} &= \frac{1}{2}\left(3p(1-p)^2\frac{1}{2} + 3p^2(1-p)\frac{2}{3} + p^3\frac{3}{4}\right) \\
&= \frac{1}{2}p\left(\frac{p^2}{4} - p + \frac{3}{2}\right)
\end{aligned}$$

Panel B of figure 1.7 plots the revenues as a function of p. The target chooses an auction if p is above a threshold say p_3 and a negotiation otherwise. The point of indifference p_3 solves

$$\frac{1}{2}p_3^2(3-2p_3) = \frac{1}{2}p_3\left(\frac{p_3^2}{4} - p_3 + \frac{3}{2}\right) \implies p_3 = 0.54 \tag{1.23}$$

Once again, we find that the seller benefits from the ability to commit, since the threshold above which he chooses an auction with commitment, p_3, is less than the threshold above which he chooses auctions with no commitment, p_{33}. The seller is now able to employ auctions over a greater range when p lies between p_{13} and p_{33}.

Theorem 12. *The seller can earn higher revenues if he can commit ex-ante to choosing a particular mechanism irrespective of the number of buyers that he observes. In equilibrium, the seller's choice of mechanism depends on the probability of entry p. The seller chooses auctions if the probability is greater than p_n and negotiations otherwise.*

Proof. The proof follows from the fact that without committment, the only equilibrium below p_{nn} was the seller choosing negotiations. With committment, choosing auctions strictly his revenue over the part of the range (p_n, p_{nn}). In the range $[p_{nn}, 1]$, the seller benefits from avoiding equilibria that feature mixing, which increases his revenue. $\square$

What might commitment look like in practice? Instead of eliciting the participation decision from each potential buyer and *then* informing them of his choice of mechanism, the seller would first inform the potential buyers of whether he has chosen an auction or negotiation and then ask them whether they want to participate in the sale. In other words, with commitment, the timing of the process is flipped from finding out the value of m and then choosing a mechanism, to choosing the mechanism first and then finding out the value of m.

1.4. Extensions of the Baseline Model

In this section, I present extensions to the baseline model. I assume that the seller and the buyers are risk-neutral and that $n = 2$ for simplicity.

1.4.1. Independent Private Values

The analysis so far has been restricted to the case where the buyers have common values. I now examine how the conclusions change if the value to the buyers is private and independent, rather than common.

1.4.1.1. Expected Seller Revenue from Negotiations. Let the maximum expected revenue from sequentially negotiating with m buyers be equal to V_m and the corresponding offer be x_m^*

Consider the first stage. If a price x is offered, the offer is accepted when $V > x$, which happens with probability $1 - x$. The revenue conditional on acceptance is x. If the offer is rejected, which happens with probability x, there are $m - 1$ buyers left to negotiate with. So, the expected revenue conditional on rejection is V_{m-1}.

This gives us the recursive equation

$$V_m = \underset{x \in [0,1]}{Max} \left(x(1-x) + (V_{m-1})x \right) \tag{1.24}$$

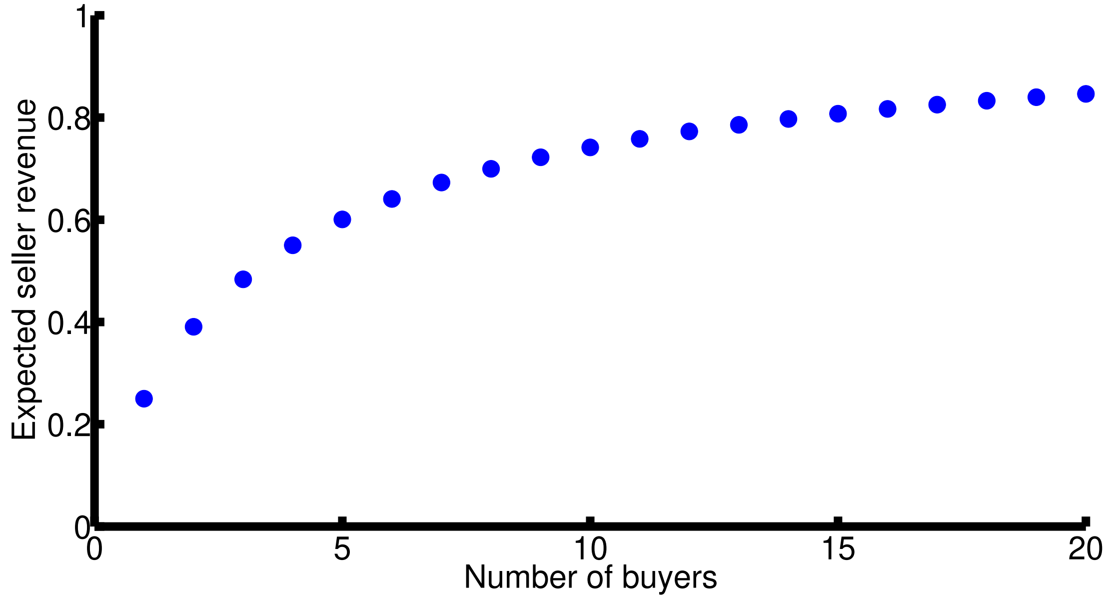

Figure 1.8. Expected seller revenue with independent buyer values $\sim U[0, 1]$

Differentiating w.r.t. x gives the optimal offer as

$$x_m^* = \frac{1 + V_{m-1}}{2} \tag{1.25}$$

Substituting in equation 1.24 , we obtain

$$V_m = \frac{(1 + V_{m-1})^2}{4} \tag{1.26}$$

$V_0 = 0$, so substituting recursively, we get the expected revenue as plotted in figure 1.8.[11] As the number of buyers increases, the seller can try offering higher amounts in the earlier stages. As $m \to \infty$, the seller is able to extract the maximum value from the transaction, which approaches 1, and the informational rent extracted by the buyers decreases.

[11]Unlike the common values case, there is no closed form solution for V_m. Instead, the solution has to be obtained numerically by recursion.

Unlike in the common values case, there is no learning in the independent private values case. The rejection of an offer of x does not affect the probability of any subsequent offer being accepted. The expected revenue increases because of two factors. First, the seller has more rounds to negotiate as the number of buyers increases. Second, the expected maximum value for the m buyers is the expected maximum of m draws from $U[0,1]$. This is given by the expression $\frac{m}{m+1}$, which approaches 1 as $m \to \infty$. Hence, there is a greater probability that a buyer will accept a higher offer.

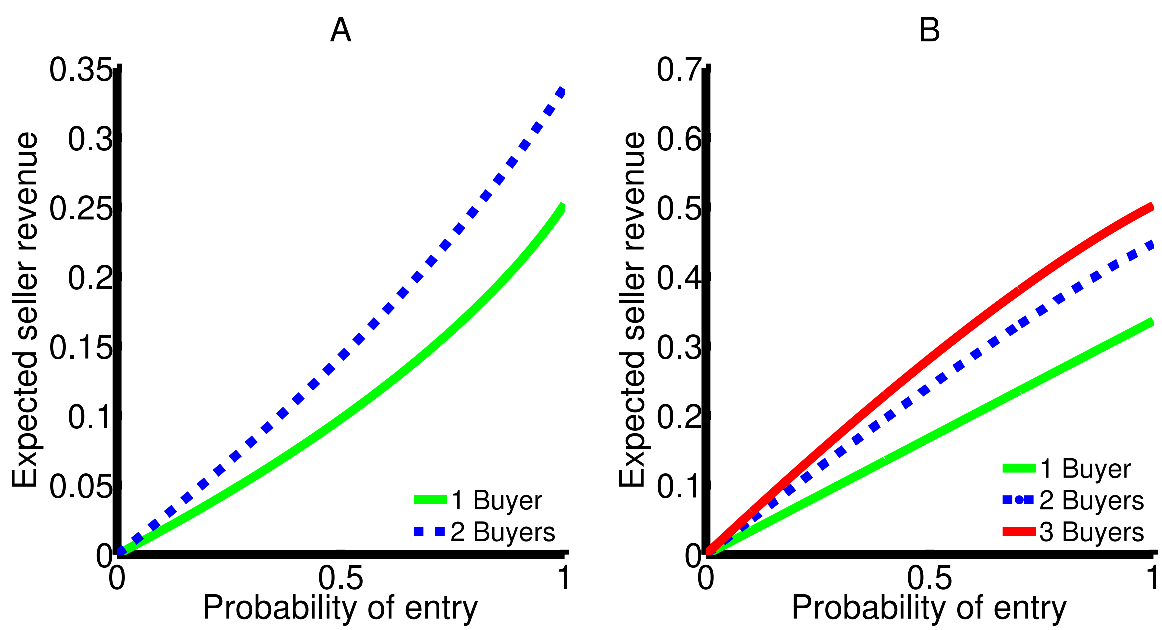

Figure 1.9. Expected seller revenue when the buyer values are independent
The figure plots the expected revenue when there is a maximum of n possible buyers whose values are independent, as a function of the probability of entry p and the number of buyers in play m. Panel A plots the graph for $n = 2$ and Panel B for $n = 3$

1.4.1.2. Expected Seller Revenue from an Auction. Previous studies such as McAfee and McMillan (1987) and Harstad et al. (1990) derive bidding strategies and seller revenues for auctions with an unknown number of buyers and independent and identically distributed buyer valuations for any arbitrary distribution of the value. I specialize their model to my setting of uniformly distributed buyer valuations to derive the expression for each buyer's bidding strategy.

With the values being independently distributed, the buyer bids are given by

$$B(V_i, n) = \frac{\sum_{r=1}^{n-1} \binom{n-1}{r} p^r (1-p)^{n-1-r} V_i^r \frac{r}{r+1} V_i}{\sum_{r=1}^{n-1} \binom{n-1}{r} p^r (1-p)^{n-1-r} V_i^r} \tag{1.27}$$

Each of the m buyers bid depending on their V_i. The expected revenue for the seller $\Pi(m, n)$ is the expected maximum bid, that is, the expected maximum of $B(V_i, n)$ for m draws of V_i. I compute this numerically as a function of m and n.

Panel A of figure 1.9 plots the expected revenue from an auction against p for $n = 2$ and $m = 1, 2$. As p approaches 1, each buyer is virtually certain that the other will enter. So, both bid $V_i/2$ and the expected revenue is $\frac{1}{3}$ as in a normal first price auction. On the other extreme, as p approaches 0, each buyer is almost sure that he is the only bidder and bids 0. Panel B shows the plot of the expected revenue from an auction against p for $n = 3$ and $m = 1, 2, 3$. Both graphs resemble the graphs for the expected revenue from an auction in the common values case.

The equilibrium characterisation and the benefits of commitment are similar to that in the common values case. The only change is that the threshold probability p_{22} is different.

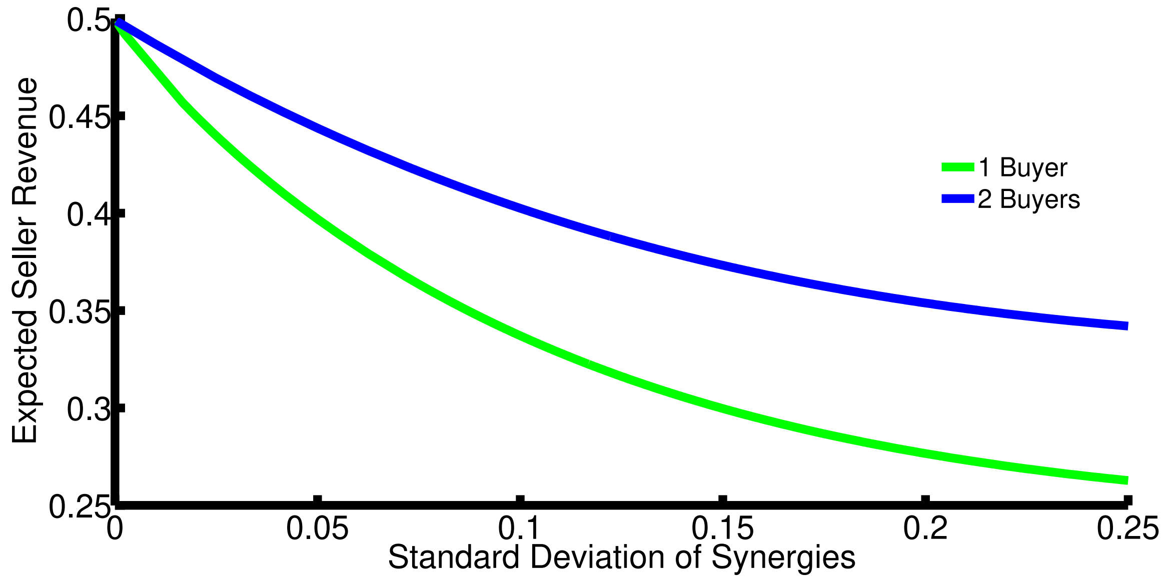

Figure 1.10. Expected revenue from negotiations as a function of the volatility of synergies
The distribution of synergies is truncated normal, with mean $\frac{1}{2}$ but various standard deviations. The revenue is plotted for negotiations when 2 buyers participate as well as when only 1 does.

1.4.2. Effect of the Volatility of the Synergies

So far, we have considered the value of the synergies to be uniformly distributed. The variance of $U[0,1]$ is $\frac{1}{2}$. To consider the effect of volatility of the synergies on negotiations, I change the distributional assumption.

I now assume that the synergies are normally distributed. I consider a series of truncated normal distributions with the same mean of $\frac{1}{2}$ but differing standard deviations.[12] I then look at how the optimal take-it-or-leave it offers in negotiations change with the volatility of the synergies. It is worth noting that the expected revenue from an auction

[12] I truncate the distributions at 0 and 1 and assign the probabilities of the value being less than 0 and greater than 1 to atoms at 0 and 1. The standard deviations refer to those of the original normal distribution before truncation.

depends only on the mean of the value of synergies, so the earlier analysis continues to hold for auctions even in this new setting.

1.4.2.1. Negotiations with 1 Buyer ($m = 1$). If a price x is offered, it is accepted with probability $1 - F(x)$ and rejected with probability $1 - F(x)$. So, the optimal offer maximizes the following expression.

$$\underset{x \in [0,1]}{Max} \left[x(1 - F(x)) \right] \tag{1.28}$$

I solve this numerically. The results are plotted in figure 1.10. The expected seller revenue increases as the volatility of the synergies decreases. In the limiting case, as the volatility becomes zero, the seller will be able to extract all of the expected synergies in the transaction i.e. $\frac{1}{2}$. This is because the rent extracted by the buyer due to asymmetric information decreases with decrease in the volatility.

1.4.2.2. Negotiations with 2 Buyers ($m = 2$). If the offer is accepted in the first stage, the expression of expected revenue is similar to that in equation 15. If the offer is rejected in the first stage, the seller still has one stage of negotiation left with an buyer. However, the value of the synergies conditional on the offer being rejected is a truncated normal with a one-sided truncation at x.

This problem is solved by backward recursion The payoff in the second stage is calculated as a function of the first stage offer. Then, the first stage offer is optimized using this payoff. The results are shown in figure 1.10. Two points are of interest. The first is that the revenue increases as the volatility on the synergies decreases. As the volatility becomes very low, the seller extracts all the synergies in the transaction. Second, the benefit of having an additional stage to learn about the value of synergies increases as the volatility increases. If the volatility is low, one stage is enough to extract most of the expected value because the asymmetry is low in magnitude. The higher the volatility is, the more the second stage of negotiation helps.

1.4.2.3. Threshold Probability of Choosing an Auction. The expected revenue from auctions remains the same independent of the volatility, but the expected revenue from negotiations changes with the volatility. So, the threshold probability above which the seller chooses an auction, p_{22}, would also change as the volatility increases. Since negotiations generate less revenue as the volatility increases, the threshold probability decreases with the volatility. Figure 1.11 shows the revenue from negotiations, auctions and threshold probability p_{22} when the volatility $\sigma = 0.1$. The threshold is higher than in the uniform distribution since the volatility is very low.

1.5. Empirical Implications

The vast amount of data available on takeovers enables the testing of empirical hypotheses generated from the model. I sketch a few of these below.

The theory specifies seller's choice of the mechanism as a function of the probability of entry p. However, sales of firms in some industries attract buyers with high probability and in some other industries with low probability. To derive empirical implications of the model in terms of the observed data on auctions and negotiations, I need to assume some distribution for p. This is because the aggregate data on auctions and negotiations pertains to a cross section of firms operating in different industries, each of which may have a different probability of entry. In the discussion that follows, I assume that $p \sim U[0, 1]$.

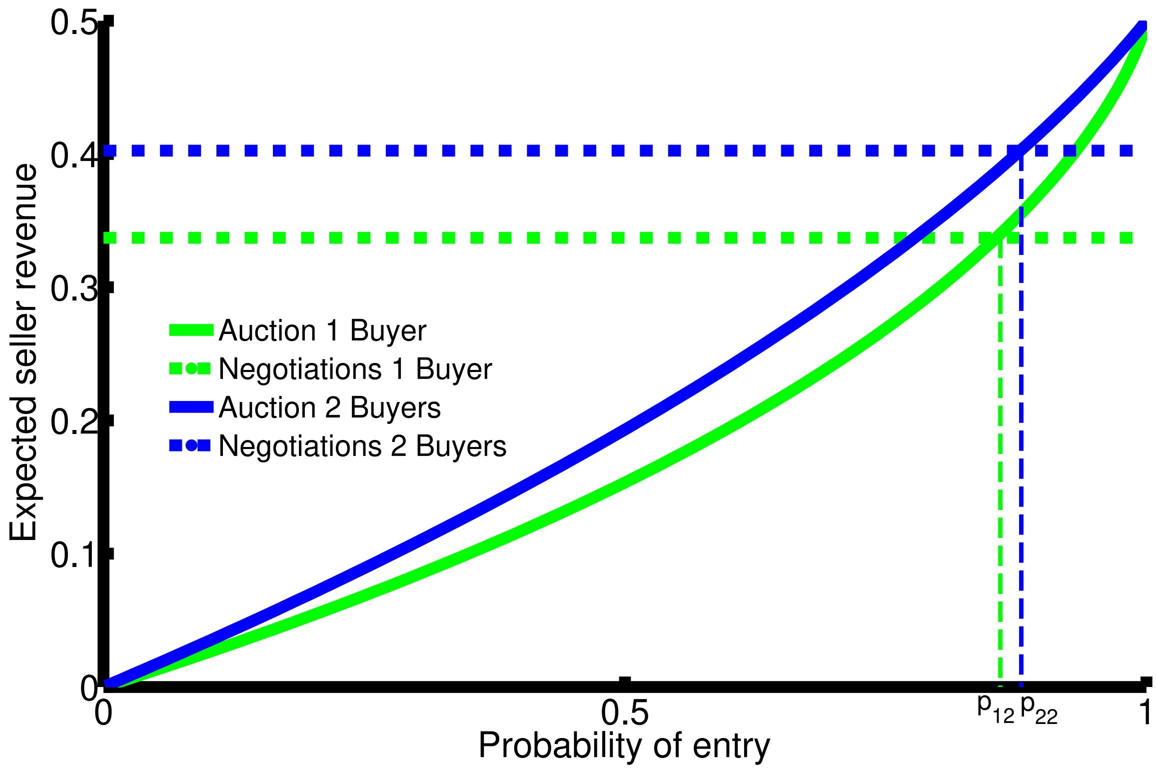

Figure 1.11. The threshold probability p_{22} when the volatility of synergies $\sigma = 0.1$

I also assume that the equilibrium that will be realised is the one where the seller gets the maximum expected utility. This corresponds to equilibrium 2 in theorem 6 , that is, if $p \geq p_{22}$, the seller chooses an auction independent of the actual number of buyers in play.

The empirical implications derived are for the observed transaction price. If the negotiation fails, the transaction will not be observed in the data. To take this into account, the expected transaction price is calculated as the expected offer made by the seller *conditional on the offer being accepted.* In the case of an auction, the firm is always sold, since the buyers always bid higher than the reserve price, which is the standalone value of the firm.

1.5.1. Mean and Volatility of the Transaction Price as a Function of the Probability of Entry

In this section, I examine how the mean and volatility of the transaction price depend on the probability of entry for a given level of the seller's and buyers' risk aversion. The relative risk aversion coefficient of the seller is assumed to be 0.5. The buyers are assumed to be risk-neutral.

The computation of expected transaction price for negotiations for a given value of p is as follows. When there is 1 buyer, the optimal offer for the seller is 0.33. So, the observed transaction price if there is 1 buyer is 0.33.

When there are 2 buyers, the optimal offer in the first round is 0.54 and in the second round 0.18. The firm is sold in the first round if the value is greater than 0.54, which happens with probability 1-0.54=0.46,[13] and in the second round if the value is between 0.18 and 0.54, which happens with probability 0.54-0.18=0.36. Conditional on the firm being sold, the probability that it was sold in the first round is $\frac{0.46}{0.46+0.36}$ and in the second round $\frac{0.36}{0.46+0.36}$. So, the observed transaction price is

$$\frac{0.46}{0.46+0.36}(0.54)+\frac{0.46}{0.46+0.36}(0.18)=0.39.$$

The last step is to multiply the observed prices conditional on there being 1 or 2 buyers by the respective probabilities that there are 1 or 2 buyers, which are $\frac{2-2p}{2-p}$ and $\frac{p}{2-p}$. So,

[13]The probability that the value V is greater than 0.56 is 1-0.56 because $V \sim U[0,1]$.

the observed transaction price for negotiations is given by the function

$$0.33\left(\frac{2-2p}{2-p}\right)+0.39\left(\frac{p}{2-p}\right) \tag{1.29}$$

For auctions, the observed transaction price is the expected revenue from commitment, which was already calculated in equation 1.23 as $\frac{1}{2}p^2$, divided by the probability of there being at least one buyer, $1-(1-p)^2$.

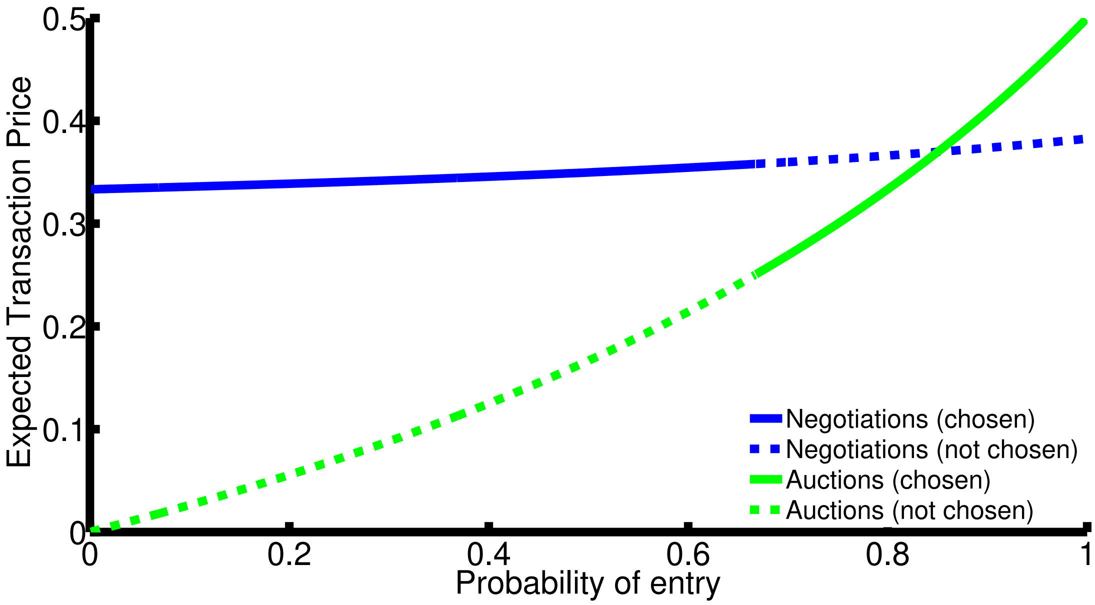

Figure 1.12. Mean of the transaction price as a function of the probability of entry
The solid part of the line and the curve correspond to regions where the mechanism is chosen; the dotted part to regions where it is not chosen. The risk-aversion of the seller is assumed to be 0.5, and that of the buyer 0.

Figure 1.12 plots the expressions in equations 1.29 and 1.23 as a function of the probability of entry for both auctions and negotiations. The straight line corresponds to negotiations and the curve to auctions. The solid part of the line and the curve correspond to regions where the mechanism is chosen; the dotted part to regions where it is not chosen.[14]

Empirical Implication 1. *For a given level of the seller's risk aversion, the mean transaction price for both auctions and negotiations increases as the as the probability of entry p increases.*

The intuition behind this result is as follows. For negotiations, the price increases because as p increases, it becomes increasingly likely that there are 2 buyers participating ($m = 2$). So, the seller is increasingly likely to have 2 rounds of negotiations than 1, which increases the expected offer price. I call this the effect of actual competition. For auctions, there are two effects in play. First, same as in negotiations, as p increases, it becomes increasingly likely that there are 2 buyers participating. This is the effect of actual competition. Second, unlike negotiations, the expected price increases with p also because the buyers believe it is more likely that $m = 2$, which leads them to bid higher whether $m = 1$ or $m = 2$. I call this the effect of latent competition. These effects combine to raise the expected transaction price.

I have so far ignored the fact that the selection of the mechanism depends on p. In the revenue-maximising equilibrium, auctions are chosen if $p \geq p_{22}$ and negotiations are chosen if $p \geq p_{22}$. Predictions about the observed transaction price *across mechanisms*

[14]It may seem paradoxical that the seller chooses auctions even in regions where the expected transaction price is lesser than that from negotiations. To reconcile this, recall that the seller's choice depends on his expected utility from the mechanism and not the expected transaction price. The expected transaction price differs from the expected seller utility due to two reasons. First, the seller is risk-averse, so the expected utility of the seller is not the same as the expected transaction price. Second, for negotiations, even if the seller is risk-neutral, the two are different because the negotiations can fail if the seller offers a price greater than the buyers' value. The expected utility is the the product of the probability of the offer being accepted and the expected transaction price conditional on the offer being accepted. So, the expected utility is less than the expected transaction price.

must also account for selection. The observed transaction price will include only the solid line and curve in figure 1.12. It is immediately apparent that the observed transaction price has a discontinuity at the threshold probability due to the effect of selection.

Empirical Implication 2. ***The effect of selection on the mean transaction price***
For a given level of the seller's risk aversion, the mean transaction price increases as the industry becomes more competitive, till a cutoff level of competition. At the cutoff, the transaction price drops and then monotonically increases.

The plot for the volatility (standard deviation) of the transaction price as a function of the probability of entry is displayed in figure 1.13. The next empirical implication follows from the shape of the graph.

Empirical Implication 3. *For a given level of the seller's risk aversion, the volatility of the transaction price for both auctions and negotiations increases as the probability of entry p increases.*

The intuition for this result is as follows. For negotiations, recall that the offer prices are 0.33 if there is one buyer and 0.54 and 0.18 in the two rounds when there are two buyers. For very low values of p, that is as $p \to 0$, there is almost certainly only one buyer and the seller always offers 0.33. So, the observed transaction price is always 0.33. The standard deviation of the transaction price is 0. For very high values of p, that is as $p \to 1$, there are almost certainly two buyers.[15] The seller offers 0.54 and 0.18 in the two

[15]Of course, the seller may never choose negotiations at such high values of probability p.

rounds and the standard deviation is 0.18. For values of p between 0 and 1, the offer can be any of 0.18, 0.33 or 0.54 and the volatility lies between 0 and 0.18.[16]

For auctions, the variance of the transaction price for a given value of p arises from 2 components, the within V variance and across V variance. For a given realisation of V, there is a range of bids since each buyer randomizes over an interval leading to within V variance. Then there is variance of the bids across Vs. For very low values of p, that is as $p \to 0$, there is almost certainly only one buyer and each buyer's bid approaches 0 for any value of V. Thus, the bid is constant and the variance is 0. For very high values of p, that is as $p \to 1$, there are almost certainly two buyers. So, each buyer's bid approaches V. The within V variance is 0 in the limit. The across V variance is just the variance of the maximum bid, which is V, when $V \sim U[0,1]$. The variance is $\frac{1}{12}$, and the corresponding standard deviation is 0.29. For values of p between 0 and 1, the variance lies between 0 and $\frac{1}{12}$.

Once we account for selection of the mechanism, that auctions are chosen if $p \geq p_{22}$ and negotiations are chosen if $p \leq p_{22}$, the only relevant portions of the graph are the solid lines, not the dotted ones. The graph tells us that although the volatility monotonically increases, there is a discontinuity at the threshold probability $p \geq p_{22}$.

Empirical Implication 4. ***The effect of selection on the volatility of the transaction price***

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[16] Conditional on the offer being accepted, the probability that the transaction price 0.54 is $\frac{0.46}{0.46+0.36}$ and that the transaction price is 0.18 is $\frac{0.36}{0.46+0.36}$. The mean is 0.39 as calculated earlier in the section. The variance is

$$\frac{0.46}{0.46+0.36}(0.54-0.39)^2 + \frac{0.36}{0.46+0.36}(0.18-0.39)^2 = 0.032 \tag{1.30}$$

and the standard deviation is 0.18.

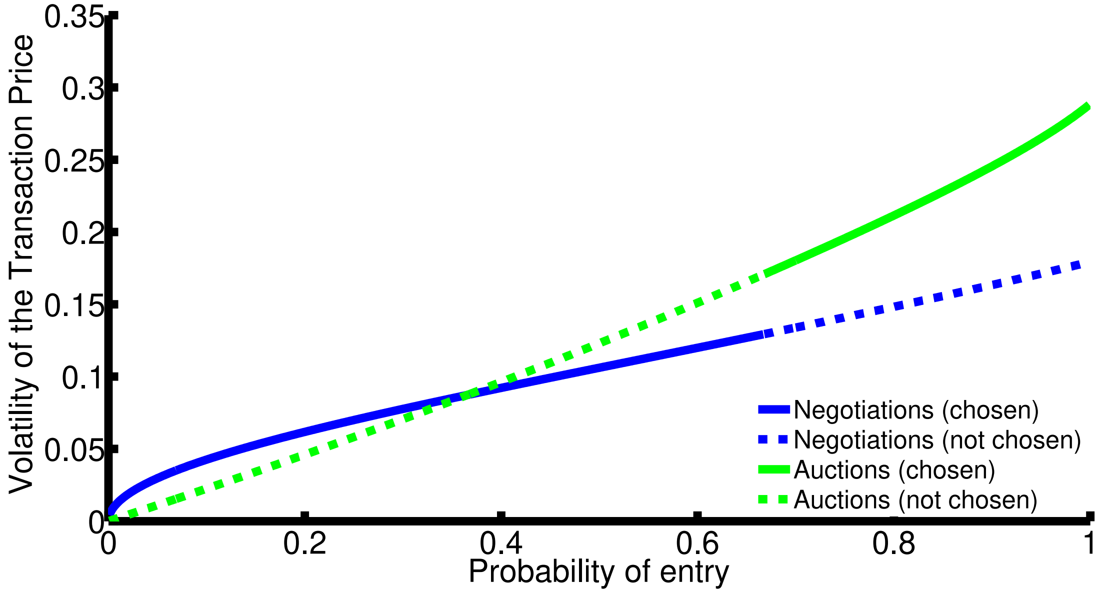


Figure 1.13. Volatility of the transaction price as a function of the probability of entry
The solid part of the line and the curve correspond to regions where the mechanism is chosen; the dotted part to regions where it is not chosen. The risk-aversion of the seller is assumed to be 0.5, and that of the buyer 0.

For a given level of the seller's risk aversion, the volatility of the transaction price increases as the industry becomes more competitive. The volatility is continuous in p at all points except at the threshold probability where the seller starts choosing auctions.

1.5.2. Mean and Volatility of the Transaction Price as a Function of the Seller's Risk Aversion

In this section, I examine how the mean and volatility of the transaction price depend on the seller's risk aversion. The buyers are assumed to be risk-neutral. The probability of entry p is assumed to be $\sim U[0,1]$.

For each value of the seller's risk aversion, the average transaction price is computed by taking the expectation of the average transaction price for a given value of p across the entire ranges of p for which that mechanism is chosen.[17] The result is plotted in figure 1.14.

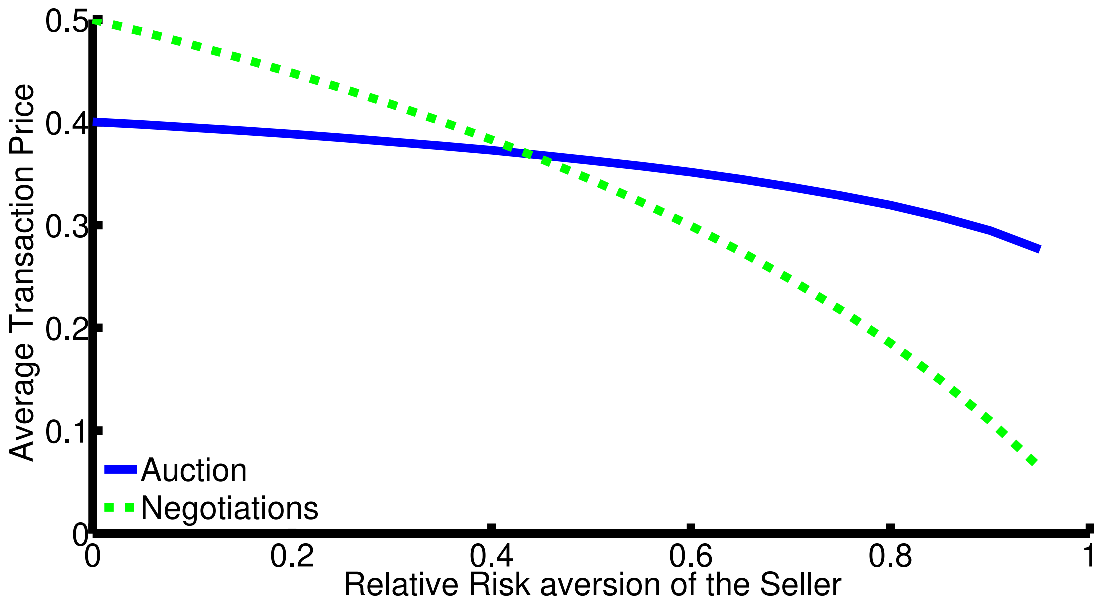


Figure 1.14. Mean of the transaction price as a function of the risk aversion of the seller

The buyers are assumed to be risk-neutral. The probability of entry p is assumed to be $\sim U[0,1]$

Empirical Implication 5. *The average transaction price for both auctions and negotiations decreases as the risk aversion of the seller increases.*

The reason why the average transaction price decreases as γ_s increases is different for auctions and negotiations. For negotiations, decrease is because the seller makes lower offers as his risk aversion increases. However, in the case of auctions, the buyers bid the same irrespective of the seller's risk-aversion. This will change the expected

[17]From figure 1.12, for auctions, this corresponds to the area under the graph from $[p_{22}, 1]$ scaled up by dividing by $1 - p_{22}$. For negotiations, it is the area under the graph from $[0, p_{22}]$ scaled up by dividing by p_{22}.

utility of the seller as he becomes more risk-averse, but why does it change the expected transaction price? The reason is purely that the range of choosing auctions increases as the seller becomes more risk-averse. Since the average transaction price conditional on p is increasing in p, the average decreases across the range $[p_{22}, 1]$ as p_{22} decreases.

Also, from figure 1.14, the average transaction price for auctions and negotiations exhibits the property of single crossing in the seller's risk aversion. This leads to the next empirical implication.

Empirical Implication 6. *Above a threshold level of the seller's risk-aversion, the expected transaction price from auctions is higher than that from negotiations. Below the threshold, the the expected transaction price from negotiations is higher than that from auctions.*

Next, I extend this analysis to compute the volatility of the transaction price. As the seller becomes more risk-averse, p_{22} decreases which affects the interval over which the computation of the volatility is done.[18] The result is plotted in figure 1.15.

Empirical Implication 7. *The volatility of the transaction price for auctions decreases monotonically as the risk aversion of the seller increases. The volatility for negotiations is non-monotonic in the risk aversion of the seller.*

[18]Unlike the unconditional mean of the transaction price, the unconditional variance cannot be computed by taking the expectation of the conditional variance given p. To understand the computation of the volatility, refer back to figure 1.12 and 1.13 which give the volatility and mean conditional on p. The unconditional volatility has two components since

$$Var(Price) = \mathbb{E}[Var(Price \mid p)] + Var(\mathbb{E}[Price \mid p]).$$

For auctions, the expectation is computed in the interval $[p_{22}, 1]$ where auctions are chosen. For negotiations, the expectation is computed in the interval $[0, p_{22}]$ where negotiations are chosen.

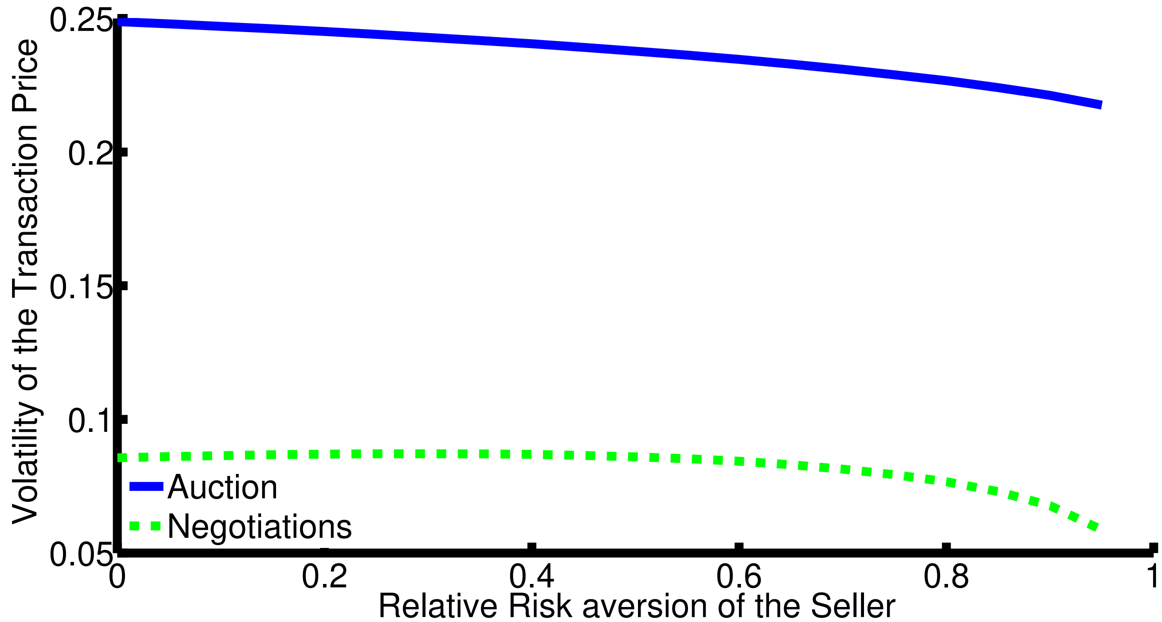


Figure 1.15. Volatility of the transaction price as a function of the risk aversion of the seller

The buyers are assumed to be risk-neutral. The probability of entry p is assumed to be $\sim U[0,1]$

As the risk aversion of the seller approaches 1, the volatility of the transaction price for negotiations drops rapidly since the seller makes very low offers. This has two effects. First, the transaction almost always goes through and second, conditional on the transaction going through, the offer made is not very volatile.

However, for auctions, as the risk aversion of the seller approaches 1, the volatility of the transaction price does not drop to 0, since the bids of the buyers are not affected by the seller's risk-aversion. The bids vary depending on the realised value of synergies unlike the seller's offers in negotiations which are independent of the realised value of synergies. So, the following empirical implication holds for all values of the coefficient of relative risk aversion of the seller in (0,1).

Empirical Implication 8. *The volatility of the transaction price for auctions is always greater than that for negotiations.*

1.6. Concluding Remarks

Recent empirical research has shown the importance of recognizing the intricacies of the sales process in analyzing corporate takeovers. Yet, prevailing models have not captured an important source of buyer uncertainty: buyers may not know how many other buyers are participating in the process. This assumes added significance in sales of firms because the average number of buyers is not just uncertain but very low. This is the first study to explicitly model how this uncertainty affects the seller's choice of mechanism between auctions and negotiations.

The main finding is that if the seller cannot commit to choosing the mechanism before he sees the number of buyers, the choice of the mechanism in the revenue-maximising equilibrium or any pure strategy equilibrium depends only on the buyers' prior beliefs about competition. It is independent of the actual competition in terms of the number of buyers who participate in the sale.

The seller can increase the revenue by committing to a choice of mechanism that depends only on the potential number of buyers and the likelihood of each entering, not on the actual number of buyers that participate. I show that the seller's choice can also be influenced by the volatility of synergies and whether the synergies are independent or correlated.

The model can be used to generate testable empirical hypotheses about the mean and volatility of the observed transaction prices. The model can also be extended to study situations that arise while selling assets rather than the whole firm. For example, consider a firm trying to spin off its assets. The firm has a choice of selling related subsidiary businesses together or separately in a sequential process. It is possible that the set of potential buyers is the same for all the subsidiaries. Whether the subsidiaries are sold separately can depend on how the buyers' beliefs about the competition in the later rounds of the sale change based on the transaction prices observed in the earlier rounds. If the competition is likely to be low and the buyers can use the prices to update their prior beliefs, a firm might prefer to sell the businesses together rather than separately. Empirical data on spinoffs can be analyzed to verify the predictions of the extension.

The theory developed in this paper sheds light on how firms are sold and goes some way towards explaining the various factors that underlie the choice of the sale mechanism. A lot of future work awaits, both in extending the model to make it richer and testing it empirically.

CHAPTER 2

Optimal M&A Advisory Contracts

2.1. Introduction

Mergers and acquisitions are a significant mechanism of allocating assets to their most productive users. A merger transaction need not be a zero-sum game; it can leave both the buyer and the seller better off. Mergers can have real effects by increasing the market power of firms and affecting ownership patterns in the economy. A well-functioning market for corporate control can serve as a barometer of the health of the economy and its effectiveness in driving weaker firms out of business.

In the US, there have historically been periods of frantic merger activity, commonly referred to as merger waves, interspersed with periods of relative calm marked by lesser consolidation. The first merger wave at the turn of the nineteenth century was marked by a series of horizontal consolidations instrumental in the emergence of the large modern corporation. Five other merger waves followed, each with its own unique characteristics. There are indications that we might be experiencing a seventh merger wave. In 2015, M&A activity reached very high levels, with $4.28 trillion worth of deals worldwide. There were 4,786 deals amounting to $1.97 trillion (approximately 11% of GDP) in the US alone.[1] Both the global and American total deal values were the highest recorded till then.

[1]Source: "Global and regional M&A: 2015", report by *Mergermarket*. Web. Accessed 23 October 2016.

Many of these deals featured M&A advisory firms advising the seller, the buyer or both. In fact, some of the prominent deals featured multiple investment banks helping the same party. The advisory fees from completed transactions alone were estimated to be $29.4 billion.[2]

If advisory firms are so common in acquisitions, what are the functions they perform? Servaes and Zenner (1996) compare acquisitions completed with and without investment banks to see why banks are hired and what functions they perform. They argue that investment banks decrease transaction costs by being able to analyze acquistions at a lower cost and thus reduce the asymmetric information inherent in any merger transaction.

Asymmetric information between the seller and the buyer regarding the synergies of the transaction are pervasive in acquisitions. The buyer and the seller often have drastically different assessments of the synergies in the transaction. This can lead to a socially efficient transaction failing or an inefficient transaction going through. A recent example is the failure of Microsoft to acquire Salesforce. Although Microsoft was willing to offer roughly $55 billion for the company, Microsoft's offer was met with counteroffers from Salesforce which were as high as $70 billion.[3] Another example of a possibly momentous merger failing due to disagreements beween the target and bidder over the price occured when negotiations between Uber and Lyft fell through in 2014.[4]

Since investment banks decrease the asymmetry of information, their presence may make the difference between the success or failure of the merger, both in terms of the

[2]Source: "Mergers & Acquisions Review: Financial Advisors. Full Year 2015", report by *Thomson Reuters*. Web. Accessed 23 October 2016.
[3]Kedmey, Dan. "Heres Why Microsoft Didn't Buy Salesforce." *Time Magazine* 22 May 2015. Web. Accessed 23 October 2016.
[4]Shen, Lucinda. "Uber Confirms 2014 Negotiations to Buy Lyft." *Fortune Magazine* 2 September 2016. Web. Accessed 23 October 2016.

transaction happening and in terms of the later performance of the target firms. However, the presence of an investment bank introduces another friction into the dynamics of the transaction, namely moral hazard. This is because exerting effort to value the synergies is costly for the investment banks.

Since the effort is costly and frequently not observable or contractible, the investment bank has an incentive to report a value of synergies without doing any investigation. The seller and buyer must incentivise their advisors to put in effort by structuring the wage contract with him appropriately. Hunter and Walker (1990) find that gains from a merger are associated with the banks exerting effort.[5]

McLaughlin (1990) points out that making fees contingent on the success of the transaction can lead to conflicts of interest between banks and bidding firms. These kind of contracts are not optimally constructed to solve the moral hazard problem. The bidder's bank may ask the bidder to bid high to ensure the transaction going through and pocket the fees. Even with optimal contracts, it is not obvious that the first best can be realized. McLaughlin (1992) finds evidence from examining tender offers that the effectiveness of fee contracts in solving agency problems in tender offers is mixed.

It is very much possible that an expert is hired to provide a valuation for the target rather than an investment bank. A case in point is PE funds in India, who are hiring industry veterans. An expert mentions that providing information on target valuations is the primary function he performs:

[5]However, their proxy for effort is the ease with which the merger negotiations were conducted. It is not clear that this is the best proxy for effort . For instance, a merger can happen if the buyer's advisor reports the highest possible value of synergies without putting in any effort because the buyer will bid the highest possible value.

> Sumit Banerjee, a cement hotshot for over a decade, is the go-to man for PE and strategic players whenever a new target comes into the salemarket. Banerjee, in the last two years, has advised at least three potential suitors Apollo, Blackstone and even Piramal Enterprises on as many occasions as they went after Lafarge's India operations and Reliance Cement... "For each of the evaluations, the funds had the bandwidth to do modelling and financial projections. All they needed was someone to validate their assumptions on market growth, prices and people. My desire was to advise them on the fundamental strength and weaknesses of the target which have a direct bearing on valuations."[6]

In practice, M&A advisors may also perform a number of other services like legal services, post-merger integration consulting services, searching for bidders or targets, advice on restructuring the target and helping to raise capital to finance the acquisition. In this paper,we focus only on the valuation services provided by the advisors. As we have argued, this is one of the most important functions of the advisor, if not the most important. In addition, nothing prevents the seller or buyer from offering separate contracts to the advisors for the other functions and the valuation services.

Many of the advisory contracts seen in practice are linearly increasing in transaction value. Two justifications for this both turn out to be false under closer examination. First, a fee increasing in value is reminiscent of Hölmstrom (1979). The resemblance is superficial. In Hölmstrom (1979), the wage depends on output because effort is not

[6]Barman, Arijit & Layak, Suman. "Why private equity funds and banks are wooing former head honchos of India Inc to manage their investments". *The Economic Times* 18 September 2016. Web. Accessed 23 October 2016.

observable, but output, which is increasing in effort, is. However, the value of synergies is completely independent of the advisor's effort. Hence, that is not the mechanism underlying the optimal contract. Second, investigating the value of a big firm might involve a higher cost of effort than for a small firm, so one would expect the payment to the advisor to be increasing in value. The fallacy in this argument is that we are comparing the payments across two transactions, one the sale of a small firm and another a large one. This paper is about why the fee paid in a *given* transaction depends on the value of the transaction. It is a comparison among different realizations of value in a given transaction than different expected values *across* transactions.

We model the sale as happening through a take-it-or-leave-it offer made by the seller to the buyer. We also assume that the seller and the buyer choose to hire an advisor or not without knowing whether the other party has a hired an advisor or not. As already mentioned, the friction is that the advisor can report a value of the synergies without having exerted the effort to find out the value.

The research question I address in this paper is the structure of the optimal contract between the acquirer/target and their M&A Advisor to overcome this moral hazard problem. This can be split into a number of sub-questions.

First, what observable parameters does the contract depend on? Does it, for example, depend on the value of the transaction or whether the transaction was a success or not? Is it increasing in the value of the transaction? Or is the contract a flat fee? Are the optimal contracts unique?

Second, if effort is costly for the advisors, for what range of the effort costs are the advisors hired? Intuitively, there must be an upper bound above which the seller or the buyer do not wish to hire the advisor since the cost is greater than the value of knowing the synergies precisely. What is the upper bound, given that neither party knows whether the other is informed or not? Third, do the advisors report truthfully in equilibrium? Do they extract information rents in equilibrium? In other words, will the payment to either advisor be higher than their cost of effort?

Fourth, are there pure strategy equilibria where the buyer/seller always hires an advisor or does nott? Can we have mixed strategy equlibria too, with the seller and the buyer mixing between hiring and not hiring the advisor)? If so, what are the probabilities with which the advisors are hired and how do they depend on the effort costs? Fifth, what are the strategies of the seller and the buyer when they are informed and uninformed i.e. when they have hired an advisor or have not? Does the seller always charge the fair price when he is informed? Does the buyer always accept it?

Sixth, what are the implications of the moral hazard problem on efficiency? How close do we get to first-best? Assuming that the synergies are always greater than zero and it is optimal to sell the firm to the acquirer, how often does the transaction fall through in spite of it being optimal? How does this affect the total surplus in the transaction?

We solve the problem in three steps, starting with a simple set up and gradually making it more complex. In part I, we consider an equilibrium where the buyer is always informed and the seller has to minimize the payment to his advisor subject to incentivising the advisor to exert effort. In other words, this part considers the optimal contract under the simpler case where the buyer always hires an advisor. In part II, we argue that this may not be a realistic assumption and solve for a mixed equilibrium where the buyer and seller mix between hiring their advisor and not hiring him. This part treats the payment to the advisor as an exogenous parameter i.e. does not endogenise the contract. In part III, we combine parts I and II. We attack the grand question of how the optimal contracts look like when both the buyer and seller optimally choose whether to hire an advisor or not.

The main results are as follows. First, the optimal contract for both the buyer and the seller depends on whether the transaction succeeded or not and, if it did, the value of the transaction. There is no payment to the advisor if the transaction failed. The seller's optimal contract with his advisor is unique and the advisor's compensation is monotonically increasing in the transaction value. The buyer's optimal contract with his advisor can take a variety of forms. However, all of these contracts share the feature that the advisor is not paid if he reports the same value to the synergies as the maximum offer the uninformed buyer would have accepted. Intuitively, the advisor is not paid if he made no difference to the buyer's decision whether to accept the offer.

Second, there is a wide range of effort costs for which the equilibria exist. We make the simplifying assumption that the costs for both the advisors are the same and solve for the equilibrium strategies as a function of the exogenous effort costs. Third, the contract incentivises the advisors to exert effort and report truthfully in equilibrium. The advisors do not extract information rents in equilibrium. Fourth, there is a range of values for which we observe mixed equilibria. I fully characterize the strategies of the buyer and seller i.e. their probabilities of hiring their advisor in terms of the exogenous effort costs of the advisor. The buyer's propensity to acquire information increases when the seller doesn't do so. Fifth, there is a range of the value where the informed seller charges a fair price, but there are ranges where he undercharges and overcharges as well.

Sixth, the first best is never obtained. Even though the advisors do not obtain information rents, there are two other sources of inefficiency. The first is that in equilibria where one party ends up informed and the other uninformed, the asymmetry of information leads to the transaction failing. The probability of the transaction succeeding decreases as the advisor's cost of effort increases. The second source of inefficiency is that even if the transaction is a success, the advisor has to be a paid its cost of effort which destroys some of the surplus in the transaction. The total surplus in the transaction is less than the expected value of the synergies.

The primary contribution I make in this paper is to characterize the optimal contract in a mergers and acquisition setting. Although there have been a few empirical studies on advisory contracts in mergers, there have not been theoretical justifications of the same. I solve this contracting problem along with the related problem of costly acquisition of socially inefficient information.The optimal contract with the advisor depends on whether the other party hires the advisor or not. To solve for the contract, one needs to know when the parties involved decide to acquire the information and vice versa.

The rest of the paper is organized as follows. Section 2.2 reviews relevant related literature. Section 2.3 provides the finer details of the model and discuss the assumptions thoroughly. Sections 2.4, 2.5 and 2.6 solve the model in three parts as mentioned above. Section 2.7 summarizes the results by giving a complete characterization of the Equilibria. Section 2.8 analyzes the implications of the equilibria for efficiency. Section 2.9 generates testable empirical hypotheses and validates empirical work already done on the topic. Section 2.10 concludes.

2.2. Related Literature

Servaes and Zenner (1996) was among the earliest papers to shed light on the factors affecting the hiring of investment banks. They compare 99 transactions from 1981 to 1992 which featured an investment bank to 198 transactions that did not. They find that banks are more likely to be hired for complex transactions and if the targets operate in many industries. This leads them to conclude that transaction costs and, in part, contracting costs and information asymmetries affect the decision to hire a bank. Owsley and Kaufman (2005) is a good description of the role of investment banks in bankruptcy. They make the case that handling distressed purchase and sale transactions require more skill than sales involving solvent companies.

Few empirical studies have looked at the actual contract between the advisors and the seller or buyer.[7] McLaughlin (1990) examines 195 tender offers between 1978 and 1985 and describes incentive problems associated with the various kind of contract designs. The contract fees fall into three basic categories: fixed fees, shares-based fees (used by buers), and value-based fees (mostly used by sellers). McLaughlin (1992) finds that fee contracts are used by both firms and bankers to solve agency problems, but they do not eliminate them. There are associations between firm objectives and contract incentives and between incentives and offer outcomes in some tests, but not in others. Hunter and Walker (1990) find that in their sample, the investment bank contracts were mostly fixed fee contracts or were based on the transaction price, contingent on the success.

There have been many studies looking at the association between the decision to hire an investment bank and the outcome of the merger. Hunter and Walker (1990) was one of

[7]This may be due to data limitations. Even for public deals, the fees paid to the advisor are not always disclosed since the disclosure is not mandatory

the earliest studies to find a positive relationship between fees, merger gains and banker effort. They conclude that investment banking merger fee contracts are designed with the aim of incentivizing optimal banker effort. Servaes and Zenner (1996) do not find any difference between the returns earned by acquirers which hired a bank and those that didn't. Daniels and Phillips (2009) show that hiring a financial advisor is associated with increases the transaction value in REIT mergers because advisors reduce the asymmetric information between the target and the bidder. Golubov et al. (2012) argue that the effect of the hiring the advisor may vary depending on the listing status of the target.

In recent years, a host of studies, too numerous to recount exhaustively, have analysed the role of bank's reputation on the gains from the merger. Kale et al. (2003) find evidence that the reputation of the financial advisor affects wealth gain in the transaction and how it is split between the target and the acquirer. Hunter and Jagtiani (2003) document a negative relationship between reputation and merger outcomes. The synergistic gains realized by the acquirers declined when top advisors were used. However, the contingent fees played a significant role in expediting the deal completion. Ismail (2010) point out that in their sample, acquirers advised by tier-one advisors lost more than $42 billion, but those advised by tier-two advisors gained $13.5 billion. They attribute this to the large loss deals advised by tier-one advisors, citing the differing incentives of banks in large and small deals as the reason. Rau (2000) finds that the post-acquisition performance of the bidding firm is negatively related to contingent fees in tender offers, porssibly because the banks just try to complete the deal.[8] Golubov et al. (2012) use a sample of U.S. acquisitions of public, private, and subsidiary firms from 1996 to 2009. They find that

[8]This is exactly the moral hazard problem we model in this paper.

top-tier advisors are associated with higher bidder returns in public acquisitions because they garner a greater share of synergies for the bidder. The effect is dampened when the target advisor is also top-tier.

Another strand of related literature looks at how information acquisition can destroy efficiency. A recent example is Glode et al. (2012) where two risk-neutral traders exchange an asset. Acquiring expertise is neutralized in equilibrium because the counterparty also acquires it.[9] Shavell (1994) considers a scenario where buyers and sellers can acquire and disclose information prior to sale. He concludes that if disclosure is voluntary rather than mandatory, there are socially excessive incentives to acquire information.

The contribution we make in this paper is to tie together these two strands of research. I argue that the two problems are connected. Incentives to acquire information depend on the other party's cost of acquiring information, which in turn is a function of the optimal contract. So, it is impossible to describe the optimal contract without knowing the probabilities with which the buyer and seller are informed. Similarly, it is impossible to know the probability with which the buyer and seller are informed without knowing the cost of the information if the contract is optimal.

[9]Their model differs from ours in quite a few ways. First, their value is binary, whereas we consider a uniformly distributed value. Second, they give the bargaining power to the buyer and we to the seller. Third, they assume that synergies are known and value unknown; we assume that the value is known and synergies unknown. Lastly and most significantly, they assume an exogenous cost of acquiring information whereas we endogenize the cost through optimal contract design.

2.3. Modelling the M&A Process

2.3.1. The Framework

The target (seller) and the acquirer (buyer) are both risk-neutral. The stand-alone value of the firm and the distribution of the synergies are common knowledge. However, the realized value of the synergies V is unknown unless an M&A advisor exerts effort to discover it.

The synergies are always positive, so it is socially optimal to trade. The seller and the buyer thus have to decide how to split the uncertain synergies between them.[10] This creates a problem of asymmetric information if either of them knows the value and the other doesn't. As a result, the transaction may fail even though it is socially efficient to transfer the asset.

The acquisition process can proceed through various mechanisms, for instance, an auction, many rounds of bargaining etc. To simplify the analysis, we assume that the seller has the bargaining power and the sale happens through a take-it-or-leave-it offer made by the seller to the buyer.

The seller and the buyer have the option of hiring an M&A advisor who can exert effort to find the realized value. An example would be hiring an investment bank.[11] Neither party observes whether the other has hired an advisor or not. In other words, the seller, when making the offer, is unaware of whether he is facing an informed buyer or an uninformed one. Similarly, the buyer, when deciding whether to accept the offer, is unaware of whether the offer was made by an uninformed seller or an informed one.

[10] From here on, we use "value" to denote the realized value of the synergies.

[11] In the rest of the paper, we use the terms "bank" and "advisor" interchangeably.

The advisor has a cost of exerting effort c. If the advisor exerts effort, he gets to know the value exactly. If he reports truthfully to the party which hired him, they get to know the exact value as well.

Both the seller and the buyer face a problem of moral hazard since the effort exerted by the advisor is not observable and, consequently, not verifiable. This prevents the seller and buyer from paying the advisor depending on the effort exerted by the advisor even though they would like to. I assume that the wage can only depend on the value reported by the advisor and whether the sale happened i.e. whether the buyer accepted the seller's offer. These assumptions are discussed in more detail in section 2.3.4.

The optimal contract between the seller (or buyer) and his advisor minimizes the expected payment to the advisor conditional on incentivizing the advisor to exert effort. The contract specifies a wage schedule $w : \mathbb{R}^+ \times \{0, 1\} \to \mathbb{R}^+$. The wage $w(V, 1)$ depends on the value V reported by the advisor and a binary variable which takes the value 1 when the transaction succeeds and 0 when it did not.

2.3.2. The Timing

The timing of the game is as follows:

(1) The value V is realized.

(2) The seller and buyer each decide whether to hire an advisor or not.

(3) If he hires an advisor, the seller (buyer) offers a wage contract to the advisor.

(4) The advisor accepts the contract or rejects it.

(5) If the advisor accepts the contract, he chooses whether to exert effort or report a value without exerting effort.

(6) The advisor reports a value to the party who hired him.

(7) The seller makes an offer to the buyer.

(8) The buyer accepts the offer or rejects it

(9) If the buyer accepts the offer, the sale happens at the offer price. The buyer's payoff is V - price - wage paid to the advisor and the seller's payoff is price - wage paid to the advisor.

(10) If the buyer rejects the offer, the sale doesn't happen. The payoff for the seller and the buyer is equal to - wage paid to the advisor.

2.3.3. Solution Concept

The solution concept is perfect Bayesian equilibrium. Recall that in a PBE, the strategies of the players have to be sequentially rational and the beliefs have to be consistent with the equilibrium strategies whenever possible. Specifically, we look for mixed equilibria where the buyer and seller are indifferent between hiring and not hiring the advisor.

2.3.4. Discussion of the Assumptions

In this section, we provide justifications for some of the assumptions we make about the M&A process.

2.3.4.1. The Bargaining Power. Empirical studies of mergers and acquisitions have repeatedly confirmed a puzzling trend- following a merger announcement, the target's stock price increases on average and the acquirer's stock price falls.[12] For example, when AT&T's acquisition of Time Warner was reported, Time Warner's share price rose 9%

[12]One of many such studies is Andrade et al. (2001).

while AT&T's dipped close to 3%.[13] This suggests that if there are any synergies in the transaction, the target extracts most of them. Hence, it seems reasonable to assume that in the negotiation process, the target (or seller) has the bargaining power.

2.3.4.2. The Take-it-or-leave-it Offer. In reality, firms can be sold through auctions or negotiations. In recent years, approximately 50% of the transactions are auctions and the other negotiations.[14] Even in a negotiation, the bargaining process can be qute complicated. The assumption that the seller is negotiating only with one buyer and that the sale happens through a take-it-or-leave-it offer is done for tractability.

2.3.4.3. Neither the Seller nor the Buyer Observes whether the Other has Hired an Advisor. This might seem like a strong assumption. However, a bank may be hired for many reasons. As long as both parties are unaware whether a bank has been hired by the other party specifically for valuation rather than something else, this assumption is justified. Second, it is sufficient if neither party observes the other hiring a bank till the sale is completed. Lastly, an alternative interpretation is that even if the bank is hired and exerts effort, the bank gets to know the value of the firm only with a specified probability. Even if the parties know that the other party hired a bank, they cannot be sure that the bank got to know the value. If the bank doesn't know the value, it is tantamount to the bank not being hired at all.

2.3.4.4. The Wage Does Not Depend on the Stock Market Reaction. There are two reasons for assuming this. First, we do not want to exclude the possibility that the target or the acquirer may be private. Second, in practice, contracts usually depend

[13] Rushe, Dominic and Thielman, Sam. "AT & T in advanced talks to acquire Time Warner, reports say" *The Guardian* 21 October 2016. Web. Accessed 23 October 2016.

[14] For more detailed empirical analysis of the same, see Boone and Mulherin (2007). For the theory underlying the choice, see Vasu (2015).

only on whether the transaction succeeded and what the value was, and almost never on the stock market reaction following the announcement or in subsequent years. This may have to do with the complexity of an M&A transaction. It may be the case that the market reaction is not an accurate assessment of a merger transaction, which justifies this assumption.

2.3.4.5. Both Parties Do Not Know the Value. Synergies may depend on the information that both parties possess individually, which makes it credible that neither party knows the value of synergies.

2.4. Part I - Optimal Contract when the Buyer is Always Informed

Before considering the case where both seller and buyer are uninformed, I consider the simpler case where the buyer has access to information at zero cost and always knows the realized value of the synergies. The seller does not know the value of the synergies unless he hires an advisor.

2.4.1. Three Possible Values for the Synergies

To simplify matters further, we start with the the synergies taking one of three possible values with equal probability of $\frac{1}{3}$. Without loss of generality, let $V \in \{\frac{1}{3}, \frac{2}{3}, 1\}$.

If the advisor reports the realized value truthfully, the distribution of the reported value is the same as that of the actual value.[15] Also, the transaction always goes through if the advisor reports truthfully. So, the expected payment made by the seller if the advisor reports truthfully is $\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1)$.

[15] I will later impose the constraint that the advisor has no incentive to misreport.

From the advisor's point of view, its payoff is the difference between the expected payment and the cost of effort c. However, the advisor has the option of not exerting effort and just reporting any one of the three values of synergies. If say the advisor reports the value to be $\frac{2}{3}$ and the seller makes an offer of $\frac{2}{3}$ to the buyer, the transaction would be a success if the realized value is greater than or equal to $\frac{2}{3}$ which happens with probability $\frac{2}{3}$. In this case, the advisor would get the wage $w(\frac{2}{3}, 1)$. However, if the transaction fails, which happens with probability $\frac{1}{3}$, the bank gets $w(\frac{2}{3}, 0)$. So, the advisor's payoff from not exerting effort and reporting a value of $\frac{2}{3}$ is $\frac{2}{3}w(\frac{2}{3}, 1)+\frac{1}{3}w(\frac{2}{3}, 0)$. To make the advisor exert effort, the seller has to make sure that the payoff is higher than that from not exerting effort, which leads to the equation

$$\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1) - c \geq \frac{2}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 0).$$

Similarly, one can write down two other IC constraints for the other possible values he can report, $\frac{1}{3}$ and 1.

The seller's problem is thus to minimize

$$\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1)$$

subject to the constraints

$$\begin{aligned}
&\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1) - c \geq w(\frac{1}{3}, 1)\\
&\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1) - c \geq \frac{2}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 0)\\
&\frac{1}{3}w(\frac{1}{3}, 1) + \frac{1}{3}w(\frac{2}{3}, 1) + \frac{1}{3}w(1, 1) - c \geq \frac{1}{3}w(1, 1) + \frac{2}{3}w(1, 0).
\end{aligned}$$

It is optimal to set $w(V,0)=0$ since the transaction fails only if the bank either did not exert effort or report truthfully, both of which he should be penalised for.

The seller's problem is thus to minimize

$$\frac{1}{3}w(\frac{1}{3},1)+\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)$$

subject to the constraints

$$\begin{aligned}
\frac{1}{3}w(\frac{1}{3},1)+\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c &\geq w(\frac{1}{3},1)\\
\frac{1}{3}w(\frac{1}{3},1)+\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c &\geq \frac{2}{3}w(\frac{2}{3},1)\\
\frac{1}{3}w(\frac{1}{3},1)+\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c &\geq \frac{1}{3}w(1,1).
\end{aligned}$$

This is a linear programming problem. The solution is given by

$$\begin{aligned}
w(\frac{1}{3},1) &= \frac{6}{5}c,\\
w(\frac{2}{3},1) &= \frac{9}{5}c,\\
w(1,1) &= \frac{18}{5}c.
\end{aligned}$$

The optimal wage schedule is plotted in Figure 2.1. Several features of the optimal wage schedule are noteworthy.

First, the expected payment to the advisor is given by $\frac{11}{5}c$. Since the bank's cost of effort is c, the advisor makes an expected profit equal to $\frac{6}{5}c$. In other words, the advisor is able to extract information rents from the seller.

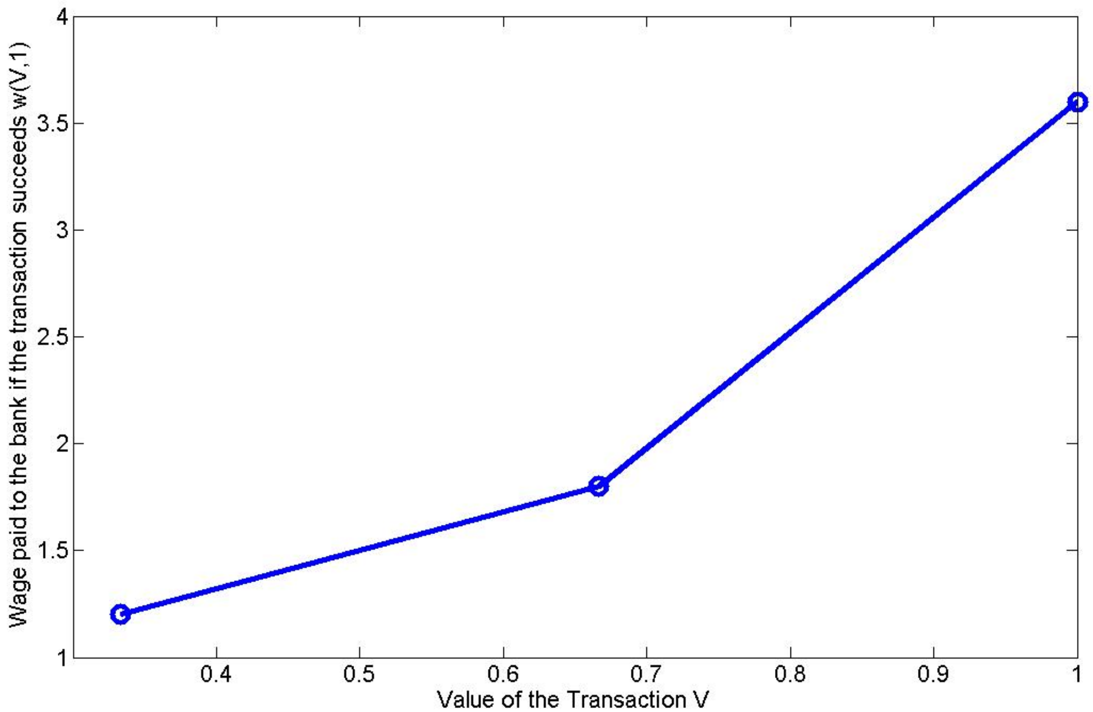


Figure 2.1. The wage schedule with 3 possible values of the synergies
The figure shows the optimal wage schedule when the synergies are $\frac{1}{3}, \frac{2}{3}$ or 1 with equal probability. The advisor's cost of effort c is assumed to be 1 (Alternatively, the y axis is scaled by c). The wage is an increasing function of the transaction value.

For what range of c will the seller hire the advisor? if he doesn't hire the advisor, the seller gets an expected payoff of $\frac{1}{4}$.[16] If he hires the the advisor, the seller can charge a fair price and get an expected payoff equal to the expected value i.e. $\frac{1}{2}$. So, he is willing to hire the advisor as long as the benefits exceed the cost, i.e.

$$\frac{1}{2} - \frac{1}{4} \geq \frac{11}{5}c \implies c \leq \frac{5}{44}.$$

[16]The seller offers Q to maximize $Q(1-Q)$, Q being the payoff if the buyer accepts the offer and $1-Q$ being the probability with which the buyer accepts the offer. The expression is maximized at $Q = \frac{1}{2}$ and the maximum value is $\frac{1}{4}$.

Second, not only is the expected profit of the bank greater than 0, but each of the wages is greater than the cost c. In other words, the bank always extracts information rents and not just in expectation.

Third, note that each of the inequalities is satisfied with equality since

$$\frac{1}{3}(w(\frac{1}{3},1)+w(\frac{2}{3},1)+w(1,1))-c=w(\frac{1}{3},1)=\frac{2}{3}w(\frac{2}{3},1)=\frac{1}{3}w(1,1)=\frac{6}{5}c.$$

This is a general feature of such contracts, a characteristic we will revisit later.

Fourth, the advisor has no incentive to misreport once he has exerted the effort. Since the wages are increasing in the value reported, misreporting to a lower value would lead to the transaction going through and the advisor getting a lower wage than if he had reported truthfully. Misreporting to a higher value would lead to the transaction failing i.e. a wage of 0.

Intuition might suggest that the optimal contract should set $w(\frac{1}{3},1)=0$. Just for illustrative purposes, if we were to set $w(\frac{1}{3},1)=0$, the optimal contract would need to solve

$$\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)$$

subject to the constraints

$$\begin{aligned}
&\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c\geq 0\\
&\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c\geq \frac{2}{3}w(\frac{2}{3},1)\\
&\frac{1}{3}w(\frac{2}{3},1)+\frac{1}{3}w(1,1)-c\geq \frac{1}{3}w(1,1).
\end{aligned}$$

The solution to this problem is given by

$$w(\frac{1}{3},1) = 0, w(\frac{2}{3},1) = 3c, w(1,1) = 6c.$$

The expected payment to the bank is given by $3c$ which is greater than $\frac{11}{5}c$ derived earlier. A comparison between the optimal wage schedule and this one is plotted in Figure 2.2.

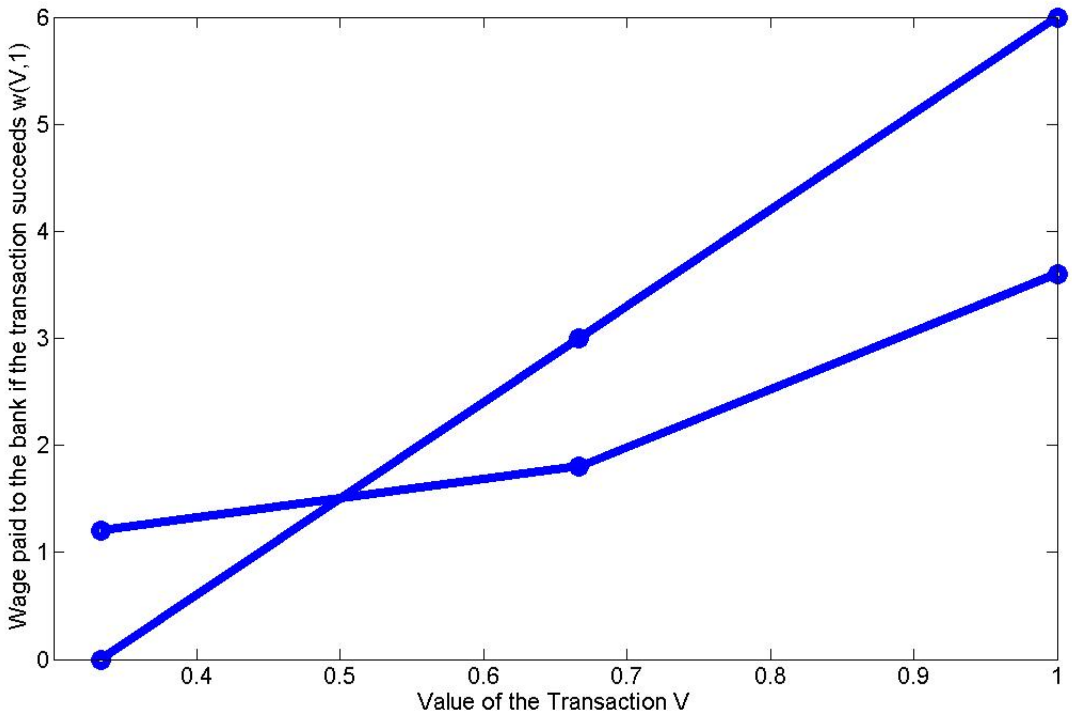


Figure 2.2. A comparison of the optimal wage schedule and one where the minimum wage is set to zero

The figure shows the comparison between the optimal wage schedule and the best wage schedule when the minimum wage is set to 0. The synergies are $\frac{1}{3}$, $\frac{2}{3}$ or 1 with equal probability. The advisor's cost of effort c is assumed to be 1. While it is true that the optimal wage schedule leads to a nonzero payment when the minimum value is reported, the lesser payment if the value reported is $\frac{2}{3}$ or 1 more than cancels out the gains from setting the minimum wage to 0.

2.4.2. Discrete Uniform Distribution of Synergies

Now consider the case where the synergies have a discrete uniform distribution taking one of n possible values with equal probability $\frac{1}{n}$. Without loss of generality, let $V \in \{\frac{1}{n}, \frac{2}{n}, ..., 1\}$.

The seller's problem is thus to minimize

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]$$

subject to the n constraints

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c \geq w(\frac{1}{n},1)$$

....

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c \geq \frac{n-k+1}{n}w(\frac{k}{n},1)$$

....

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c \geq \frac{1}{n}w(1,1).$$

The optimal wage contract satisfies each of the n constraints with equality. The n wages in the optimal contract are given by

$$w(\frac{1}{n}, 1) = \left(\frac{1}{\frac{1}{2} + \frac{1}{3} ... + \frac{1}{n}}\right) c$$

....

$$w(\frac{k}{n}, 1) = \left(\frac{1}{\frac{1}{2} + \frac{1}{3} ... + \frac{1}{n}}\right) \frac{n}{n-k+1} c$$

....

$$w(1, 1) = \left(\frac{1}{\frac{1}{2} + \frac{1}{3} ... + \frac{1}{n}}\right) nc.$$

For proof, see Appendix B.1. Figure 2.3 shows the shape of the contract for different values of n. The wage is monotonically increasing in the transaction value.

A noteworthy feature of the contract is that the expected payment to the advisor converges to c as $n \to \infty$. Thus, the information rents extracted by the advisor (also equal to the wage on reporting the lowest value) decrease to 0 as n becomes very large. How is the advisor able to break even? It is because the maximum wage, which corresponds to reporting the maximum possible value, increases as n increases, even as the lowest wage falls. Figure 2.4, which plots the maximum wage against n, makes this clear.

Another feature of the contract is that it is robust to misreporting once the advisor has exerted the effort. Misreporting to a higher value leads to the transaction failing and the advisor receiving a wage of 0. Misreporting to a lower value results in the transaction

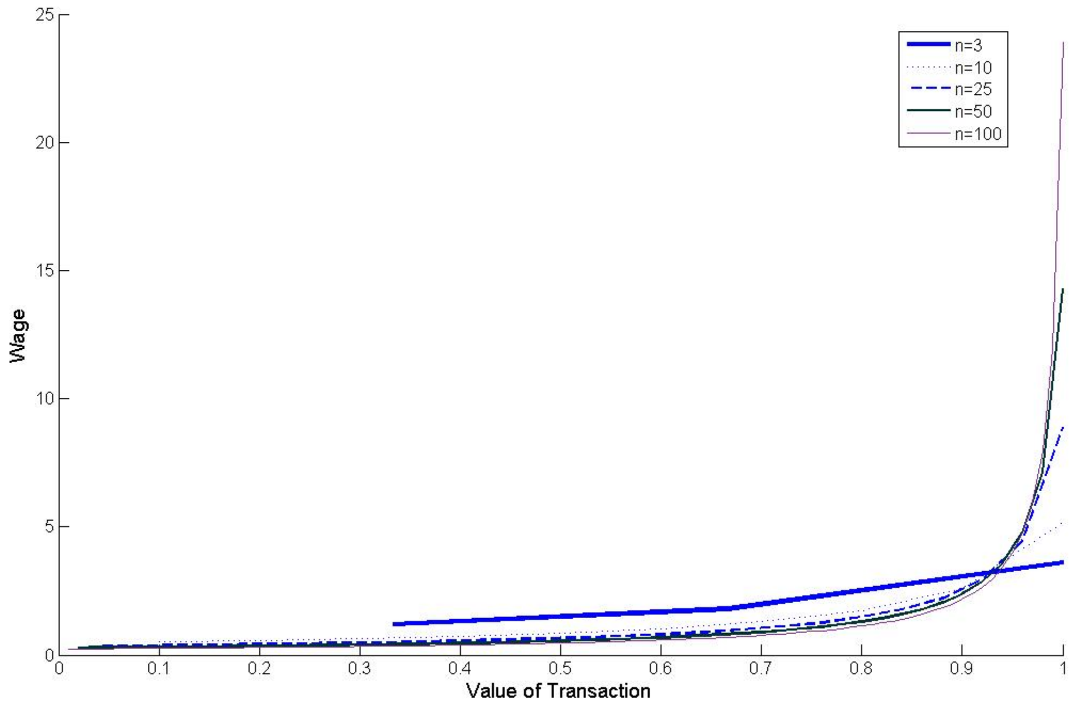


Figure 2.3. The optimal wage contract for different values of n
The figure shows the optimal wage schedule for $n = 3, 10, 25, 50$ and 100. The advisor's cost of effort c is assumed to be 1 (Alternatively, the y axis is scaled by c). The wage is an increasing function of the transaction value. The information rents extracted by the advisor (also equal to the wage on reporting the lowest value) decrease to 0 as n becomes very large. In other words, the expected payment to the advisor approaches the cost of effort. Also, the maximum wage, which corresponds to reporting the maximum possible value, increases as n increases.

succeeding, but the advisor being paid a lower wage than it could have by just reporting truthfully.

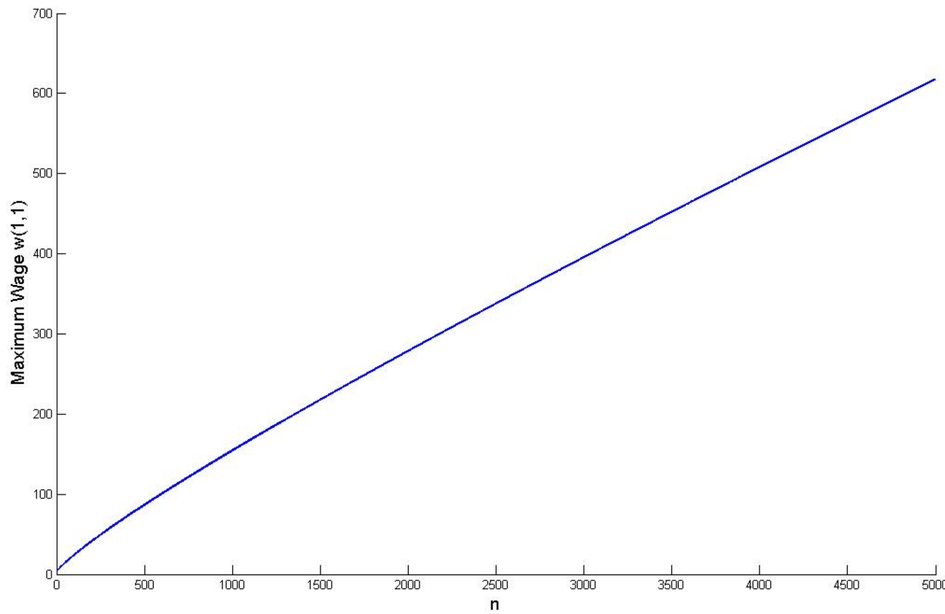


Figure 2.4. The maximum wage paid to the advisor
The figure shows the maximum wage that the advisor can possibly obtain as a function of n. This corresponds to the wage if the advisor reports the highest possible value i.e. $w(1,1)$.

2.4.3. Synergies $\tilde{V} \sim U(0,1)$

Now consider the case where the synergies are uniformly distributed in $[0,1]$. Set $w(V,0)$ to 0. Reasoning as above yields the seller's problem to be

$$\text{Minimise} \int_0^1 w(V,1)dV - c \text{ s.t.}$$

$$\int_0^1 w(V,1)dV - c \geq (1-V)w(V,1) \;\forall V \in [0,1]$$

To solve this, I consider a probability distribution for $\tilde{V}$, U_ϵ which is obtained by the following transformation of $U[0,1]$:

- Leave the distribution unchanged in the interval $[0, 1-\epsilon]$.
- Redistribute the probability mass from $[1-\epsilon, 1]$ to an atom of mass ϵ at $1-\epsilon$.

This distribution can be made arbitrarily close to the uniform distribution by letting $\epsilon \to 0$. The uniform distibution and U_ϵ are plotted in Figure 2.5.

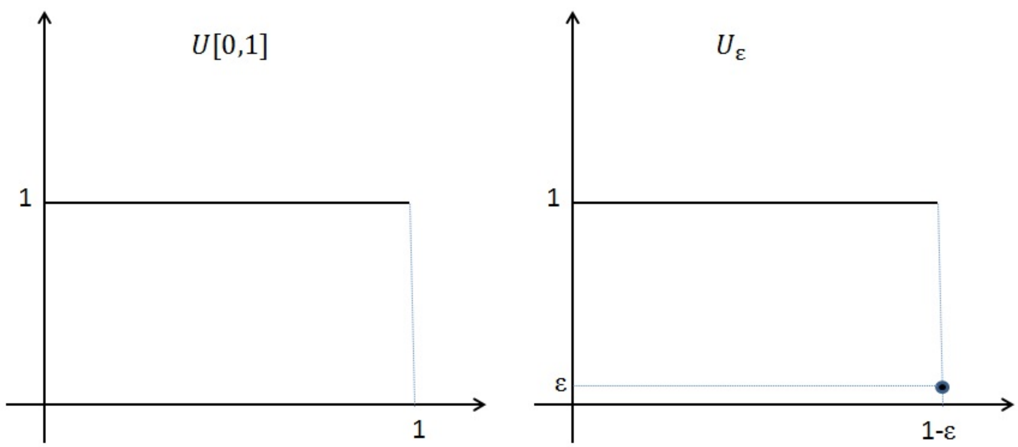


Figure 2.5. The uniform distibution and U_ϵ
The uniform distribution $U[0,1]$ and the modified uniform distribution which we refer to as U_ϵ. The modification consists of shifting the probability mass from the interval $[1-\epsilon, 1]$ to the point $1-\epsilon$, thus creating an atom of mass ϵ at $1-\epsilon$. As ϵ approaches 0, U_ϵ approaches $U[0,1]$.

The optimal wage contract satisfies each of the constraints with equality. The contract is given by

$$w(V,1) = \frac{1}{1-V}\frac{c}{ln(\frac{1}{\epsilon})}.$$

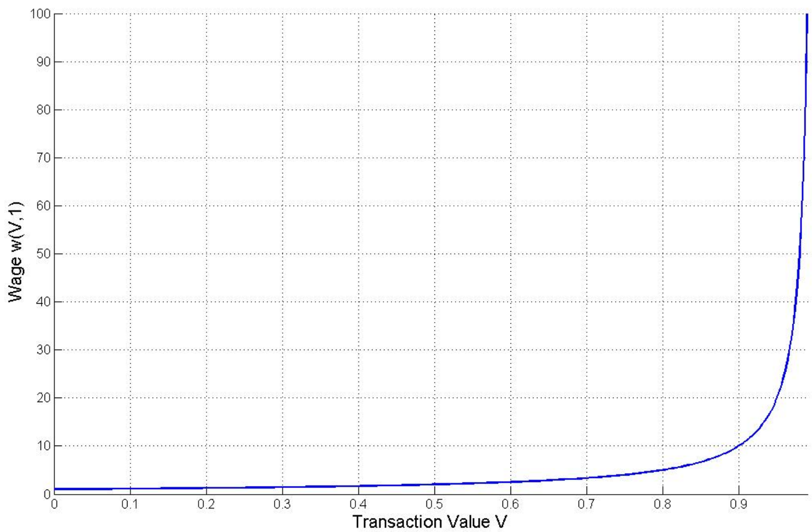


Figure 2.6. The wage schedule for a continuous uniform distribution
The figure shows the wage paid to the bank $w(V, 1)$ as a function of the value V reported by it when the values are uniformly distributed in [0,1].

The wage as a function of reported value is plotted in Figure 2.6. The wage is monotonically increasing in the value reported. The wage increases sharply as the value reported approaches 1. As a result of the optimal wage schedule, the bank is kept to its reservation utility of 0.

The expected payment to the bank is given by

$$c_s = c\left(1 + \frac{1}{ln(\frac{1}{\epsilon})}\right)$$

Just as in the discrete uniform distribution case above where $c_s \to c$ as $n \to \infty$, in this case $c_s \to c$ as $\epsilon \to 0$. In other words, the expected payment to the bank is not greater than the cost of effort. The proof is given in Appendix B.2.

The seller is willing to hire the advisor as long as the benefits of being fully informed exceed the cost. Following the calculation in section 2.4.1,

$$\frac{1}{2} - \frac{1}{4} \geq c \implies c \leq \frac{1}{4}.$$

2.5. Part II - Equilibria when Both the Seller and Buyer Hire Advisors

2.5.1. Pure Strategy Equilibria

In part I, we assumed that the buyer's bank could acquire information at zero cost. This corresponds to a pure strategy equilibrium where the buyer always hires the bank. Pure strategy equilibria can exist only if the effort costs of the advisors are zero or larger than $\frac{1}{8}$. This becomes apparent if we consider the two possible pure strategy equilibria for the buyer.

2.5.1.1. Buyer Always Informed. In part I, we assumed that the buyer could acquire information at zero cost. However, if he has to make a non-zero payment to the advisor, he would never choose to always hire the advisor. Why? Because if the buyer always hires the advisor, the seller would also always hire the advisor (unless he has to pay the advisor more than $\frac{1}{4}$ as we saw inpPart I above) so as to charge the buyer the fair value. But if the seller always hires the advisor, his offer would be equal to the value. The buyer would get zero profit from the transaction, but will have to pay his advisor. He is clearly better off not hiring the advisor. Thus, the buyer playing a pure strategy of always hiring the advisor is not an equilbrium.[17]

[17]This is subject, of course, to the reasonable caveat that the seller pays less than $\frac{1}{4}$ to the advisor.

2.5.1.2. Buyer Never Informed. If the buyer is never informed, the seller has no incentive to be informed since he can just offer $\frac{1}{2}$ without hiring the advisor.[18] However, if the seller is never informed, the buyer can benefit by being informed and accepting offers only when the value of the firm is greater than $\frac{1}{2}$. This would give him an expected payoff of $\frac{1}{8}$.[19] Hence, he would not want to always be uninformed unless his expected payment to the advisor is greater than $\frac{1}{8}$. Thus, the buyer playing a pure strategy of never hiring the advisor is not an equilbrium.[20]

2.5.2. Mixed Strategy Equilibria

Clearly, the decision by either party to hire an advisor depends on the probability of the other party being informed. I begin by searching for equilibria where both the seller and the buyer mix between hiring and not hiring an advisor. Let p_s and p_b be the probabilities with which the seller and the buyer hire the sell-side and buy-side advisors respectively. In addition, let c_s and c_b be the expected fees paid to the advisor by the seller and the buyer respectively. These are assumed to be exogenous for now. In the next section, I endogenize these parameters.

2.5.3. The Buyer's Strategy

The buyer accepts or rejects the offer made by the seller. If the buyer is uninformed, he will accept any offer less than l where $l \in [0, 1]$ is the expected value of the firm conditional

[18] Intuitively, why find out the value if the buyer never conditions his strategy on the value of the firm and it is always optimal to sell the firm?

[19] He accepts with $\frac{1}{2}$ probability and gets a profit $\frac{1}{4}$ (equal to the difference between expected value in $[\frac{1}{2}, 1]$ and the payment of $\frac{1}{2}$) if he accepts.

[20] This is subject, of course, to the reasonable caveat that the buyer pays less than $\frac{1}{8}$ to the advisor.

on the seller offering l. If the buyer is informed, he will accept any offer less than the value V.

2.5.4. The Uninformed Seller's Strategy

An offer of Q is accepted if the buyer is uninformed and Q is less than l or the buyer is informed and Q is less than V.

So, the probability of an offer of Q being accepted is

$$(1 - p_b)\mathbf{1}(Q \leq l) + p_b\mathbf{1}(Q \leq V).$$

It follows that the expected utility from quoting Q is

$$Q((1 - p_b)\mathbf{1}(Q \leq l) + p_b(1 - Q)).$$

The optimal offer depends both on l and p_b and is given by

$$Q_u = \begin{cases} l & \textit{if } p_b \in [0, \frac{l}{l^2+\frac{1}{4}}] \\ \frac{1}{2} & \textit{if } p_b \in [\frac{l}{l^2+\frac{1}{4}}, 1] \end{cases} \text{if } l \leq \frac{1}{2} \tag{2.1}$$

$$Q_u = \begin{cases} l & \textit{if } p_b \in [0, \frac{1}{2l}] \\ \frac{1}{2p_b} & \textit{if } p_b \in [\frac{1}{2l}, 1] \end{cases} \text{if } l > \frac{1}{2}. \tag{2.2}$$

For proof, see Appendix B.3. Intuitively, if the buyer hires the advisor with very low probability, the seller is facing an uninformed buyer most of the time. So, he offers the maximum amount the uninformed buyer will accept i.e. l. On the other hand, if there is a high chance that the buyer is informed, the seller's offer will not depend on the threshold of the uninformed buyer. The seller will choose an offer which maximizes the expected utility i.e. the product of the payoff from the offer and the probability of it being accepted.

2.5.5. The Informed Seller's Strategy

The expected utility from quoting Q is

$$Q\left((1-p_b)\mathbf{1}(Q \leq l) + p_b\mathbf{1}(Q \leq V)\right).$$

However, since the informed seller knows V, his optimal offer can also depend on V in addition to l and p_b. It is given by

$$Q_i = \begin{cases} l & \textit{if } V \in [0,(1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l,1] \end{cases} \quad \text{if } p_b \leq l \tag{2.3}$$

$$Q_i = \begin{cases} l & \textit{if } V \in [0,(1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l,\frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b},1] \end{cases} \quad \text{if } p_b > l. \tag{2.4}$$

For proof, see Appendix B.4. The optimal offer by the seller is not always equal to the value reported by the advisor even though the advisor reports truthfully. There are

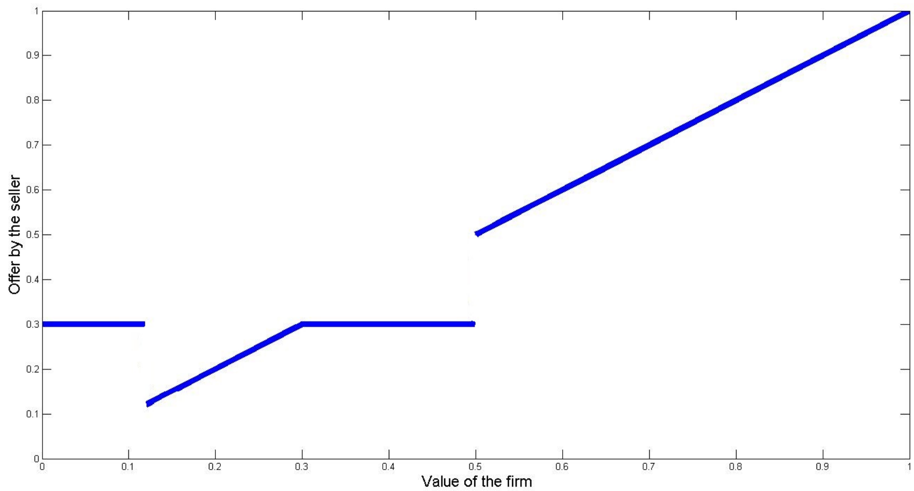


Figure 2.7. Informed seller's offers

The figure shows the offers made by the informed seller. The informed seller's offer depends on the value V that the bank reports, the threshold l of the uninformed buyer and the probability with which the buyer is an informed one p_b. The graph is for $l = 0.3$ and $p_b = 0.6$ The optimal offer by the seller is not always equal to the value reported by the bank even though the bank reports truthfully. There are intervals where the seller overcharges, charges a fair price and undercharges.

intervals where the seller overcharges, charges a fair price and undercharges. The intuition is that if the advisor reports a very low value, the seller is better off taking a gamble that the buyer is uninformed rather than offering the very low value. As the value increases, the seller charges a fair price because the aforementioned gamble is no longer optimal. Once the value reported by the advisor crosses the threshold of the uninformed buyer, the advisor does not want to risk charging a fair price because the uninformed buyer will reject it. Hence, there is an interval where he undercharges. However, as the value increases further, he is willing to gamble on the fact that the buyer is in fact informed. The optimal offer is graphed in Figue 2.7 as a function of the value reported by the advisor.

2.5.6. Consistent Equilibria after Imposing Additional Constraints

We need to impose subgame perfection of the buyer as an additional constraint. The buyer cannot be left with surplus from any offer he accepts, because the seller, knowing this, could then increase the offer price. Similarly, the buyer should reject anything that gives him a negative payoff.

The informed buyer conditions his strategy on the value of the firm. The uninformed buyer conditions his strategy on the expected value of the firm *given the seller's offer.* He accepts the offer if the expected value is geater than or equal to the offer, and rejects if it is less than or equal to the offer.[21]

Once we impose these constraints, the set of equilibria correspond to the following:

The uninformed buyer always accepts any offer $\in [0, l]$ and rejects any other offer. The informed buyer accepts any offer $\in [0, V]$. The uninformed seller offers l. The informed sellers's strategy depends on V, l and p_b and is given by

[21] At first glance, it may seem that l should always be $\frac{1}{2}$ in any equilibrium. After all, the expected value is $\frac{1}{2}$, so wouldn't sequential rationality dictate the buyer to set $l = \frac{1}{2}$? Not if we consider the impact of the informed seller's behaviour on the uninformed buyer.
To illustrate this, let us consider an uninformed buyer who does set $l = \frac{1}{2}$. The uninformed seller would offer $\frac{1}{2}$ irrespective of p_b since this is optimal whether the buyer is informed or uninformed (In other words, offering $\frac{1}{2}$ is the dominant strategy for the unnformed seller. Remember that $Q(1-Q)$ is maximized at $\frac{1}{2}$). However, we know that the *informed seller* would also quote $\frac{1}{2}$ for some values reported by his bank. To be precise, the informed seller's strategy is

$$Q_i = \begin{cases} \frac{1}{2} & \textit{if } V \in [0, (1-p_b)\frac{1}{2}] \\ V & \textit{if } V \in [(1-p_b)\frac{1}{2}, \frac{1}{2}] \\ \frac{1}{2} & \textit{if } V \in [\frac{1}{2}, \frac{1}{2p_b}] \\ V & \textit{if } V \in [\frac{1}{2p_b}, 1] \end{cases}$$

It should be clear now that the uninformed buyer, when offered $\frac{1}{2}$, must also take into account that it may be the informed seller offering him $\frac{1}{2}$ after finding out that $V \in [0, (1-p_b)\frac{1}{2}]$ or $V \in [\frac{1}{2}, \frac{1}{2p_b}]$. This is what restricts the value of p_b if $l = \frac{1}{2}$.

$l \in [0.15, \frac{1}{2}]$

$$p_b \in [0.54, \frac{l}{l^2 + \frac{1}{4}}]$$

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

$$p_s = \frac{(2l-1)p_b(1+p_b)}{(1-p_b^3)l - (p_b^2 + p_b)(1-l)}$$

$l \in [\frac{1}{2}, 0.54]$

$$p_b \in [\sqrt{\frac{2l-1}{l^2}}, l]$$

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1] \end{cases}$$

$$p_s = \frac{(2l-1)(1-p_b l)}{p_b l(p_b l + 1 - 2l)}$$

$$p_b \in [l, 0.54]$$

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

$$p_s = \frac{(2l-1)p_b(1+p_b)}{(1-p_b^3)l - (p_b^2+p_b)(1-l).}$$

For proof, see Appendix B.5. For plots of the possible values of l and p_b in equilibria, see Figure 2.8.

2.5.7. The Seller's Cost of Hiring the Advisor

For the seller to mix between hiring an advisor and not hiring one, his payoffs from the two have to be equal.

For equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

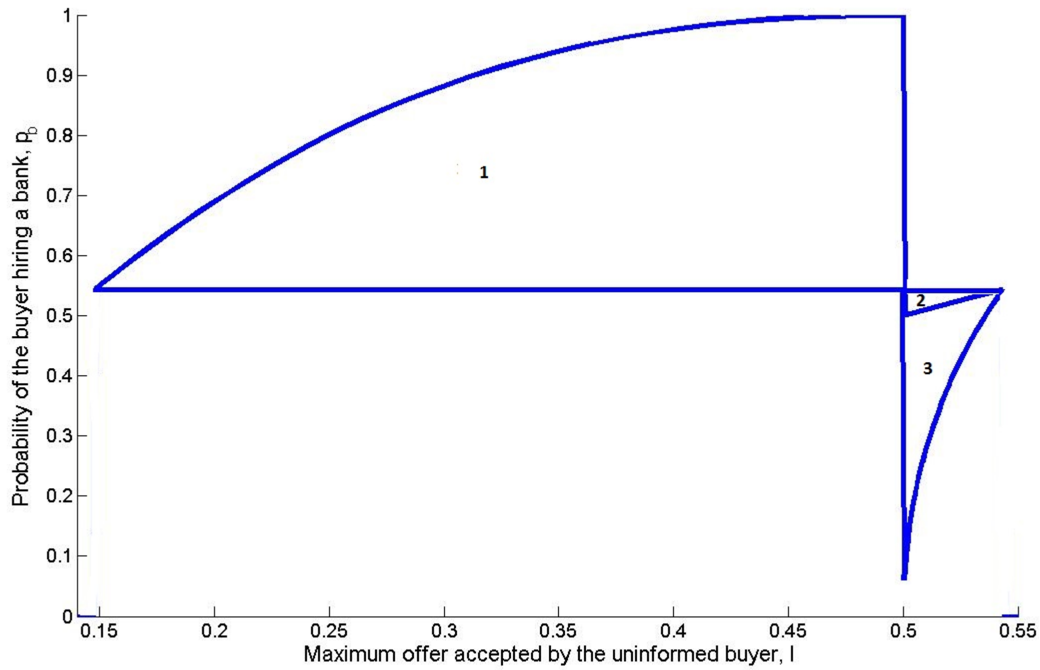


Figure 2.8. Range of l and p_b in all equilibria

The figure shows the ranges of l, the maximum offer accepted by the uninformed seller, and p_b, the probability of the buyer hiring a bank, for all possible equilibria. Areas 1 and 2 correspond to equilibria of the form $lVlV$. Area 3 corresponds to equilibria of the form lVl.

the seller's cost of hiring the advisor is given by

$$c_s = \frac{1}{2}l^2 p_b^2 + \frac{1}{2}(\sqrt{p_b} - \frac{l}{\sqrt{p_b}})^2.$$

For equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1] \end{cases}$$

the seller's cost of hiring the advisor is given by

$$c_s = \frac{1}{2} l^2 p_b^2.$$

For proof, see Appendix B.6. Note that for any l, c_s is increasing in p_b.

2.5.8. The Buyer's Cost of Hiring the Advisor

For the buyer to mix between hiring an advisor and not hiring one, his payoffs from the two have to be equal. Since he has no bargaining power, both have to be equal to zero. We have already considered the case of the uninformed buyer above. In this section, we focus on the informed buyer to get a condition on the buyer's cost of hiring the advisor.

For equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

the buyer's cost of hiring the advisor is given by

$$c_b = \frac{1}{2}(1-l)^2 + \frac{1}{2}p_s\left(\frac{l^2}{p_b}(\frac{1}{p_b} - 2) + 2l - 1\right).$$

For equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1] \end{cases}$$

the buyer's cost of hiring the advisor is given by

$$c_b = \frac{1}{2}(1-l)^2.$$

For proof, see Appendix B.7. What are the range of payments to the advisors that we see in equilibrium? Figure 2.9 plots how increasing l and p_b affects c_s and c_b which support the equilibrium for region 1 in Figure 2.8. The graphs are reminiscent of indifference curves. Changing l corresponds to a shift of the curve outwards, while changing p_b leads to a movement upwards *along* the curve. Hence, given any pair (c_s, c_b) in this range, there is a unique l and p_b and, by extension, a unique p_s.

Figure 2.10 plots all values of c_s and c_b for which equilibria exist. Note that at this stage, neither of these are exogenous. Rather, they depend on the true exogenous parameter, the advisor's cost of effort, and the information rents on top of that which the advisors may or may not be able to extract in the optimal contract. I endogenize the payment to the advisors as a function of these exogenous parameters next.

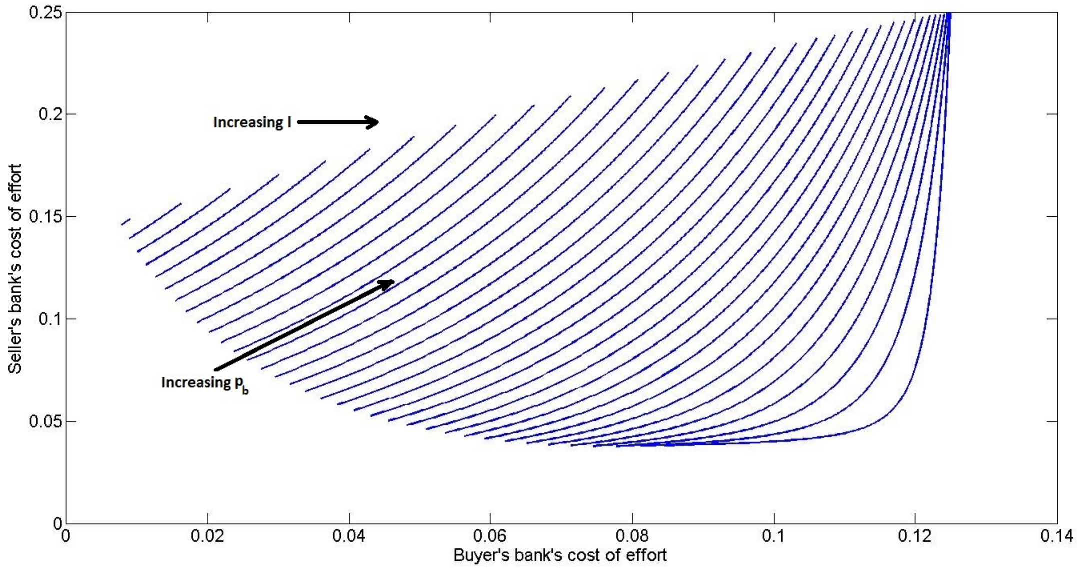


Figure 2.9. Existence of a unique mapping from (c_s, c_b) to (l, p_b, p_s)

The figure shows how the buyer's and seller's cost of hiring an advisor change as l and p_b increase for equilibria corresponding to region 1 in Figure 2.8. Increasing l leads to a shift of the curve outwards, while increasing p_b leads to a movement upwards *along* the curve. There is a unique mapping between any (c_s, c_b) and (l, p_b, p_s).

2.6. Part III - Optimal Contracts when Both the Buyer and Seller Hire Advisors

2.6.1. Seller's Contract with the Advisor

So far, we have assumed that the cost c_s is exogenous. However, c_s is the outcome of a contract between the seller and the advisor. Now, we turn to the issue of how c_s is related to c, the cost of effort the advisor incurs. It turns out that the the seller's ability to hold the advisor to an information rent of zero depends on which equilibrium is realised.

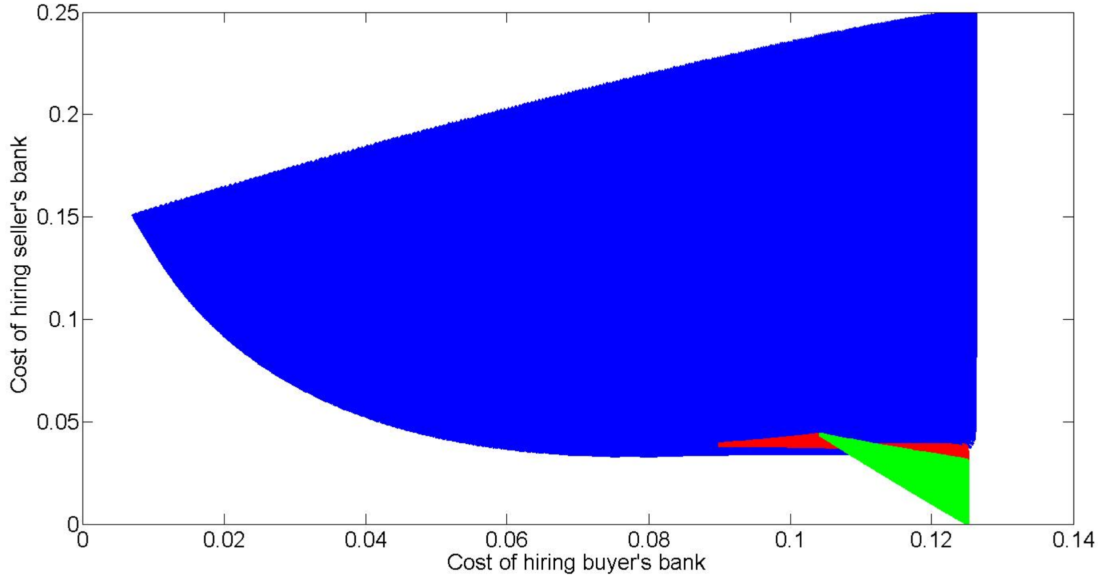


Figure 2.10. The seller's and buyer's costs of hiring an advisor for which a mixed equilibrium exists

The figure shows all possible values of the buyer's and seller's cost of hiring an advisor for which a mixed equilibrium exists. The blue and red areas correspond to $lVlV$ equilibria and the green to lVl equlibria

Start by considering equlbria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1]. \end{cases}$$

Let us consider the various options available to the advisor if he decides to report a value without exerting any effort. If the seller's offers are the same for two or more reported values, the corresponding wages have to be the same. If the wages are different, the advisor would always report that value for which the wage is the highest. This doesn't affect the probability of the transaction going through, but increases the wage conditional on it going through.

If the advisor reports a value in either of the intervals $[0,(1-p_b)l]$ or$[l,\frac{l}{p_b}]$ without putting in effort, the seller offers the buyer l. The uninformed seller accepts it and the informed seller accepts it if the realized value $\leq l$, i.e. with probability $1-l$. So, the transaction goes through with probability $(1-p_b)+p_b(1-l)$ or $1-lp_b$. So, the intermediary gets an expected payoff of $w(l)(1-lp_b)$ without exerting effort.

If the advisor reports a value V in the interval $[(1-p_b)l,l]$ without putting in effort, the seller offers the value reported. In this range, the value is less than l, so the uninformed seller would accept the offer. The transaction goes through with probability $(1-p_b)+p_b(1-V)$ or $1-Vp_b$. So, the intermediary can secure himself an expected payoff of $w(V)(1-Vp_b)$ without exerting effort for all $V \in [(1-p_b)l,l]$

The option left is to report a value V in the interval $[\frac{l}{p_b},1]$ without putting in effort. The seller offers the value reported. In this range, the value is greater than l, so the uninformed seller would reject the offer. The offer will only go through if the buyer is informed and the actual value is less than the value reported, i.e. with probability $p_b(1-V)$. So, the intermediary can secure himself an expected payoff of $p_b(1-V)w(V)$ without exerting effort for all $V \in [\frac{l}{p_b},1]$.

To motivate the advisor to exert effort, his payoff on exerting effort and reporting the value truthfully must be greater than 0. If he exerts effort and the value $V \in [0,(1-p_b)l]$, the seller offers l. Since the actual value is less than l, the transaction goes through only if the buyer is uninformed i.e. with probability $1-p_b$. So, expected payoff conditional on the value being in this range is $(1-p_b)w(l)$. If he exerts effort and the value $V \in [(1-p_b)l, l]$, the seller offers V. The transaction always goes through since $V \leq l$. If he exerts effort and the value $V \in [l, \frac{l}{p_b}]$, the seller offers l. Since the value is greater than l, the transaction always goes through. So, expected payoff conditional on the value being in this range is $w(l)$. If he exerts effort and the value $V \in [\frac{l}{p_b}, 1]$, the seller offers V. Since $V \geq l$, the transaction only goes through if the seller is informed i.e. with probability p_b.

Hence, the expected payoff if he exerts effort is

$$(1-p_b)l(1-p_b)w(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + \left(\frac{l}{p_b} - l\right) w(l) + p_b \int\limits_{\frac{l}{p_b}}^{1} w(V)\,dV - c.$$

Simplifying this expression and equating the payoff from exerting effort to be greater than any payoff from not exerting effort gives the seller's problem and the family of constraints he faces.

Minimise the expected payment to the advisor

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int_{(1-p_b)l}^{l} w(V)\, dV + p_b \int_{\frac{l}{p_b}}^{1} w(V)\, dV - c$$

subject to the constraints

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int_{(1-p_b)l}^{l} w(V)\, dV + p_b \int_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq (1 - p_b l)w(l)$$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int_{(1-p_b)l}^{l} w(V)\, dV + p_b \int_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq (1 - V p_b)w(V)$$

if $V \in [(1 - p_b)l, l]$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int_{(1-p_b)l}^{l} w(V)\, dV + p_b \int_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq p_b(1 - V)w(V)$$

if $V \in [\frac{l}{p_b}, 1]$.

We solve this for the modified uniform distribution U_ϵ introduced in Section 2.4.3. The optimal contract is given by

$$w(V,1) = \begin{cases} k' c \frac{1}{1-p_b V} & \text{if } V \in [(1-p_b)l, l] \\ k' c \frac{1}{p_b(1-V)} & \text{if } V \in [\frac{l}{p_b}, 1-\epsilon] \end{cases}$$

where k' is a constant. The payment made to the advisor is given by $c(1 + k')$. As $\epsilon \rightarrow 0$, the payment is just the cost of effort c, so the advisor extracts no information rents. The proof is in Appendix B.8.

Figure 2.11 shows the wage paid to the seller's advisor as a function of the value reported by the advisor. Remember that in these equilibria, the value at which the transaction takes place is not necessarily the value reported by the advisor because there are intervals in which the seller undercharges or overcharges the buyer. The incentives to the advisor are determined by the seller's offer and not his report since the seller's offer determines the probability of the transaction happening. For reports of low values and values in the interval above l, the advisor offers l so that the wage corresponds to $w(l, 1)$. This is why the wage is not monotonically increasing in the value reported by the advisor.

We still have to check whether the advisor has an incentive to misreport the value *after* exerting the effort. We need to check this for misreporting from each of the intervals to each of the other 3 intervals. I illustrate this for one interval below.

If the advisor finds that $V \in [0, (1 - p_b)l]$ and reports the value truthfully to the seller, the seller makes the buyer an offer of l. The advisor gets $w(l)$ only if the buyer is uninformed.

- If instead, the advisor misreports the value to $V' \in [(1 - p_b)l, l]$, the seller offers V'. Since $V' < l$, the transaction still goes through only if the buyer is uninformed and the bank gets $w(V) < w(l)$ if it goes through. So, the advisor has no incentive to do this.

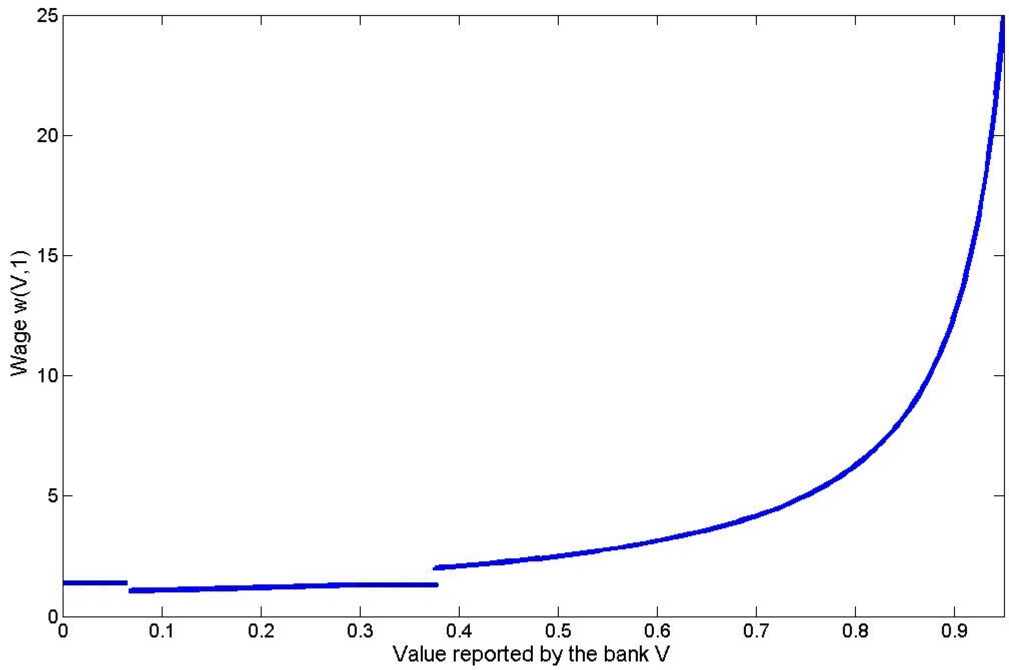


Figure 2.11. Wage paid to the seller's advisor as a function of the value reported for $p_b = 0.8$ and $l = 0.3$

The figure shows the wage paid to the seller's advisor as a function of the value reported by the advisor. This corresponds to the equilibrium where the buyer hires an advisor with probability $p_b = 0.8$ and accepts any offer less than or equal to $l = 0.3$ if he hasn't hired an advisor. The wages have been scaled by $ck^{'}$ where c is the advisor's cost of exerting effort and $k^{'}$ is a constant.

- Misreporting to $V^{'} \in [l, \frac{l}{p_b}]$ is pointless since the seller offers l in that range, same as without misreporting, which changes neither the fee nor the probability of the transaction going through.
- Lastly, misreporting to $V^{'} \in [\frac{l}{p_b}, 1]$ leads to the seller offering $V^{'}$. In this case, the transaction fails for sure because both the informed and uninformed seller will reject the offer since $V^{'} > V$ and $V^{'} > l$ respectively. So, there is no incentive to misreport to this range.

Hence, there is no incentive to misreport from $V \in [0, (1 - p_b)l]$ to any of the other three intervals. Similarly, it can be shown that the contract is misreporting-proof i.e. the advisor has no incentive to misreport the value after discovering that the true value lies in each of the other three intervals as well. The proof is given in Appendix B.9. The contract is plotted in Figure 2.12 for $l = 0.3$ and $p_b = 0.8$.

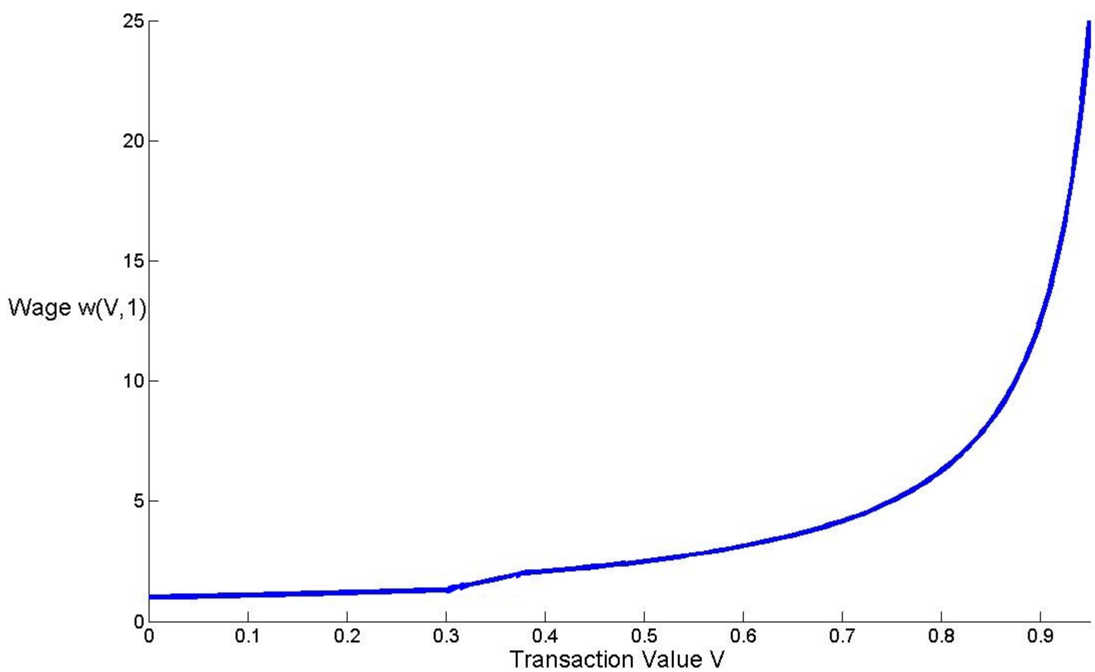


Figure 2.12. Optimal contract with the seller's advisor for $p_b = 0.8$ and $l = 0.3$

The figure shows the wage paid to the seller's bank as a function of the transaction value. This corresponds to the equilibrium where the buyer hires a bank with probability $p_b = 0.8$ and accepts any offer less than or equal to $l = 0.3$ if he hasn't hired a bank. The wage is increasing monotonically in the transaction value. Some of the wages are off the equilibrium path and will not be observed in equilibrium. The wages have been scaled by ck' where c is the bank's cost of exerting effort and k' is a constant.

Now consider equilbria of the form

$$Q_i = \begin{cases} l & \text{if } V \in [0, (1-p_b)l] \\ V & \text{if } V \in [(1-p_b)l, l] \\ l & \text{if } V \in [l, 1]. \end{cases}$$

The optimal contract is given by

$$w(V,1) = k^{''} c \frac{1}{1-p_b V} \ \text{if } V \in [(1-p_b)l, l]$$

where $k^{''}$ is a constant. The payment the intermediary is given by $c(1+k^{''})$. The payment is always greater than the cost of effort c, so the advisor extracts information rents in this range. Also, this contract is misreporting-proof. These results are proved in Appendix B.10

2.6.2. Buyer's Contract with the Advisor

The buyer's advisor reports a value to the buyer. His contract can only depend on whether the offer corresponding to the value he reports is equal to the actual offer made by the seller.

First, consider equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1]. \end{cases}$$

It is clear that the buyer has no way of distinguishing between the lower and upper intervals since the seller offers l in both cases. So the wage in these two intervals has to be the same. Else, the buyer's advisor would never report a value in the interval corresponding to the lower wage. Let this wage be $w(l, 1)$.

If $V \in [(1-p_b)l, l]$ or $V \in [\frac{l}{p_b}, 1]$, the seller offers V. If the buyer's advisor does not exert any effort, he guesses V right with probability 0. So, not putting effort and reporting a value in either of these intervals gives him zero utility.

The only case in which the buyer's advisor can get an expected utility greater than 0 without exerting effort is if he reports $V \in [0, (1-p_b)l]$ or $V \in [\frac{l}{p_b}, 1]$. In both these cases, he gets paid $w(l)$ if the seller offers l. The seller offers $w(l)$ if he is uninformed or if he is informed and the value falls into those intervals i.e. with probability $\Big[1 - p_s + p_s(\frac{1}{p_b} - p_b)l\Big]$. So, the buyer's advisor can get $(1 - p_s + p_s(\frac{1}{p_b} - p_b)l)w(l, 1)$ without putting in any effort.

If the advisor does exert effort, he discovers $V \in [0, (1-p_b)l]$ or $V \in [\frac{l}{p_b}, 1]$ with probability $(\frac{1}{p_b} - p_b)l$. The seller offers l whether he is informed or uninformed. So, with probability $(\frac{1}{p_b} - p_b)l$, the bank gets $w(l, 1)$. If $V \in [(1-p_b)l, l]$ or $V \in [\frac{l}{p_b}, 1]$, his reported value matches the seller's offer only if the seller is informed.

The buyer tries to minimize the expected payment to the bank

$$(\frac{1}{p_b} - p_b)lw(l,1) + p_s \left[\int_{(1-p_b)l}^{l} w(V,1)dV + \int_{\frac{l}{p_b}}^{1} w(V,1)dV \right]$$

subject to the advisor's IC constraint for exerting effort

$$(\frac{1}{p_b} - p_b)lw(l,1) + p_s \left[\int_{(1-p_b)l}^{l} w(V,1)dV + \int_{\frac{l}{p_b}}^{1} w(V,1)dV \right] - c \geq$$
$$\left[1 - p_s + p_s(\frac{1}{p_b} - p_b)l\right] w(V_l,1)$$

which simplifies to

$$p_s \left[\int_{(1-p_b)l}^{l} w(V,1)dV + \int_{\frac{l}{p_b}}^{1} w(V,1)dV \right] - c \geq [1 - p_s] \left[1 - (\frac{1}{p_b} - p_b)l\right] w(l,1).$$

Decreasing $w(l,1)$ decreases the objective function and strictly loosens the constraint. So, the optimal contract sets $w(l,1)$ to 0. The constraint thus binds. The expected payment to the advisor is always c. This means that the advisor extracts no informational rents.

In general, the buyer's optimal contract can take various forms. However, all contracts are subject to two restrictions. First, the wage corresponding to a report of l, the threshold that the buyer would have used had he not hired the advisor, must be zero.[22] Second, the advisor makes the cost of effort c in expectation and gets no information rents. Subject to

[22] Intuitively, hiring the advisor has not really helped the buyer in this case and so the advisor is paid zero.

these constraints, the contract can be flat, increasing or decreasing. One way to implement this is through a flat fee $= \frac{c}{p_s(1-\frac{l}{p_b}+lp_b)}$ in $V \in [(1-p_b)l, l]$ or $V \in [\frac{l}{p_b}, 1]$. In general, any wage structure $w(V,1)$ which satisfies

$$\int_{(1-p_b)l}^{l} w(V,1)dV + \int_{\frac{l}{p_b}}^{1} w(V,1)dV = \frac{c}{p_s} \text{ and}$$

$$w(l,1) = 0$$

will work.

It is easy to verify that the advisor will not misreport once he has been incentivized to put in the effort. If the advisor misreports from $V \in [0,(1-p_b)l]$ or$V \in [l, \frac{l}{p_b}]$, where the seller offers l, to $V' \in [(1-p_b)l, l]$ or $V' \in [\frac{l}{p_b}, 1]$, where the seller offers V, he has zero probability of getting the value right. Hence, he will not report a value in this range unless he finds the value to be in the range. If instead, the advisor misreports from $V' \in [(1-p_b)l, l]$ or $V' \in [\frac{l}{p_b}, 1]$, where the seller offers V, to $V \in [0,(1-p_b)l]$ or $V \in [l, \frac{l}{p_b}]$, he gets paid $w(l,1)$ i.e. 0 at best. There is no incentive to do this either. Hence, this contract is misreporting-proof.

Figure 2.13 shows 3 possible optimal contracts. Since the contract is not uniquely pinned down, there are many others.

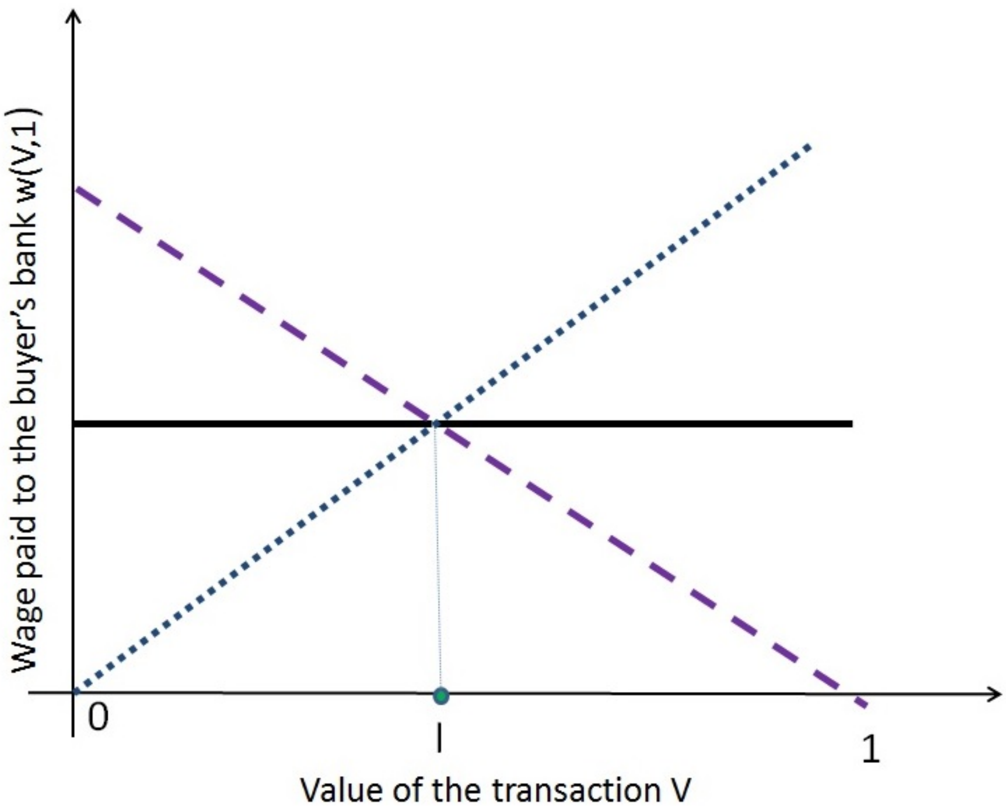


Figure 2.13. Optimal contract with the buyer's advisor
The figure shows the wage paid to the buyer's bank as a function of the transaction value. In general, the buyer's optimal contract can take various forms subject to two restrictions. First, the wage corresponding to a report of l, the threshold that the buyer would have used had he not hired the bank, must be zero. Second, the bank makes the cost of effort c in expectation and gets no information rents. Subject to these, the contract can be flat, increasing or decreasing. Three such contracts are given in the figure.

For equilibria of of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1] \end{cases}$$

it is apparent by arguments almost exactly the same as above that the optimal contract sets

$$\int_{(1-p_b)l}^{l} w(V,1)dV = \frac{c}{p_s} \text{ and}$$

$$w(l,1) = 0.$$

Once again, the advisor is paid c in expectation, i.e. extracts no information rents, and the contract is misreporting-proof.

2.6.3. The Range of c for Which the Mixing Equilibria Exist

We have demonstrated that the buyer's advisor never gets information rents and the seller's advisor doesn't for the $lVlV$ equilibria. For the rest of the paper, we consider only equilibria where the cost of effort c is the same for both the buyer's advisor and the seller's advisor. Figure 2.10 makes it clear that the lVl equilibria cannot have both the advisors' costs of effort equal to c. This is because for these equilibria, $c_b = c$ and $c_s > c$ since the seller's advisor extracts information rents. However, from the graph, we see that this is not possible. By the same logic, the $lVlV$ corresponding to the areas shaded red can also be ignored. We are left only with the blue area of the graph. This is shown in Figure 2.14

2.7. Complete Characterization of the Equilibria

I fully characterise the buyer's and seller's strategies in equilibria given the cost of effort c of their advisors. A strategy is described below in terms of the ordered triple

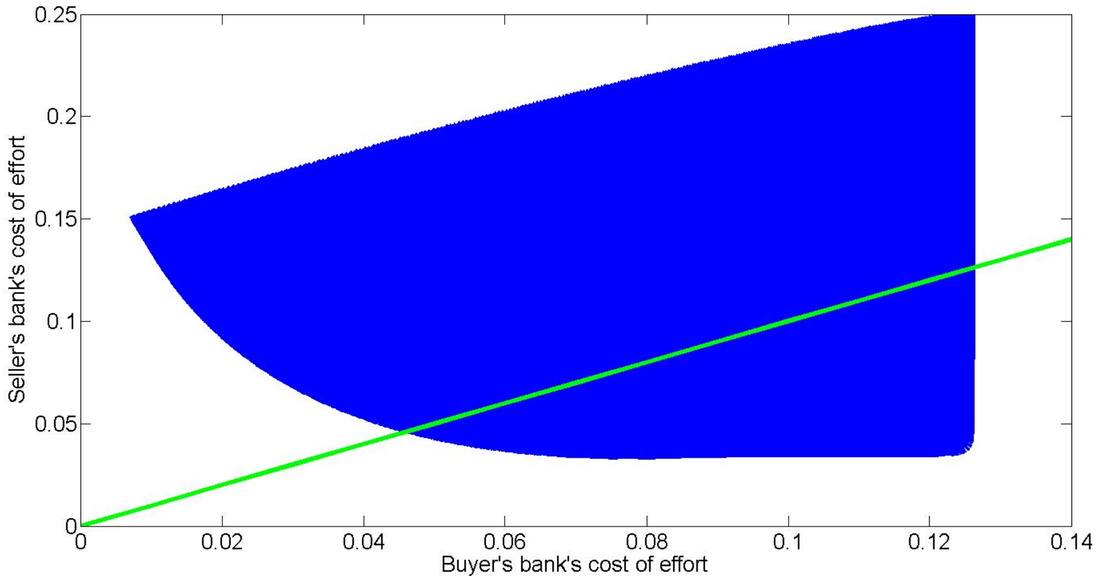


Figure 2.14. Equilibria and the cost of effort of the advisors
The figure shows the ranges of the cost of effort of the advisors for which mixed equilibria exist under the assumptions that both the advisors have the same cost of effort c. The set of equlibria correspond to the intersection of the blue region in Figure 2.10 with the $x = y$ line. In the equilibria, both advisors get paid the effort cost. As can be seen from the figure, equilibria exist for a wide range of c.

(l, p_b, p_s) given c. In keeping with the requirements of a perfect Bayesian equilibrium, the beliefs of either party are consistent with the strategy of the other. I also describe the contracts between both parties and their advisors. The advisors exert effort and report truthfully in all the equilibria.

Buyer's Strategy

If uninformed, the buyer accepts any offer less than or equal to a threshold l. If informed, the buyer accepts any offer less than or equal to the value V. The buyer becomes informed i.e. hires an advisor with probability p_b. The buyer's contract with his bank is given by, for eg, a flat fee $= \frac{c}{p_s(1-\frac{l}{p_b}+lp_b)}$ in $V \in [(1-p_b)l, l]$ or $V \in [\frac{l}{p_b}, 1]$ and $w(l,1) = 0$

Seller's Strategy

If uninformed, the seller offers l.

If informed, the seller offers

$$Q_i = \begin{cases} l & \textit{if } V \in [0,(1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1]. \end{cases}$$

The 3 unknowns l, p_s and p_b are determined from the three equations

$$\begin{aligned} p_s &= \frac{(2l-1)p_b(1+p_b)}{(1-p_b^3)l-(p_b^2+p_b)(1-l)} \\ c &= \frac{1}{2}l^2p_b^2 + \frac{1}{2}(\sqrt{p_b} - \frac{l}{\sqrt{p_b}})^2 \\ c &= \frac{1}{2}(1-l)^2 + \frac{1}{2}p_s\left(\frac{l^2}{p_b}(\frac{1}{p_b}-2)+2l-1\right). \end{aligned}$$

The seller's contract with his bank is given by

$$w(V,1) = \begin{cases} k'c\frac{1}{1-p_bV} & \textit{if } V \in [(1-p_b)l, l] \\ k'c\frac{1}{p_b(1-V)} & \textit{if } V \in [\frac{l}{p_b}, 1-\epsilon]. \end{cases}$$

2.8. Analysis of the Equilibria

Numerical solutions of the above equations for various values of c show that the solution (l, p_b, p_s) is unique for each value of c. I now turn to the question of how each of these parameters depend on c and how these affect the efficiency and the probability of the transaction going through. Figure 2.15 shows how l, p_s and p_b vary with c. As the cost of effort increases, the buyer hires an advisor more often. He also accepts more offers if uninformed because his threshold of accepting is higher. However, the seller hires the advisor less often.

The transaction goes through with probability

$$p_{success} = p_s p_b + (1 - p_s)(1 - p_b) + p_s(1 - p_b)(\frac{l}{p_b}) + (1 - p_s)(p_b)(1 - l)$$

The proof is in Appendix B.11. As the cost of effort increases, the probability of sale decreases. The graph is shown in Figure 2.16. With no advisors, the seller offers $\frac{1}{2}$ and the buyer always accepts it. With advisors, the transaction happens less frequently, which affects the efficiency.

How does this affect the expected utility of the seller? Since the seller is indifferent between hiring and not hiring the advisor, the expected utility of the seller is calculated easily as the expected utility of the uninformed seller, which we derived in Appendix to be equal to $l(1-lp_b)$. The graph is shown in Figure 2.17. With no advisors in the picture, the seller offers $\frac{1}{2}$ and the buyer always accepts it. So, the seller would have got a payoff of $\frac{1}{2}$. As can be seen, the payoff with the possibility of hiring advisors is always less than

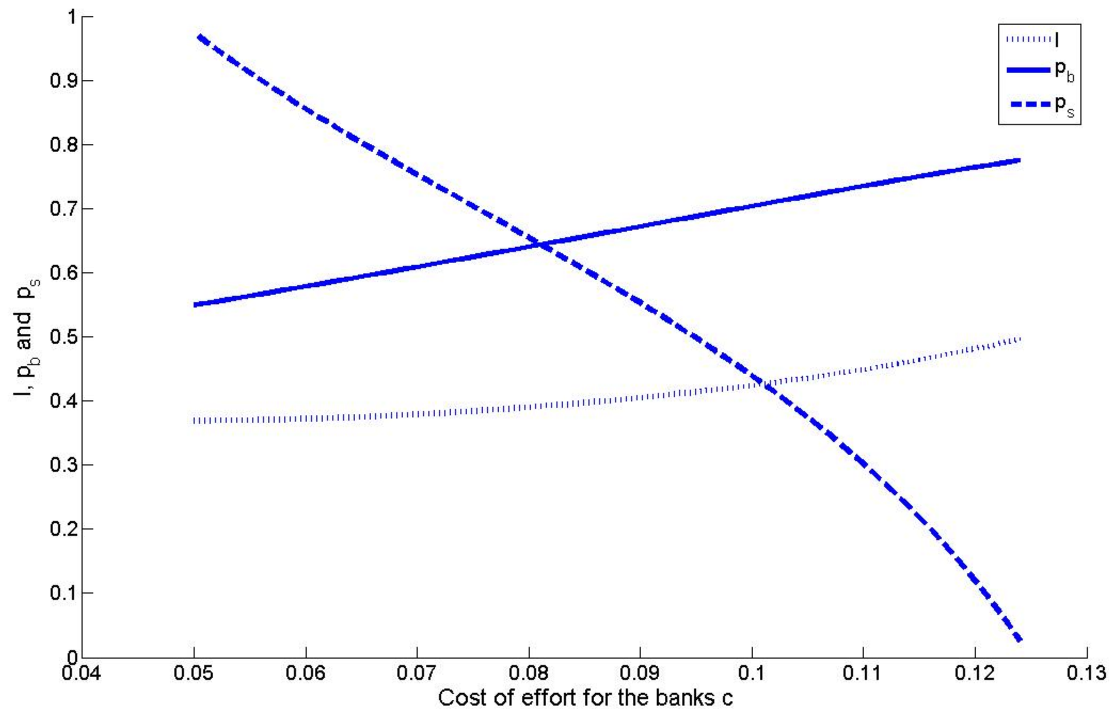


Figure 2.15. l, p_s and p_b as a function of c
The figure shows the probabilities of the seller and the buyer hiring their advisors and the maximum offer accepted by the uninformed buyer as a function of the advisors' cost of effort. As the cost of effort increases, the buyer hires an advisor more often. He also accepts more offers if uninformed (because his threshold l increases). However, the seller hires the advisor less often.

$\frac{1}{2}$. Remember that the buyer is held to his reservation utility of 0 in all equilibria since the seller has the bargaining power. So, this is also the total surplus.

The possibility of information acquisition thus has two effects. First, it destroys the total surplus in the transaction when the transaction happens. Second, the transaction happens less frequently.

So, we see that the efficiency is destroyed in two ways, namely the transaction not always going through, and the bank exerting costly effort to acquire the information.

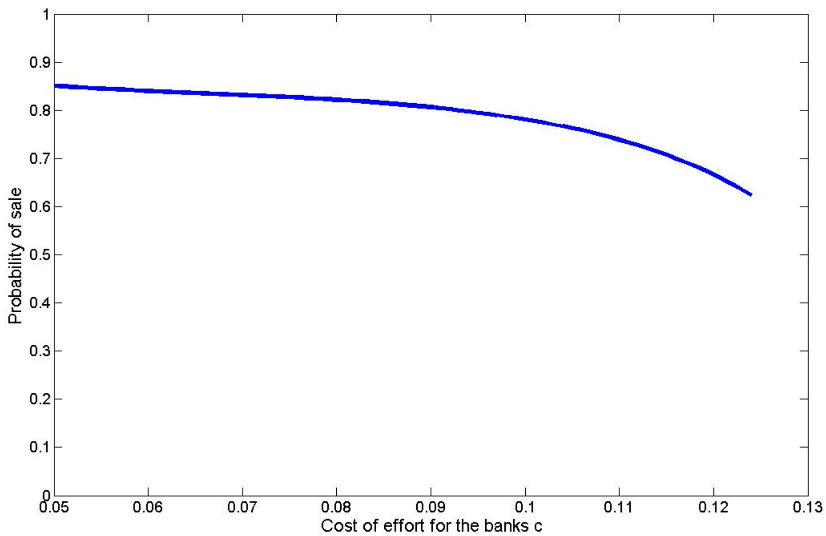


Figure 2.16. Probability of the sale as a function of c
The figure shows the probability of the transaction going through as a function of the advisors' cost of effort. The transaction happens less frequently than the benchmark case with no advisors, where it always goes through.

Interestingly, the banks do not extract rents, which would have decreased the efficiency even further.

2.9. Empirical Implications

In this section, we reconcile the predictions from the model with some of the empirical studies.

We have shown that equilibria exist for a wide range of effort costs. McLaughlin (1990) finds that on an average, the fees paid to the banks were 0.77% of acquisition value for target firms and 0.55% for bidder firms. Servaes and Zenner (1996) report a value of 1%. In the model, we find the range to be about 4% to 12.5% of the synergies. Synergies

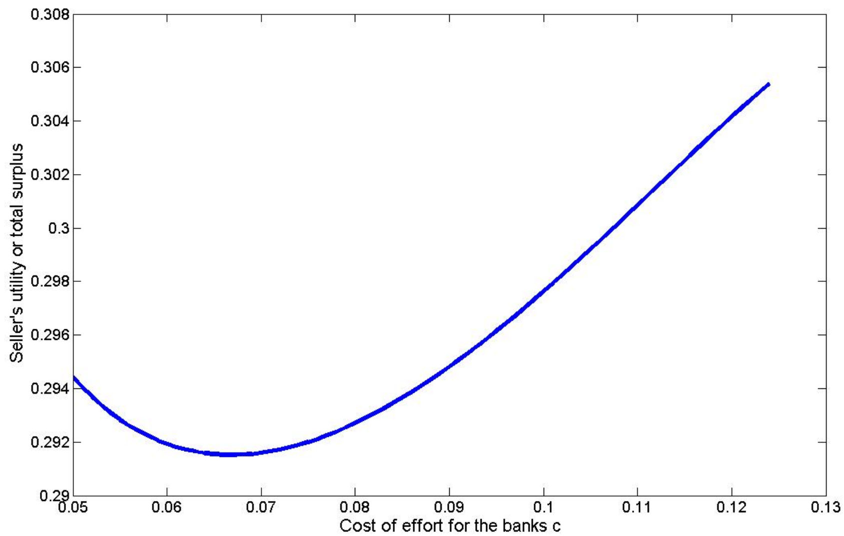


Figure 2.17. The seller's payoff (or total surplus) as a function of c
The figure shows the seller's payoff (or total surplus) as a function of the advisors' cost of effort. The seller's surplus is always lesser than the benchmark value of 0.5 with no advisors.

themselves can be anywhere from 2% to 10% of the transaction value. So the estimates are well within our predictions.

A more straightforward prediction of the model can be crosschecked with the estimate provided by Hunter and Walker (1990) who state that that on average, only about 6% of the merger gains were captured by the investment bankers in the form of fees. Once again, 6% falls well within our permitted range of c in equilibrium.

Another empirical implication is the difference in fees charged between high quality and low quality banks. Golubov et al. (2012) find that top-tier advisors charge a mean advisory fee of 0.55% of the transaction value and non-top-tier advisors charge 0.72%.[23]

[23]They also find differences in the characteristics of firms who hire top-tier advisors and non-top-tier advisors. The former are public firms. Public firms have more information and financial statements publicly available, so they are less costly to investigate.

On the returns to the acquirer, Servaes and Zenner (1996) find that the returns earned by acquirers are independent of whether the bank is hired or not. Our model predicts this almost by construction since the buyer always gets a payoff of 0. Regarding returns to the target, Asquith (1983) provide evidence that increases in the probability of merger benefit the stockholders of target firms. This is true for some ranges of c, the low values to be precise.

On the likelihood of the transaction going through, Hunter and Jagtiani (2003) suggest that the payment of larger advisory fees do not play an important role in determining the likelihood of completing the deal. Our model in fact predicts a negative relation between the two.

2.10. Concluding Remarks

In this paper, we solve for the optimal contract that a seller or buyer should offer an information intermediary who can precisely know the value of the asset if he exerts effort. Using a simple model, we first solve for an equilibrium choice of firms to get informed as a function of the intermediary's cost of effort. We then characterize the features of the optimal contract, comment on the impact on efficient allocation of assets and generate empirically testable hypotheses.

In light of the recent merger wave, mergers and acquisitions conjure up images of high value transactions (upwards of even $100 billion in many cases) between two big, public firms. However, it would be a mistake to focus only on such deals in any discussion of asymmetric information problems.[24] There is an active market for the sale of smaller

[24] It is also well-documented that the stock market reaction to acquirers of private firms is on an average positive while that to acquisitions of public targets is zero or even negative. This suggests that acquisitions of private firms are value enhancing for the shareholders of both the acquirer and the target. Empirical

firms, which account for a large proportion of mergers in many industries.[25] If the target is a smaller private firm, the buyer and the target face even more uncertainty regarding the synergies in the transaction since there is less publicly available information about the target.[26] In addition, innovative start ups with intangible assets which are difficult to value are inevitably private firms. For these reasons, the analysis in the paper is particularly relevant for the sale of small, private targets.

In conclusion, it is worth noting that although we have specialized the discussion to an M&A setting, the paper can be applied to most other settings involving the sale of an asset of unknown value.The model suits any context where advisors can be hired to provide information on the value of the asset, as long as their effort is costly and not verifiable. It is applicable, for example, to the sale of a real estate property or the negotiation between a seller of a rare painting and an art collector. The optimal contract in all these settings is of the form described above.

studies investigating this are many, Capron and Shen (2007), Faccio et al. (2006) or Officer (2007) to name just three.

[25]See for example Ho, Catherine. *"Law firm mergers continue to target small firms"*. The Washington Post. 6 July 2014. Web. Accessed 24 October 2016.

[26]For more on the costs and benefits of acquiring small firms, see for example Shen and Reuer (2005) or Moeller et al. (2004).

CHAPTER 3

Product Market Relatedness, Antitrust and Merger Decisions (Joint with Kirti Sinha)

3.1. Introduction

We examine the effects of product market relatedness between two firms on the probability of a merger transaction between them. There are reasons to believe that this effect can be positive or negative. First, synergies, the excess value created when a firm merges with another, are often cited as a reason for the merger. Synergies in turn may be related to the similarity in product offerings of the two firms. Second, antitrust laws may block the merger of two firms if they are competitors operating in the same product market. If firms anticipate this, the similarity of product markets between them may lead to lesser chance of merger incidence. We investigate the direction and magnitude of the effect before delving into the possible causes and consequences of it.

Synergies are closely related to how similar the product markets of the acquirer and target are. For example, Kaplan et al. (2000) state that "While there is considerable disagreement as to whether mergers create value in general (which we describe below), there is something of a consensus that combinations of related companies can realize synergies and are, therefore, more valuable than unrelated combinations." Morck et al. (1990) and Berger and Ofek (1995) look at the effect of a firm entering an unrelated market on the firm value and document a negative relationship. However, this view has been disputed by many studies which point out the advantages of acquiring a firm that has complementary assets rather than similar ones. Indeed, similarity in product markets may lead to cannibalization of sales and redundancies whereas complementarity can allow the firm to redeploy resources or expand into another market.[1]

Even if we establish the direction of the effect of product market relatedness (henceforth PMR) on synergies, higher or lower synergies by themselves do not necessarily result in a higher or lower chance of the merger happening. Higher synergies, if they are a result of pricing power after the merger, may lead to the merger being blocked for violating the antitrust guidelines. The Federal Trades and Commission website states that "During a merger investigation, the agency seeks to identify those mergers that are likely either to increase the likelihood of coordination among firms in the relevant market when no coordination existed prior to the merger, or to increase the likelihood that any existing coordinated interaction among the remaining firms would be more successful, complete, or sustainable." Overall, it is not clear whether the probability of a transaction is increasing, decreasing or even monotonic in the product market similarity between the potential acquirer and target.

Previous studies have looked at effects of acquisitions of firms in the same industry versus diversifying acquisitions on acquirer profitability. While this is an informative distinction, even firms in the same industry may differ in the similarity of their product markets across time and in the cross section. To address our research question, it is crucial to use a continuous variable to measure product market relatedness. For this purpose, we use the firm pair-year level PMR measure, based on the similarity between the product

[1]See Capron et al. (1998), Wang and Zajac (2007), Cassiman and Veugelers (2006) and Fulghieri and Sevilir (2012).

description sections of the 10-K forms of the two firms in that year, as in Hoberg and Phillips (2010).

Our findings are as follows. We find a nonlinear inverted U-shaped relationship between the probability of a merger and the product market relatedness between the acquirer and the target. This relationship is robust to controlling for deal fixed effects and a host of acquirer and target characteristics. We also control for technological similarity between the firms to ensure that it does not drive our results. Not only is the effect of PMR robust to controlling for technological overlap, but the effect of technology overlap is itself in the shape of an inverted U.

This finding can either be because of the synergies themselves having a non-monotonic relationship with the PMR, or because the probability is less due to other factors, for example antitrust investigations blocking the merger of firms which are very similar. We attempt to disentangle these measures in two ways. We first show that in regimes where the antitrust regulatory intensity is higher and in markets which are more concentrated, the inverted U-shape is more pronounced. This leads us to conclude that the decrease in likelihood can be partially attributed to stronger antitrust measures.

We then examine the effect of the PMR on the synergy, premium paid and the cumulative abnormal returns of both acquirer and target in the [-10,0] window around the merger announcement date. We find that the synergies, premium and the acquirer CAR have an inverted U-shaped relationship with the PMR as well. This suggests that in addition to the anti-takeover laws, synergy considerations were indeed significant in determining merger firm pairings. Acquirers are less likely to merge with targets that are very similar or very dissimilar to themselves. The former offer synergies arising from economies of scale and the latter those from economies of scope. Firms prefer intermediate levels of both kinds of synergies than a very high level of one and low level of the other.

We make significant contributions to the existing literature on firm similarity, antitrust, synergies and likelihood of mergers. First, to the best of our knowledge, this is the first study to show a non-monotonic relationship between acquirer-target product market similarity and the probability of a merger. Second, we find that technological similarity has a similar effect on merger likelihood even when controlling for the product market similarity. Third, we demonstrate that the effect is different in different antitrust regimes. Finally, we show that the inverted U relationship holds between the synergies and the PMR as well.

Our paper is most closely related in certain aspects to two previous studies Hoberg and Phillips (2010) and Bena and Li (2014). However, our analysis differs from theirs in the following ways.

In terms of the research question, Hoberg and Phillips (2010) look at how the similarity between firms in an industry affect the chance of one of them being part of a merger transaction, but do not look at the bilateral similarity between the acquirer and potential targets. Bena and Li (2014) look at the effect of technological overlap between two firms whereas we look at the effect of product market overlap. In addition, they do not examine the effect of technological similarity on any of the deal financial variables like market reactions or synergies.

In terms of methodology, Hoberg and Phillips (2010) do not control for a given acquirer-target pair since they do not focus on bilateral similarity. Bena and Li (2014) employ a control sample of potential acquirers and targets which did not merge for each merger in their sample. In contrast, our control sample is the acquirer-target pair in the years previous to the acquisition. This specification has the advantage that it enables us to remove the effects of time-invariant acquirer and target characteristics.

Last, in terms of results, unlike them, our main result across most of our specifications and a host of dependent variables is a non-monotonic relationship between acquirer-target similarity and the decision to merge. None of their findings involve such a relationship.

The rest of the paper is organized as follows. Section 3.2 discusses the research design and the empirical specifications. Section 3.3 describes the data sources. Section 3.4 summarizes the empirical results on the relationship between merger incidence and PMR. Section 3.5 explores our findings in greater depth and offers two possible explanations. Section 3.6 concludes.

3.2. Empirical Methodology

Since we are interested in the effect of the PMR between the acquirer and the target on the likelihood of the merger, our dependent variable is an indicator that takes a value of 1 if the two firms merged and 0 otherwise. If we only use the data for the acquirer and target in the merger years, we will have no variation in the dependent variable since it always takes a value of 1. To obtain our estimates, we need a set of control firms for each deal.

We use the AT&T - BellSouth $86 billion deal announced in 2006 to illustrate the rationale behind our empirical methodology. Figure 3.1 shows the PMR and the technology overlap between the two firms in the years leading to the acquisition. Ideally, we would want a control sample of the targets which were considered by AT&T but with whom the merger did not happen. In settings similar to ours, choosing an appropriate control sample is tricky. Since we do not know the consideration set of targets, there are two possible approaches to choose controls.

One approach is to generate matched sample of firms based on some criteria like size, year and industry, and consider them as potential targets which were not chosen, as Bena and Li (2014) do. If we were to use this specification and then include a deal fixed effect, we would exploit the variation among the PMRs of the target with each of the matched firms. However, what matters for the acquisition decision might be the change in PMR between the acquirer and the eventual target over time. The time series variation in the PMR for a selected target may be more relevant than the cross sectional variation in PMR between firms that are selected and not selected, particularly if target selection is driven by time-invariant firm-specific effects that we cannot control for.

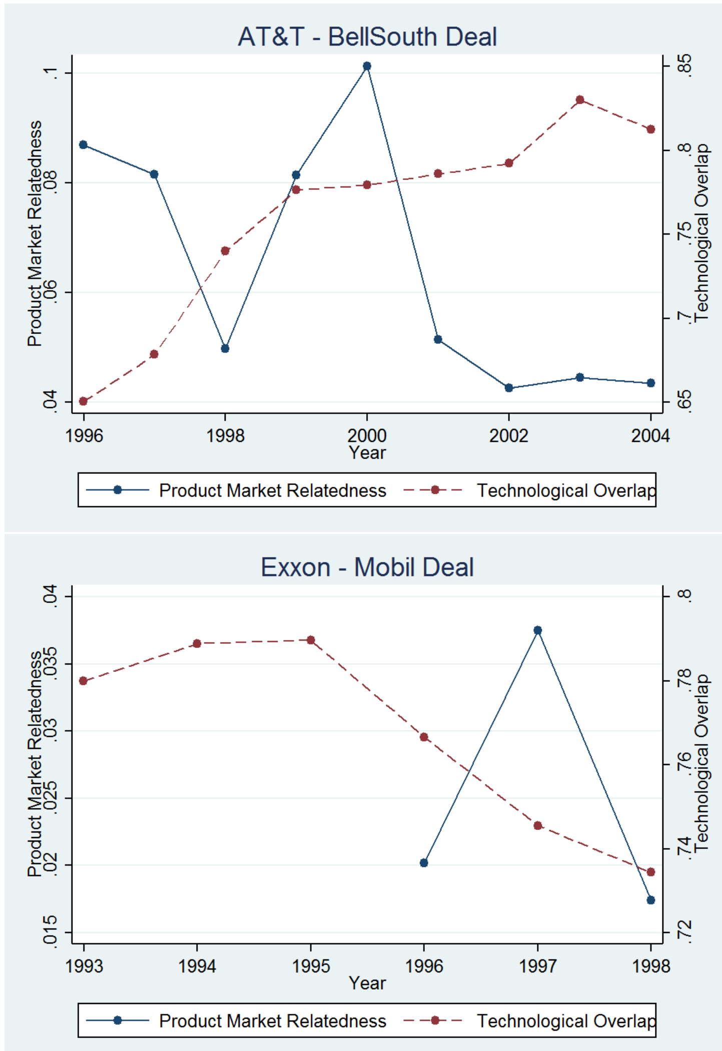


Figure 3.1. Product market relatedness and technology overlap
The figure shows the PMR and Technology Overlap between the acquirer and target firms in the years leading up to the deal for two different deals. The PMR measure is from Hoberg and Phillips (2010) and is based on the degree to which two firms use the same words in their 10-K product descriptions. We construct the Technology Overlap measure based on the similarity of the patent portfolios of the two firms as described in the Appendix.

Motivated by this logic, we assume that the appropriate control sample for the target firm in the year of acquisition is the target firm in the years leading to the acquisition.

In terms of our example, the assumption is that AT&T viewed BellSouth as a potential candidate in the years prior to the acquisition before finally acquiring it when it matched the criteria for acquisition.[2] This is similar to Blonigen and Pierce (2016), who, in one of their difference-in-difference specifications, compare plants acquired in a year with a control group made up of plants that will be acquired in subsequent years.[3] The advantage of this specification is that all time-invariant acquirer and target characteristics which affect the deal will be captured by our deal-specific fixed effects and thus will not bias our estimates.[4]

Our main regression specification is

$$Merger_{i,t} = \alpha_i + \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{it} \tag{3.1}$$

where $Merger_{i,t}$ is an indicator variable that takes a value 1 if the deal happened in that year and 0 if it did not. PMR is the product market relatedness defined in more detail in section 3.3.3. X is a vector of controls defined in the Appendix. Notice that in this specification, we add a fixed effect for each deal. In essence, our results are thus driven by variation within a deal. In addition to the linear term, we use the quadratic term to check whether the effect of PMR is non-linear or indeed even non-monotonic. We cluster the standard errors at the deal level. In regressions where we test for the effect of innovation

[2]In other words, AT&T acquired BellSouth in a year not because BellSouth had a "more suitable" PMR than (fictitious control firms) BellNorth, BellEast and BellWest, but because it had a "more suitable" PMR than itself in previous years.
[3]They argue that "this is a valid strategy if the attributes that make a plant more likely to be a target exist for a few years before a successful match with an acquirer is made." Our argument is exactly the same.
[4]In addition, we can interpret our results as either the choice of the acquisition decision or of acquisition timing.

overlap, the specification we adopt is similar

$$\begin{aligned}\text{(3.2)}\quad Merger_{i,t} &= \alpha_i + \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \beta_3 TechnologyOverlap_{i,t-1} \\ &+ \beta_4 TechnologyOverlap^2_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{it}\end{aligned}$$

where $TechnologyOverlap$ is the measure of technological overlap described in section 3.3.3. We employ a linear and quadratic term for the innovation overlap too, exactly like for the PMR.

Next, we look at possible mechanisms through which the PMR affects transaction incidence. First, we explore the effects of antitrust regulation on the relationship between the merger decision and PMR by interacting the PMR variable and its square with the Herfindahl Hirschman Index (HHI) for the market. The specification is[5]

$$\begin{aligned}\text{(3.3)}\quad Merger_{i,t} &= \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \beta_3 HHI + \beta_4 PMR_{i,t-1} * HHI \\ &+ \beta_5 PMR^2_{i,t-1} * HHI + \gamma X_{i,t-1} + \epsilon_{it}\end{aligned}$$

To evaluate a second potential channel through which PMR affects merger decisions, we look at how the synergies (and the premium paid) in the deal vary with the PMR. For these regressions, we cannot use deal fixed effects since there is only one observation per deal. The regressions we run take the form:

$$\text{(3.4)}\quad SynergyMeasure_i = \beta_1 PMR_i + \beta_2 PMR^2_i + \gamma X_i + \epsilon_i$$

The PMR and the controls correspond to the year before the merger.

[5]We do not include deal-specific fixed effects in this specification because there isn't enough variation within a deal to look at the differential effect of PMR on the merger likelihood depending on the value of HHI.

3.3. Data and Summary Statistics

3.3.1. Sample Construction

The sample includes mergers and acquisitions of US listed corporations from 1996 to 2015. We compile our data from several databases. We obtain the mergers and acquisitions (M&A) information from the Securities Data Company (SDC) M&A database. We collect PMR and market concentration (HHI) data from Hoberg-Phillips's Data Library website[6] and the patent data from National Bureau of Economic Research (NBER) Patent Citation database. To construct the corporation-level control variables, we obtain financial statement items from Compustat Industrial Annual Files and security prices from the Center for Research in Security Prices (CRSP) database.

We begin with a total of 28,395 deals from SDC and then apply the following filters to the data:

(1) Keep only the transactions where both the target and acquirer are publicly listed US firms. This is because we have the PMR data only for publicly listed US firms. This filter decreases our sample to 12,420 deals.

(2) We also exclude any recapitalization, exchange offer and buybacks. This step removes 1,655 deals from the sample.

(3) We remove 139 duplicates where the details to a specific entry was difficult to ascertain, leaving us with 10,482 deals.

(4) We keep deals with announcement date between January 1, 1996 and December 31, 2015. This is because the PMR data starts from 1996. This reduces our sample by 2,232 deals.

[6] http://hobergphillips.usc.edu/

This leaves us with a sample of 8,250 deals. We then merge this data with firm-year level financial data (for both acquirer and target) from Compustat and the stock price data from CRSP. We remove any deals with missing Compustat and CRSP identifier, leaving us with 3,616 merger and acquisition deals. We exclude deals where PMR and other controls variables are missing in the merger year. We also ensure that each deal has at least two years of observations. The final sample includes 896 deals and 4,767 deal-years.

For our tests related to technological overlap, we combine our acquirer-target sample with the NBER patents dataset. The NBER dataset is only available until 2006 and further limits our combined PMR and technology overlap sample to 135 deals and 561 deal-years. Table 3.1 summarizes the year-wise merger announcements for the entire sample, non-missing PMR sample and non-missing PMR and technology overlap sample.

3.3.2. Dependent Variables

3.3.2.1. Merger. For our 4,767 deal-years, our dependent variable is an indicator variable, Merger, which takes a value of 1 if the merger between the two firms was announced in that year and 0 otherwise.

3.3.2.2. Synergies. For our deal-level regressions, we look at merger synergies and related variables. To compute the combined synergies of the transaction and the share captured by the target, we use the methodology in many previous studies, for example Golubov et al. (2012), Kale et al. (2003) and Bradley et al. (1988). We begin by calculating the cumulative abnormal returns for the acquirer and the target in the window [-10,0].

Table 3.1. Corporate acquisitions over time

Year	Total Acquirer-Target Sample	Acquirer-Target with non-missing PMR Sample	Acquirer-Target with non-missing PMR and Technology Overlap Sample
1996	301	NA	NA
1997	379	100	9
1998	386	110	26
1999	406	122	16
2000	331	67	13
2001	216	49	14
2002	129	35	15
2003	150	41	9
2004	148	44	13
2005	138	29	12
2006	139	33	8
2007	156	38	NA
2008	106	33	NA
2009	88	24	NA
2010	83	25	NA
2011	64	24	NA
2012	87	27	NA
2013	92	26	NA
2014	98	32	NA
2015	119	37	NA
	3616	896	135

This table reports the number of announced corporate acquisitions by the year of the bid announcement over the period January 1, 1996 to December 31, 2015. We require that both acquirer and target are US public companies. A deal enters the *Acquirer-Target Sample* if both the acquirer and the target firm are covered by Compustat/CRSP. For the *Acquirer-Target Sample with PMR*, we require that the acquirer and the target both have a valid *PMR* measure and other control variables, particularly in the year of the bid announcement. For the *Acquirer-Target Sample with PMR and Overlap*, we require that the acquirer and the target both have a valid *PMR* and *Technology Overlap*. The PMR measure, from Hoberg and Phillips (2010), is based on the degree to which two firms use the same words in their 10-K product descriptions and is available only from 1996. *Technology Overlap* is a measure based on the overlap in the patent portfolios of the two firms, calculated as described in the Appendix, and is available only till 2006.

The abnormal returns are calculated using the market adjusted model where CRSP value-weighted index return is the market return. We then multiply the CARs for each firm by the respective market capitalization at $t = -10$ to get the extra value created by the merger i.e. the synergies. The synergies are then added together to get the combined synergies.

$$\begin{aligned} Synergies(Dollars) =& CAR_{Acquirer} \times MarketCap_{Acquirer} \\ &+ CAR_{Target} \times MarketCap_{Target} \end{aligned}$$

However, it is difficult to compare dollar synergies across deals. So, we divide the synergies in dollar terms by the sum of the market capitalizations of both firms at $t = -10$ to get the combined synergies.

$$Synergies(\%) = \frac{CAR_{Acquirer} \times MarketCap_{Acquirer} + CAR_{Target} \times MarketCap_{Target}}{MarketCap_{Acquirer} + MarketCap_{Target}}$$

The share of the synergies that go to the target's shareholders (TSOS) is calculated as:

$$TSOS = \begin{cases} \frac{CAR_{Target} \times MarketCap_{Target}}{Synergies(Dollars)} & \text{if } Synergies(Dollars) \geq 0 \\ 1 - \frac{CAR_{Target} \times MarketCap_{Target}}{Synergies(Dollars)} & \text{if } Synergies(Dollars) < 0 \end{cases}$$

The premium is calculated as a fraction of the target's market capitalization at $t = -10$:

$$Premium = \frac{Payment}{MarketCap(t = -10)_{Target}}$$

3.3.3. Independent Variables

3.3.3.1. Product Market Relatedness (PMR). The product market relatedness (PMR) data were downloaded from the Hoberg-Phillips's Data Library website. We use the larger TNIC Database (which is calibrated to have the same granularity as two-digit

SIC codes) so that we can increase the number of matches in our sample. For each deal, we retain the acquirer-target pairwise similarity from 1996 to the acquisition date. This gives us the time series of the product similarity between the acquirer and target for each transaction from the SDC database.

3.3.3.2. Technology Overlap. It is calculated (based on Jaffe (1986)) as :

$$\frac{S_{acq}S'_{tar}}{\sqrt{S_{acq}S'_{acq}}\sqrt{S_{tar}S'_{tar}}}$$

where the vector $S_{acq} = (S_{acq,1}, ...S_{acq,J})$ captures the innovation activity of the acquirer firm and vector $S_{tar} = (S_{tar,1}, ...S_{tar,J})$ captures the innovation activity of the target firm, and J denotes the technology class. S_{acq} is calculated as the ratio of number of patents awarded in technology class J till date to the total number of patents awarded till date in all technology classes to the acquirer. S_{tar} is defined similarly.

3.3.3.3. Herfindahl Hirschman Index (HHI). We obtain HHI from Hoberg-Phillips's Data Library website.[7] It is a proxy for the level of competition in the market. Specifically the Text-based Network Industry Classifications Herfindahl index in Hoberg and Phillips (2016) has been calculated using a dynamic industry classification based on each firm's product descriptions from annual 10-K filings.

The summary statistics of the variables used in our regressions are presented in Table 3.2.

[7]http://hobergphillips.usc.edu/industryconcen.htm

Table 3.2. Summary statistics

Panel A: Deal-Year Variables

	Count	Mean	SD	Min	P25	P50	P75	Max
Acquirer Book to Market	4767	0.558	0.718	-7.750	0.252	0.437	0.680	13.918
Target Book to Market	4767	0.634	0.958	-23.284	0.288	0.516	0.811	29.923
Acquirer Leverage	4767	0.229	0.202	0.000	0.065	0.189	0.340	2.071
Target Leverage	4767	0.234	0.226	0.000	0.046	0.190	0.362	3.044
Acquirer Stock Runup	4767	5.865	38.538	-91.477	-14.241	1.211	17.920	501.780
Target Runup	4767	3.337	53.333	-96.419	-19.742	-2.084	16.858	1405.284
Acquirer Volatility	4767	0.026	0.016	0.005	0.015	0.021	0.032	0.185
Target Volatility	4767	0.032	0.020	0.006	0.018	0.027	0.040	0.227
Acquirer Cash to Assets	4767	0.138	0.180	0.000	0.024	0.059	0.170	0.956
Target Cash to Assets	4767	0.158	0.208	0.000	0.021	0.059	0.214	0.974
Acquirer Return on Assets	4767	0.020	0.160	-3.491	0.008	0.031	0.069	1.336
Target Return on Assets	4767	-0.027	0.262	-5.130	-0.004	0.017	0.055	2.154
Acquirer Total Assets	4767	10.371	27.765	0.003	0.563	2.212	8.337	707.121
Target Total Assets	4767	2.801	6.746	0.001	0.142	0.562	2.263	102.580
PMR	4767	0.119	0.110	0.000	0.054	0.096	0.153	0.939
HHI	4767	0.136	0.131	0.016	0.056	0.094	0.162	1.000
Technology Overlap	561	0.217	0.256	0.000	0.000	0.102	0.356	1.000

Panel B: Deal Characteristics

	Count	Mean	SD	Min	P25	P50	P75	Max
Deal Value	896	2.455	7.478	0.000	0.057	0.246	1.262	89.168
Premium	896	1.398	1.053	0.000	1.047	1.332	1.725	15.014
Acquirer CAR	896	-0.009	0.098	-0.579	-0.056	-0.010	0.036	0.640
Target CAR	896	0.179	0.310	-0.546	0.008	0.116	0.267	3.881
Synergies	896	0.020	0.099	-0.329	-0.031	0.015	0.068	0.839
Form	896	0.824	0.381	0.000	1.000	1.000	1.000	1.000
All Cash Deal Indicator	896	0.249	0.433	0.000	0.000	0.000	0.000	1.000
Mixed Deal Indicator	896	0.443	0.497	0.000	0.000	0.000	1.000	1.000
Hostile Takeover Indicator	896	0.038	0.191	0.000	0.000	0.000	0.000	1.000
Tender Offer Indicator	896	0.102	0.302	0.000	0.000	0.000	0.000	1.000
Reverse Takeover Indicator	896	0.035	0.183	0.000	0.000	0.000	0.000	1.000
Acquirer Market Capitalization	896	14.084	44.465	0.007	0.475	1.616	6.753	492.460
Target Market Capitalization	896	1.873	5.769	0.001	0.068	0.233	1.107	65.386
PMR	896	0.142	0.129	0.000	0.062	0.116	0.179	0.939
HHI	896	0.141	0.139	0.016	0.059	0.094	0.172	1.000
Technology Overlap	135	0.223	0.253	0.000	0.014	0.116	0.323	1.000

This table reports summary statistics for the dependent, independent and control variables used in our regressions. Panel A reports the statistics for variables defined at the deal-year level and Panel B for the variables defined only in the year of the merger. Definitions of the variables are provided in the Appendix.

3.4. Results

3.4.1. Product Market Relatedness and Acquisition Likelihood

We start by examining how the Product Market Relatedness affects the likelihood of the merger.

Table 3.3. Main results

	(1) Merger	(2) Merger	(3) Merger	(4) Merger	(5) Merger	(6) Merger
PMR	0.750***	0.885***	2.414***	4.898***	5.957***	18.197***
	(6.78)	(7.84)	(7.03)	(7.08)	(8.18)	(6.81)
PMR^2	-0.629***	-0.780***	-1.484**	-4.358***	-5.489***	-14.583***
	(-3.65)	(-4.28)	(-2.01)	(-4.50)	(-5.27)	(-3.16)
Constant	0.115***	0.057***	-0.134**	-1.956***	-2.324***	
	(11.61)	(2.98)	(-2.49)	(-27.00)	(-18.25)	
Observations	4,767	4,767	4,767	4,767	4,767	4,767
R-squared	0.0132	0.0322	0.1931	0.0132	0.0323	0.283
Deal FE	NO	NO	YES	NO	NO	YES

Our main regression specification is:

$$Merger_{i,t} = \alpha_i + \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{it} \quad (3.5)$$

where $Merger_{i,t}$ is an indicator variable that takes a value 1 if an M&A transaction between the two firms occurred in that year and 0 if it did not. The PMR variable is a measure of the product market relatedness between the two firms based on the product descriptions in the 10-K forms as in Hoberg and Phillips (2010). It lies in the interval (0,1) and a higher similarity measure implies that the two firms have product descriptions more closely related to each other. X is a vector of controls. Columns (1)-(3) provide results for the OLS regression estimation, columns (4)-(5) for the logistic regressions and column (6) for the conditional logistic regression. The sample covers mergers announced between January 1, 1996 and December 31, 2015. Definitions of the variables are provided in the Appendix. T-stats (based on clustered standard errors) are reported in parentheses; *, **, and *** denote significance at the 10%, 5% , and 1% respectively.

Table 3.3 presents the results. In column (1), we pool all the observations together and do not add any deal fixed effects. Our results suggest that the relationship between likelihood of the merger and the PMR is not monotonic, but an inverted U. The likelihood of the merger decreases when the PMR increases above a threshold. Even below the threshold, the marginal effect of an increase in the PMR on the likelihood decreases as the PMR increases. In other words, at low values of PMR, a small increase in PMR leads to a larger increase in likelihood of merger than at high values.

Column (2) represents the same specification with the control variables for the target and acquirer added. Comparing the coefficients on the linear and quadratic terms of the PMR variable, we find that the estimates including the controls are similar to those without them.[8]

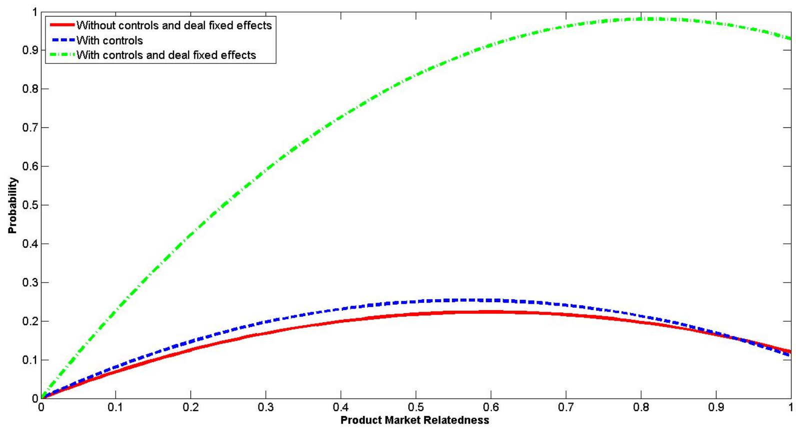
In column (3), we add deal fixed effects to the specification in column (2). The fixed effects are added to see whether our results change once we account for the average PMR between the target and acquirer across years. It might be the anticipated increase or decrease in the PMR that matters rather than the absolute level. Deal fixed effects enables us to remove the effect of the average PMR from the probability of the transaction and thus concentrate purely on the within-deal variation. The coefficients of the linear and quadratic term change when we include the deal-specific fixed effects. However, the

[8]This is not surprising. The only way omitted variables can bias the regression estimates is if they are correlated with both the outcome variable (probability of the transaction) and the dependent variables (PMR). While the control variables doubtless affect the outcome, there is no reason to suppose that the PMR between two firms would be related to any of their financial variables like cash, book to market ratio etc. This points to another advantage of the measure and why it can be considered plausibly exogenous. The measure is affected by the decisions of two firms, so that in the absence of coordination between them, one would not expect it to be related to the financial variables of either firm.

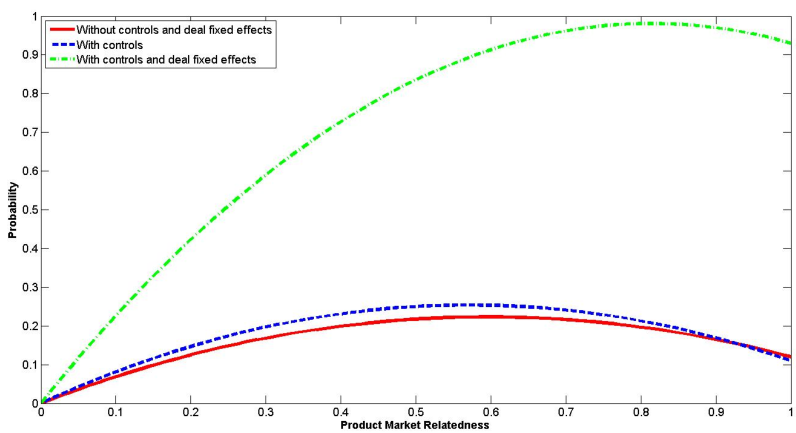
statistical significance of our estimates at 1% and the inverted U-shape persist under all three specifications.

Figure 3.2. The non-monotonic effects of PMR on transaction incidence

Panel A: OLS regressions



Panel B: Logistic regressions



The figure shows the effects of PMR on the probability of transaction incidence based on the regressions in Table 3.3. Panel A corresponds to the OLS specification and Panel B to the logistic regression specification. For the OLS regressions, we only plot the linear and quadratic terms in PMR. For the logistic regressions, we assume that the control variables are at the mean levels. For the conditional logistic regression, we assume a suitable value for the deal-specific fixed effect to plot the graph since the fixed effects are not estimated in a conditional logistic regression.

Columns (4) and (5) employ a logistic regression specification rather than an OLS regression. This change doesn't affect the statistical significance of our coefficients. Column (6), the conditional logistic regression, allows us to include deal-specific fixed effects for each transaction. This is our most preferred specification.

Across all the specifications in Table 3.3, both the linear and the quadratic terms retain their signs and continue to be highly significant. We conclude that the non-monotonic inverted-U relationship between PMR and transaction likelihood is robust to various regression specifications and controls. The peaks of the graphs, which are the points after which an increase in PMR decreases the likelihood of the merger, are 0.60, 0.57 and 0.81 for the OLS regressions and 0.56, 0.54 and 0.62 for the logistic regressions. Thus, the peak is stable across the regressions too.

Determining the magnitude of the effect is tricky due to the presence of the quadratic term in all our regressions. In Figure 3.2, we plot the marginal effect of the PMR on the likelihood of the merger. For the linear regressions, we plot the marginal effects due to the linear and quadratic terms of equation 3.1 since the other terms only affect the level. For the logistic regression without controls and deal-fixed effects, we plot the probabilities directly as a function of the PMR using the exponential specification of the logistical regression. For the regression with controls but without the deal fixed effects, we calculate the probability assuming that each control variable takes the median value in the sample.

3.4.2. Technological Overlap and Acquisition Likelihood

Table 3.4. PMR and technology overlap

	(1) Merger	(2) Merger
PMR	34.037***	36.991***
	(3.20)	(3.16)
PMR^2	-50.073***	-53.693***
	(-2.75)	(-2.61)
Technology Overlap		29.921***
		(3.43)
Technology Overlap^2		-21.023***
		(-2.67)
Observations	561	561
Deal FE	YES	YES
R-squared	0.364	0.446
Controls	YES	YES

The conditional logit specification to test the effect of PMR and Technology Overlap on the probability of merger incidence is as follows:

$$
\begin{aligned}
Merger_{i,t} &= \alpha_i + \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} \\
&\quad + \beta_3 TechnologyOverlap_{i,t-1} + \beta_4 TechnologyOverlap^2_{i,t-1} + \gamma X_{i,t-1} + \epsilon_{it}
\end{aligned}
$$

where $Merger_{i,t}$ is an indicator variable that takes a value 1 if an M&A transaction between the two firms occurred in that year and 0 if it did not. The PMR variable is a measure of the product market relatedness between the two firms based on the product descriptions in the 10-K forms as in Hoberg and Phillips (2010). *Technology Overlap* is the measure of technological innovation overlap between the acquirer and the target. Both measures of similarity lie in the interval (0,1) and a higher similarity measure implies that the two firms have product descriptions / patent portfolios that are more closely related to each other. X is a vector of controls. Definitions of the variables are provided in the Appendix. All specifications include deal fixed effects. T-stats (based on clustered standard errors) are reported in parentheses; *, **, and *** denote significance at the 10%, 5% , and 1% respectively.

So far, we have looked at one measure of firm similarity i.e. product market relatedness.

We now examine another dimension along which two firms can be similar, the technological overlap between them. Bena and Li (2014) show that technological overlap between the acquirer and target affects the likelihood of the transaction. So, there may be a concern that our results so far are driven by the fact that firms with high PMR may also have high technological overlap. To check whether this is the case, we see whether the effect of product market overlap persists after controlling for the technological overlap.

Table 3.4 displays the results of our analysis. In column (1), we run the same regression specification as in column (6) of Table 3.3 to ensure that any difference in this sub-sample results is not an artifact of the sub-sample selection. The result of the regression indicates that the relationship between transaction incidence and PMR is broadly the same in this sub-sample as for the whole sample. Both the linear and the quadratic terms are significant, and the signs remain the same.

Column (2) reports the results once we add the technological overlap and its square as independent variables. We find that the effect of the PMR remains, that is the PMR matters over and above the technological similarity. Also, the coefficient on the PMR variable and its squared terms in column (2) are very similar to those in column (1), confirming that it is not the correlation between PMR and technology overlap that drove our earlier results.

Further, we confirm the Bena and Li (2014) result that technology overlap is independently significant in determining the merger likelihood. Unlike them, however, we find that the effect is nonlinear. The square of technological relatedness is negative and statistically significant at the 1% level, so the inverted U shaped relationship we find between merger likelihood and the PMR holds for the technology overlap too.[9]

[9] Ahuja and Katila (2001) shows a nonlinear impact of technological overlap on the post-acquisition innovation output of the acquiring firm. However, their study differs from ours in numerous ways. Their

3.5. Possible Mechanisms

So far, we have established that both the PMR and the innovation overlap have non-monotonic effects on the likelihood of a firm acquiring a target. We now examine the mechanism through which these effects manifest themselves.

High firm similarity may dissuade firms from merging due to two reasons. Consider first the antitrust explanation. If two firms are very similar, specifically in the product market, antitrust regulators may block the merger from happening. Anticipating this, firms may decide not to merge with the firm till the PMR decreases in the future. The alternative mechanism is through the effect of PMR on synergies rather than antitrust investigations. The likelihood of the firms merging varies in proportion to the synergies. The synergies may have an inverted U relationship with PMR which leads to the likelihood of merger incidence having the same relationship.

sample only includes the chemical industry, their overlap measure differs from ours, their effect is on subsequent innovation rather than on merger incidence, and their result only holds for technological acquisitions within the industry. Bena and Li (2014) do not find a nonlinear effect even on the post-acquisition innovation output.

3.5.1. Antitrust Regulation

Table 3.5. PMR and market concentration

	(1) Merger	(2) Merger	(3) Merger	(4) Merger
PMR	0.557***	0.682***	3.977***	4.866***
	(3.24)	(4.09)	(3.67)	(4.44)
PMR2	-0.354	-0.445*	-2.984*	-3.657**
	(-1.31)	(-1.81)	(-1.91)	(-2.55)
HHI	-0.006	-0.241	0.080	-1.412
	(-0.07)	(-1.64)	(0.14)	(-1.36)
HHI × PMR	2.138**	2.134**	10.748*	11.548*
	(2.12)	(2.12)	(1.81)	(1.81)
HHI × PMR^2	-2.852**	-3.361***	-14.729*	-18.424**
	(-2.05)	(-2.60)	(-1.78)	(-2.29)
Constant	0.110***	0.088***	-2.003***	-2.136***
	(6.95)	(2.97)	(-17.22)	(-10.45)
Observations	4,767	4,767	4,767	4,767
R-squared	0.0161	0.0384	0.0159	0.0378
Deal FE	NO	NO	NO	NO
Controls	NO	YES	NO	YES

The table reports results from the following regression specification

$$Merger_{i,t} = \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \beta_3 HHI + \beta_4 PMR_{i,t-1} * HHI + \beta_4 PMR^2_{i,t-1} * HHI + \gamma X_{i,t-1} + \epsilon_{it}$$

where $Merger_{i,t}$ is an indicator variable that takes a value 1 if an M&A transaction between the two firms occurred in that year and 0 if it did not. The PMR variable is a measure of the product market relatedness between the two firms based on the product descriptions in the 10-K forms as in Hoberg and Phillips (2010). HHI is a measure of market concentration based on Hoberg and Phillips (2016). X is a vector of controls. This specification does not include deal fixed effects. Columns (1)-(2) employ a linear regression specification and columns (3)-(4) employ a logistic regression specification. Definitions of the variables are provided in the Appendix. T-stats (based on clustered standard errors) are reported in parentheses; *, **, and *** denote significance at the 10%, 5% , and 1% respectively.

Section 7 of the Clayton Act bars mergers if "in any line of commerce or in any activity affecting commerce in any section of the country, the effect of such acquisition may be substantially to lessen competition, or to tend to create a monopoly."[10] The line of commerce referred to constitutes a product market. The FTC guidelines on horizontal mergers states that "When a product sold by one merging firm (Product A) competes against one or more products sold by the other merging firm, the Agencies define a relevant product market around Product A to evaluate the importance of that competition."[11]

The definition of the relevant product market is crucial in determining whether the merger is blocked or not. Two contrasting recent examples are the Staples and Office Depot merger in 1997 which was blocked whereas the Whole Foods Market's acquisition of Wild Oats Market went ahead in 2007.[12] In one case, the market was defined narrowly whereas in the other case, it was defined more broadly.[13] As the PMR between two firms increases, the firms are viewed as competitors and the chances of the merger being blocked thus increases.

The second important factor in determining whether a horizontal merger should be allowed is the industry concentration. The FTC guidelines state that the Herfindahl-Hirschman Index, a measure of market concentration, is a factor that the agencies take into account. If antitrust plays a key role in merger decisions, it is likely to do so in markets where the acquirer's market concentration is high. So, the effect of PMR on merger decisions should be particularly pronounced in markets which are concentrated.

[10] 15 U.S. Code §18.

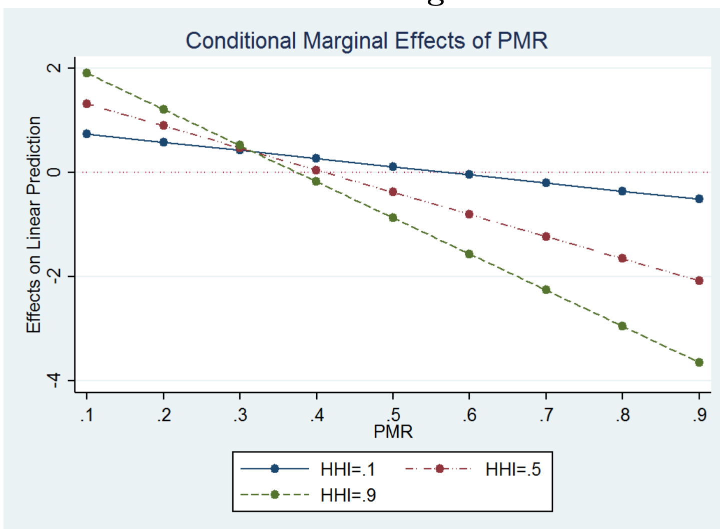
[11] As quoted in the horizontal merger guidelines at the FTC site, available online at www.ftc.gov/sites/default/files/attachments/mergers/100819hmg.pdf, last accessed on March 31$^{\text{th}}$, 2017.

[12] This is not just a recent phenomenon. Writing a few decades ago, Manne (1965) expressed the opinion that "Antitrust problems in the merger field seem more and more to be confined to discussions of relevant product and geographic markets and perhaps to the issue of quantitative substantiality."

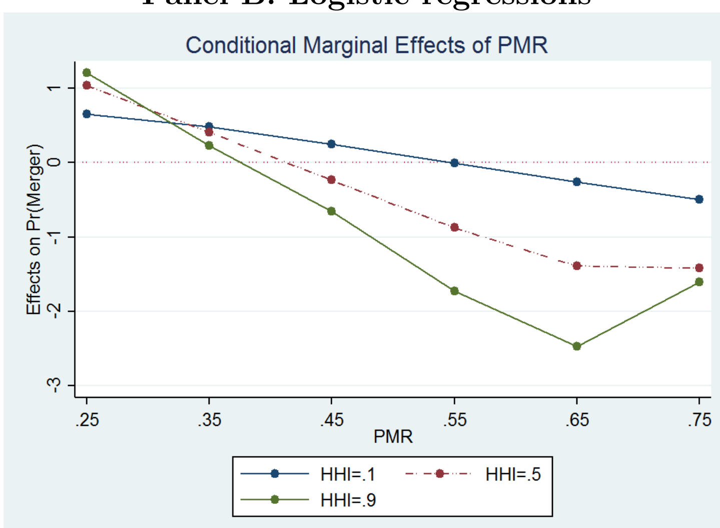
[13] See Varner and Cooper (2007) for more details on these two proposed mergers and the role the definition of the relevant product market played in both of them.

Figure 3.3. The marginal effects of PMR on transaction incidence

Panel A: OLS regressions



Panel B: Logistic regressions



The figure shows how the marginal effects of PMR on the probability of transaction incidence vary with the market concentration. The estimates are based on the regressions in Table 3.5. We plot the marginal effects for low, medium and high values of market concentration (0.1, 0.5 and 0.9). All control variables are assumed to be at their mean levels. Panel A corresponds to the OLS regression specification and Panel B to the logistic regression specification.

We test this hypothesis by adding interaction terms for the PMR and its square with the HHI. The results are given in Table 3.5. In columns (1) - (2), we employ a linear regression specification and in columns (3)-(4) a logistic regression specification. We do not include deal-fixed effects since there is not enough variation within a deal to capture the differential effect of PMR depending on HHI. We run regressions with and without controls. Across all specifications, we find that the interaction coefficient of the squared term is negative, thus showing that high values of market concentration affect merger decisions at higher PMRs more than low values of market concentration.

Since our regressions have both linear and squared terms, we plot the marginal effects of the PMR on the probability for easy interpretation and visualization.[14] Our preferred specification is the logistical regressions with controls, column (4). Figure 3.3 shows that as HHI increases, a small change in the PMR leads to a larger drop in the probability of the transaction incidence. It is clear from the graph that the value of the PMR at which the marginal effect starts becoming negative[15] decreases as the market concentration increases. This strongly suggests that antitrust acts as a deterrent for firms to consider merging with a target which operates in very similar product markets.

To bolster our results further, we next examine whether PMR affects merger decisions more in times of higher regulatory intensity. There are reasons to believe that the antitrust regulators were more active during certain years in the two decades covered by our sample. Katz and Shelanski (2007), for example, mentions the unusually high number of merger investigations initiated by the Antitrust Division of the U.S. Department of Justice in

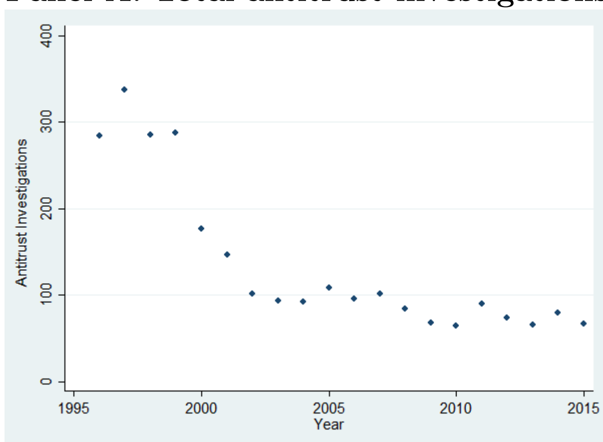
[14]This is just the partial derivative of the probability with respect to the PMR.

[15]This is the point after which an increase in PMR causes the probability of transaction to decrease.

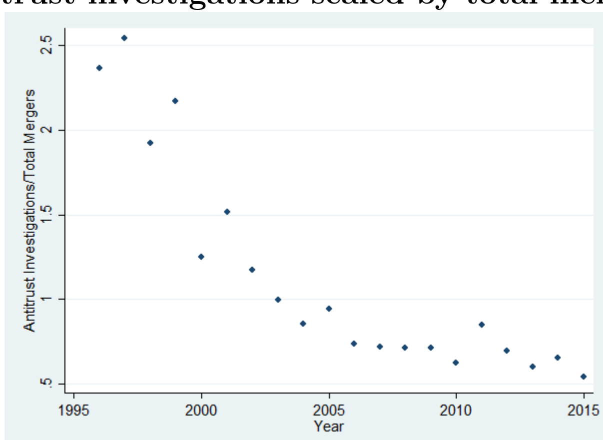
the years from 1996 to 2005. We collect the data on the investigations from the DOJ website.[16]

Figure 3.4. Antitrust investigations over time

Panel A: Total antitrust investigations



Panel B: Total antitrust investigations scaled by total merger announcements



Panel A shows the total number of antitrust investigations in each year from 1996 – 2015. Panel B shows the ratio of antitrust investigations to the total number of mergers and acquisition in that year. The data is taken from the ten year workload statistics, available at the website of the Department of Justice (https://www.justice.gov/atr).

[16]The Ten Year Workload Statistics Report can be accessed at https://www.justice.gov/atr/division-operations.

Table 3.6. Product market relatedness and antitrust intensity

	(1) 1996 - 2005 Merger	(2) 2006 - 2015 Merger	(3) 1996 - 2015 Merger	(4) 1996 - 2015 Merger
PMR	5.884***	6.098***	6.098***	7.499***
	(6.52)	(4.22)	(-4.22)	(3.43)
PMR2	-5.048***	-6.421***	-6.421***	-8.463***
	(-3.86)	(-3.48)	(-3.48)	(-2.88)
High			-1.153***	-0.194
			(-3.92)	(-0.73)
High × PMR			-0.215	-3.849
			(-0.12)	(-1.55)
High × PMR^2			1.373	7.119**
			(0.60)	(2.11)
HHI				1.045
				(1.08)
HHI × PMR				-10.333
				(-0.97)
HHI × PMR^2				12.679
				(1.05)
High × HHI				-1.743
				(-1.58)
High × HHI × PMR				32.025**
				(2.50)
High × HHI × PMR^2				-47.124***
				(-2.75)
Constant	-2.729***	-1.576***	-1.576***	-2.148***
	(-15.16)	(-7.06)	(-10.95)	(-7.07)
Observations	3,496	1,271	4,767	4,767
Deal FE	NO	NO	NO	NO
R-Squared	0.0417	0.0394	0.0461	0.0409
Controls	YES	YES	YES	YES

Columns (1) and (2) provide subsample regressions similar to our main specification for time periods 1996-2005 and 2006-2015 respectively. In columns (3) and (4), we use the following regression specification for the whole sample to test the effect of antitrust intensity (as measured by the antitrust investigations in a given year).

$$Merger_{i,t} = \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \beta_3 HHI + \beta_4 High + \beta_5 PMR_{i,t-1} * HHI$$
$$+ \beta_6 PMR^2_{i,t-1} * HHI + \beta_7 PMR_{i,t-1} * High + \beta_8 PMR^2_{i,t-1} * High + \beta_9 HHI * High$$
$$+ \beta_{10} PMR^2_{i,t-1} * HHI * High + \beta_{11} PMR^2_{i,t-1} * HHI * High + \gamma X_{i,t-1} + \epsilon_{it}$$

where $Merger_{i,t}$ is an indicator variable that takes a value 1 if an M&A transaction between the two firms occurred in that year and 0 if it did not. X is a vector of controls. Definitions of the variables are provided in the Appendix. T-stat (based on clustered standard errors) are reported in parentheses; *, **, and *** denote significance at the 10%, 5% , and 1% respectively.

To lessen concerns of reverse causality, that the investigations are higher because mergers were more likely, we also calculate the number of investigations as a fraction of the total number of mergers and acquisitions announced in the year. Figure 3.4 shows the graph of these variables. We find that in the first decade of the sample, both the number and fraction of mergers investigated was high in relation to our overall sample.

We first run the regression specifications in Table 3.3 column (5) i.e. logit regressions for the subsample from 1996 to 2005. Note that we do not include deal-fixed effects since our data for some deals covers both the low and high intensity regimes.[17] Table 3.6 provides results for these tests. Columns (1)-(2) show that the effect of PMR was present in both high and low regulatory intensity regimes.

To examine the differential effect of the two regimes, we postulate that the differential effect of PMR due to the level of market concentration is further amplified if the antitrust regulators are more active. To test this, we add a double interaction term between the PMR, antitrust intensity and HHI.

The regression with the double interaction takes the following form.

(3.6)

$$\begin{aligned} Merger_{i,t} &= \beta_1 PMR_{i,t-1} + \beta_2 PMR^2_{i,t-1} + \beta_3 HHI + \beta_4 High + \beta_5 PMR_{i,t-1} * HHI \\ &+ \beta_6 PMR^2_{i,t-1} * HHI + \beta_7 PMR_{i,t-1} * High + \beta_8 PMR^2_{i,t-1} * High \\ &+ \beta_9 HHI * High + \beta_{10} PMR^2_{i,t-1} * HHI * High \\ &+ \beta_{11} PMR^2_{i,t-1} * HHI * High + \gamma X_{i,t-1} + \epsilon_{it} \end{aligned}$$

[17]This is almost identical to the subsample for which we have the technology overlap proxy.

Where $High$ is an indicator variable that takes value 1 when the time period is 1996-2005 and 0 otherwise. HHI is a continuous variable that denotes acquirer's market concentration. All other variables have been defined previously.

The coefficients of interest here are β_{10} and β_{11}, which indicate the difference between the differential effect of PMR in markets with different market concentrations in regimes with high antitrust intensity versus those with low antitrust intensity. Column (4) shows that the interaction effect is positive for the linear PMR term (32.025) and negative for the quadratic PMR term (-47.124), which means that the negative effect dominates at high values of PMR.[18]

Overall, our findings in this section provide strong support to the hypothesis of antitrust regulation being a mechanism through which PMR affects the mergers. We thus make a contribution to the literature on the costs and benefits of antitrust regulation. For example, Stillman (1983) examines the question of whether horizontal mergers challenged in the past by the federal government would have resulted in higher product prices to consumers while acknowledging that the study looks only at challenged mergers. However, we show that it is not enough to look only at the mergers which were challenged by the regulators to calculate the welfare effects absent antitrust since even the decision of firms to merge may be affected by the antitrust policies. In addition, we demonstrate that the effect of the policies are highest exactly in those industries and markets that they are designed for.

However, from Table 3.5, we see that the PMR has a non-monotonic effect even when the HHI is low, since the coefficients of the linear and quadratic terms are significant

[18]However, column (3) demonstrates that if we do not add the double interaction term, we find no statistically significant difference between the two decades, which further strengthens our conclusion.

by themselves as well. This suggest that antitrust might not be the only factor at play. Also, in Table 3.4, we find technological overlap to have a non-monotonic effect on the transaction likelihood, which is difficult to explain purely through an antitrust mechanism. Motivated by this, we next turn to the effect of PMR on synergies as an alternative mechanism that affects merger likelihood.

3.5.2. Synergies, Premium Paid and Stock Market Reaction

Prior literature, for example, Andrade et al. (2001) and Bradley et al. (1988) has documented that on average, mergers create value for shareholders of the combined merging parties. In this section, we examine the effect of PMR on total synergies and how they are split between the acquirer and target. Our dependent variables are total synergies, target share of synergies, premium paid, and cumulative abnormal market returns for the acquirer and the target.

Acquiring a firm that operates in similar product markets may lead to synergies from the economies of scale whereas acquiring a dissimilar firm may lead to economies of scope. It is certainly possible that the combined synergies from both these effects are highest when the target is neither very similar nor very dissimilar (in terms of product markets) from the acquirer. If this were the case, one would expect that the relationship between synergies and PMR exhibits an inverted U relationship analogous to that between merger likelihood and PMR.[19]

[19] A similar argument holds for innovation overlap too. However, we have very few deals with data for innovation overlap leading to insufficient power to test the hypothesis about gains from innovation overlap.

Table 3.7. Merger synergies and premium

	(1) Synergies	(2) TSOS	(3) Premium	(4) Acquirer CAR	(5) Target CAR
PMR	0.166***	2.673**	1.325**	0.113*	-0.105
	(2.98)	(2.09)	(2.27)	(1.90)	(-0.54)
PMR^2	-0.233***	-3.493**	-1.894***	-0.189**	-0.116
	(-3.07)	(-2.23)	(-2.70)	(-2.27)	(-0.51)
Acquirer Stock Runup				0.001***	0.000
				(4.03)	(0.60)
Acquirer Volatility				-0.312	0.685
				(-1.05)	(0.94)
Acquirer Book to Market				0.010*	-0.031**
				(1.79)	(-2.55)
All Cash Deal Indicator	0.019**	-0.232	-0.334***	0.012	0.054*
	(2.14)	(-1.33)	(-4.11)	(1.38)	(1.79)
Mixed Deal Indicator	0.008	0.150	0.155**	-0.007	-0.022
	(1.08)	(0.94)	(2.01)	(-0.84)	(-0.90)
Hostile Takeover Indicator	0.012	0.790**	-0.035	-0.028**	-0.057
	(0.92)	(2.08)	(-0.20)	(-2.23)	(-1.21)
Tender Offer Indicator	0.018	0.061	0.627***	0.014	0.139***
	(1.42)	(0.39)	(2.76)	(1.21)	(3.16)
Reverse Takeover Indicator		0.365	0.008	-0.023	-0.014
		(0.65)	(0.10)	(-1.10)	(-0.46)
Deal Value	0.000				
	(0.65)				
Constant	-0.008	0.829***	1.230***	-0.019*	0.180***
	(-0.96)	(4.63)	(14.27)	(-1.72)	(5.29)
Observations	894	881	894	894	894
R-squared	0.0219	0.0177	0.0579	0.0555	0.0448
Deal FE	NO	NO	NO	NO	NO

The table presents results from the following regression specification

$$SynergyMeasure_i = \beta_1 PMR_i + \beta_2 PMR_i^2 + \gamma X_i + \epsilon_i$$

The dependent variable is the total synergies from the merger in column (1), the target's share of synergy in column (2), the premium in column (3) and the acquirer and target CARs in the [-10,0] window around the merger announcement date in columns (4) and (5) respectively. The PMR variable is a measure of the product market relatedness between the two firms based on the product descriptions in the 10-K forms as in Hoberg and Phillips (2010). X is a vector of controls. Definitions of the variables are provided in the Appendix. T-stat (based on robust standard errors) are reported in parentheses; *, **, and *** denote significance at the 10%, 5% , and 1% respectively.

The results we obtain are presented in Table 3.7. Note that unlike the earlier specifications, we now have only one observation per deal. So, we employ more control variables to isolate the effect of PMR on the transaction-level synergy variables. Also, we drop two deals where the combined synergies exceeded 100% since they are outliers in our sample.

In column (1), we present evidence that the total synergies (in percentage terms) have an inverted U relationship with the PMR. Both the linear and quadratic terms are statistically significant at the 1% level. The synergies are increasing in PMR till PMR=0.36 and decrease afterwards. In addition to the statistical significance, the effect is economically significant too. For example, an increase of PMR from 0 to 0.36 (which corresponds to the peak of the inverted parabola) increases the synergy by 2.96%.

Prior research has shown that target shareholders on average extract most of the synergies in M&A transactions. The dependent variable in columns (2) and (3) are the fraction of synergies that accrue to the target's shareholders and the premium paid to the target. The inverted parabola shape continues to hold. Our findings show that when the firms become very related in the product markets, the target extracts less of the synergies. A possible explanation could be that the bargaining power of the acquirer increases if there are no other competing acquirers.

Finally, in columns (4) and (5), we look at the stock market reactions of the acquirer and target to the merger announcement. The cumulative abnormal return of the acquirer has the same inverted U relationship with the PMR as in previous regressions. An increase of PMR from 0 to the peak of the inverted parabola (which occurs at 0.3) increases the CAR of the acquirer by 1.69%. This is consistent with both columns (1) and (4) which show that the total synergies increase, but the fraction extracted by the target increases as well. So, the acquirer's CAR increases by less than the total synergies.

Considered in their totality, the results in this section indicate that synergies are a significant alternative channel through which the PMR affects merger likelihood. Since the relationship between PMR and synergies is not monotonic, even in the absence of antitrust, firms are not keen to acquire targets either very similar or very dissimilar to themselves in terms of product market relatedness.[20]

3.6. Discussion and Concluding Remarks

In this paper, we examine the effect of product market relatedness of two firms on the likelihood of incidence of a merger transaction between them. We find an inverted U-shaped relationship between PMR and transaction likelihood. We provide two possible mechanisms underlying the effect. The first is the possibility that antitrust investigations may block the merger of two related firms, causing firms to not contemplate a merger decision. The second is through the effect of synergies in the transaction, which have the same inverted U relationship with the PMR.

While we have considered the entire sample of mergers across industries in the past two decades, there are a few caveats that are in order. First, we look only at the acquisitions of public firms since the PMR variable is only available for public firms. The acquisition of private firms is an important part of the market for corporate control in the US, particularly in innovative sectors where private firms are the drivers of path-breaking

[20]A similar effect may possibly hold for innovation overlap, but our subsample for which we have innovation overlap data is too small to test that hypothesis.

innovation. Product market relatedness may play a different role in private firms due to lesser antitrust scrutiny, although this is changing in recent years.[21]

Perhaps more importantly, we only look at horizontal mergers since the PMR data is only available for firms that have some overlap to begin with. However, the fact that we find a robust relationship even among firms which are above a minimum threshold in PMR suggests that the results may be stronger in the entire sample of firms including firms which are in completely non-overlapping product markets. Past studies have looked at diversifying acquisitions, whereas our study shows that the degree of diversification within the same industry matters as well.

We leave to future work the welfare implications of antitrust laws and whether the efficiencies due to the mergers on average outweigh the anti-competitive effects brought about by them. It will be interesting to see how the product market relatedness influences post-merger pricing decisions.

Last, we look only at the firms which eventually announced their intention to merge. The effect is likely to be higher if we included mergers with potential targets which were not eventually announced. An important implication is that any examination of antitrust policy must take into account that antitrust measures are internalized by the firms even in the case of mergers which are not blocked or withdrawn due to antitrust concerns.

[21] See for example http://antitrustconnect.com/2016/09/18/antitrust-issues-facing-private-equity-entities/

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APPENDIX A

Appendix to Chapter 1

A.1. Expected Revenue from Negotiations- a Proof using Induction

The proof uses induction. We need to prove that

$$\text{If } W_m = \frac{1}{4(1 - V_{m-1})} \text{ and } W_0 = 0, \text{ then } W_m = \frac{m}{2(m+1)}.$$

To prove this by induction, assume the statement

$$W_m = \frac{m}{2(m+1)}$$

is true for $m = l - 1$. If this is the case,

$$\begin{aligned} W_l &= \frac{1}{4(1 - W_{l-1})} \\ &= \frac{1}{4(1 - \frac{l-1}{2l})} \\ &= \frac{l}{2(l+1)}. \end{aligned}$$

So, if the statement holds for $m = l - 1$, it also holds for $m = l$.

$V_0 = 0$, which means that the statement holds for $m = 0$.

Hence, it must hold for $m = 1$. Extending the logic, it holds for all m.

A.2. Expected Revenue from an Auction

The cdf of each bid is

$$F(x) = \frac{1-p}{p}\left(\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}} - 1\right).$$

The pdf of the maximum bid is

$$G(x) = mF(x)^{m-1}F'(x).$$

Substituting for $F(x)$, this simplifies to

$$\begin{aligned}
G(x) &= m\left(\frac{1-p}{p}\right)^m \left(\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}} - 1\right)^{m-1} \frac{1-\gamma_b}{n-1}\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}-1} \frac{V}{(V-x)^2} \\
&= m\frac{1-\gamma_b}{n-1}\left(\frac{1-p}{p}\right)^m V^{\frac{1-\gamma_b}{n-1}} \left(\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}} - 1\right)^{m-1} \left(\frac{1}{V-x}\right)^{\frac{n-\gamma_b}{n-1}}.
\end{aligned}$$

The expected utility of the seller for a given V,

$$\Pi_V(m,n) = \int_0^{\left(1-(1-p)^{\frac{n-1}{1-\gamma_b}}\right)V} G(x)\frac{x^{1-\gamma_s}}{1-\gamma_s}\mathrm{d}x$$

Substituting for $G(x)$, this simplifies to

$$\Pi_V(m,n) = m\frac{1-\gamma_b}{n-1}\left(\frac{1-p}{p}\right)^m V^{\frac{1-\gamma_b}{n-1}}$$
$$\int_0^{\left(1-(1-p)^{\frac{n-1}{1-\gamma_b}}\right)V} \left(\left(\left(\frac{V}{V-x}\right)^{\frac{1-\gamma_b}{n-1}} - 1\right)^{m-1} \left(\frac{1}{V-x}\right)^{\frac{n-\gamma_b}{n-1}}\right) \frac{x^{1-\gamma_s}}{1-\gamma_s} \mathrm{d}x.$$

Now, change the variable of integration to y where y is given by

$$y = \left(\frac{V}{V-x}\right)^{\frac{1}{n-1}}$$
$$\frac{1}{V-x} = \frac{1}{V}(y)^{n-1}$$
$$x = V\left(1 - \frac{1}{y^{n-1}}\right)$$
$$\mathrm{d}x = V(n-1)\frac{1}{y^n}\mathrm{d}y$$

Substituting all these into the integral and changing the limits of integration yields

$$\Pi_V(m,n) = m\frac{1-\gamma_b}{n-1}\left(\frac{1-p}{p}\right)^m V^{\frac{1-\gamma_b}{n-1}}$$
$$\int_1^{\frac{1}{(1-p)^{\frac{1}{1-\gamma_b}}}} (y^{1-\gamma_b} - 1)^{m-1}\left(\frac{1}{V}\right)^{\frac{n-\gamma_b}{n-1}} y^{n-\gamma_b} \frac{V^{1-\gamma_s}}{1-\gamma_s}\left(1 - \frac{1}{y^{n-1}}\right)^{1-\gamma_s} V(n-1)(y)^{-n}\mathrm{d}y$$
$$= (1-\gamma_b)m\left(\frac{1-p}{p}\right)^m \frac{V^{1-\gamma_s}}{1-\gamma_s}\left(\int_1^{\frac{1}{(1-p)^{\frac{1}{1-\gamma_b}}}} (y^{1-\gamma_b} - 1)^{m-1} y^{-\gamma_b}\left(1 - \frac{1}{y^{n-1}}\right)^{1-\gamma_s}\mathrm{d}y\right)$$

A.3. Target's Expected Revenue from an Auction as a Function of p for $n = 2, 3$ and $m = 1, 2, 3$

$$\Pi(1,2) = \frac{1}{2}\frac{1-p}{p}\left(\int_{1}^{\frac{1}{1-p}}(1-\frac{1}{y})\mathrm{d}y\right)$$
$$= \frac{1}{2}\left(1+\frac{1-p}{p}ln\left(1-p\right)\right)$$

$$\Pi(2,2) = \frac{1}{2}\left(\frac{1-p}{p}\right)^{2}2\left(\int_{1}^{\frac{1}{1-p}}(y-1)(1-\frac{1}{y})\mathrm{d}y\right)$$
$$= \frac{1}{2}\left(3-\frac{2}{p}+2\left(\frac{1-p}{p}\right)^{2}ln\left(\frac{1}{1-p}\right)\right)$$

$$\Pi(1,3) = \frac{1}{2}\frac{1-p}{p}\left(\int_{1}^{\frac{1}{1-p}}(1-\frac{1}{y^2})\mathrm{d}y\right)$$
$$= \frac{1}{2}\frac{1-p}{p}\left(y+\frac{1}{y}\Big|_{1}^{\frac{1}{1-p}}\right)$$
$$= \frac{1}{2}\frac{1-p}{p}\left(\frac{p^2}{1-p}\right)$$
$$= \frac{1}{2}p$$

$$\Pi(2,3) = \frac{1}{2}\left(\frac{1-p}{p}\right)^2 2\left(\int_1^{\frac{1}{1-p}} (y-1)(1-\frac{1}{y^2})\mathrm{d}y\right)$$
$$= \frac{1}{2}\left(\frac{1-p}{p}\right)^2 2\left(\int_1^{\frac{1}{1-p}} (y-1-\frac{1}{y}+\frac{1}{y^2})\mathrm{d}y\right)$$
$$= \frac{1}{2}\left(\frac{1-p}{p}\right)^2 2\left(\frac{1}{2}y^2 - y - ln(y) - \frac{1}{y}\Big|_1^{\frac{1}{1-p}}\right)$$
$$= \frac{1}{2}\left(\frac{1-p}{p}\right)^2 \left(\left(\frac{1}{1-p}\right)^2 + 1 - \frac{2}{1-p} + 2ln\,(1-p) + 2p\right)$$
$$= \frac{1}{2}\left(2p + \frac{2}{p} - 3 + 2\left(\frac{1-p}{p}\right)^2 ln\,(1-p)\right)$$

$$\Pi(3,3) = \frac{1}{2}\left(\frac{1-p}{p}\right)^3 3\left(\int_1^{\frac{1}{1-p}} (y-1)^2(1-\frac{1}{y^2})\mathrm{d}y\right)$$
$$= \frac{1}{2}\left(3p - 11 + \frac{15}{p} - \frac{6}{p^2} + 6\left(\frac{1-p}{p}\right)^3 ln(\frac{1}{1-p})\right)$$

A.4. The Equilibria when $n = 2$

$$p^* = \frac{pq_2}{(1-p)q_1 + pq_2} \text{ if at least one of } q_1 \text{ or } q_2 \text{ is not zero.} \tag{A.1}$$

$$p^* \in [0,1] \text{ if both } q_1 \text{ and } q_2 \text{ are zero.} \tag{A.2}$$

Sequential rationality of the seller implies that if there is only one buyer in play, the posterior belief has to be greater that p_{12} for the seller to choose an auction over a negotiation. If there are two buyers in play, the posterior belief has to be greater that p_{22} for the seller to choose an auction over a negotiation.

$$q_1 = 1 \text{ if } p^* > p_{12} \tag{A.3}$$

$$q_1 = 0 \text{ if } p^* < p_{12} \tag{A.4}$$

$$q_1 \in [0, 1] \text{ if } p^* = p_{12} \tag{A.5}$$

$$q_2 = 1 \text{ if } p^* > p_{22} \tag{A.6}$$

$$q_2 = 0 \text{ if } p^* < p_{22} \tag{A.7}$$

$$q_2 \in [0, 1] \text{ if } p^* = p_{22} \tag{A.8}$$

We also know that

$$p_{22} > p_{12} \tag{A.9}$$

Now, we just consider the various strategies of the seller i.e. possible values of q_1 and q_2.

Assume for the moment that the seller doesn't always choose negotiations i.e. that at least one of q_1 or q_2 is not zero.

- Say $q_1 = 0$. This implies that $p^* = 1$. But if $p^* = 1$, $q_1 = 1$. Hence, $q_1 = 0$ is not an equilibrium
- Next, consider $q_2 = 0$. If $q_2 = 0$, $p^* = 0$. If $p^* = 0$, $q_1 = 0$. By assumption, at least one of q_1 or q_2 is not zero. So, this is not an equilibrium.
- Next, consider $q_1 \in (0, 1)$. Hence, $p^* = p_{12}$. If $p^* = p_{12}$, $q_2 = 0$. But if $q_2 = 0$, $p^* = 0$ which contradicts $p^* = p_{12}$. So, $q_1 \in (0, 1)$ is not an equilibrium.
- Next, consider $q_2 \in (0, 1)$. Hence, $p^* = p_{22}$. If $p^* = p_{22}$, $q_1 = 1$. This also means that $p > p_{22}$. This is indeed a possible equilibrium.
- If $q_1 = 1$ and $q_2 = 1$, $p^* \geq p_{22}$ and $p^* = p$ i.e. $p \geq p_{22}$. This is indeed an equilibrium.

This proves that for $p < p_{22}$, we do not have an equilibrium where at least one of q_1 or q_2 is not zero.

Now consider the case we have ignored till now, i.e. both q_1 and q_2 are zero. In this case, no Bayesian updating happens since the posterior belief is equal to the prior belief. This is an equilibrium always for any value of p with the appropriate beliefs off the equilibrium path.

A.5. Expected Revenue from an Auction for a General Probability Distribution

Let p_i denote the equilibrium probability that there are i buyers in the auction.

Let $F(x)$ denote the cdf of the bid in the interval $[0, V - p_1 V]$.

Then, the cdf of the maximum bid with m buyers is $(F(x))^m$.

The pdf is $\mathrm{d}(F(x))^m$.

The expected utility of the seller $\Pi(m,n)$ is given by

$$\int_{0}^{(V-p_1V)} \frac{x^{1-\gamma_s}}{1-\gamma_s}\mathrm{d}(F(x))^m.$$

Denote $F(x)$ by y where y is defined implicitly by the equation

$$\left(p_1 + p_2y + p_3y^2 + \cdots + p_ny^n\right)\frac{(V-x)^{1-\gamma_b}}{1-\gamma_b} = p_1\frac{V^{1-\gamma_b}}{1-\gamma_b}.$$

Rearranging this equation gives

$$x = V\left[1 - \left(\frac{p_1}{p_1 + p_2y + p_3y^2 + \cdots + p_ny^n}\right)^{\frac{1}{1-\gamma_b}}\right].$$

Substituting in the original integral and changing the variable of integration to y (including changing the limits) gives

$$\Pi(m,n) = \frac{V^{1-\gamma_s}}{1-\gamma_s}\int_{0}^{1}\left[1 - \left(\frac{p_1}{p_1 + p_2y + p_3y^2 + \cdots + p_ny^n}\right)^{\frac{1}{1-\gamma_b}}\right]^{1-\gamma_s} my^{m-1}\mathrm{d}y.$$

A.6. Expected Revenue from Committing to an Auction

A buyer's payoff if he participates in the auction is $V(1-p)^{n-1}$. So, the ex-ante payoff of each buyer is given by

$$(1-p)0 + pV(1-p)^{n-1} = pV(1-p)^{n-1}.$$

The sum of the ex-ante payoffs of the n buyers is $pnV(1-p)^{n-1}$. It must be that the sum of the seller's payoff and all the buyers' payoffs equals the gains from trade. Trade happens if there it at least one bidder, which happens with probability $(1-(1-p)^n)$, so the gains from trade are $V(1-(1-p)^n)$.

$$\text{Seller's payoff} + pnV(1-p)^{n-1} = V(1-(1-p)^n).$$

Taking expectations, we get that the seller's expected payoff is

$$\frac{1}{2}\left(1-(1+(n-1)p)(1-p)^{n-1}\right).$$

APPENDIX B

Appendix to Chapter 2

B.1. The Optimal Wage Contract when Synergies Have a Discrete Uniform Distribution

The seller's problem is to minimize

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]$$

subject to the constraints

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq w(\frac{1}{n},1)$$

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq \frac{n-1}{n}w(\frac{2}{n},1)$$

....

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq \frac{n-k+1}{n}w(\frac{k}{n},1)$$

....

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq \frac{2}{n}w(\frac{n-1}{n},1)$$

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq \frac{1}{n}w(1,1).$$

Divide both sides of k^{th} equation by $n-k+1$ and add all the equations up.

The seller's problem is to minimize

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]$$

subject to

$$\left(\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\right)\left(\frac{1}{n}+\frac{1}{n-1}+...+\frac{1}{2}+1\right)\geq \frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right].$$

Simplifying the constraint, the problem becomes to minimize

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]$$

s.t.

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c\geq \frac{1}{\frac{1}{n}+\frac{1}{n-1}+...+\frac{1}{2}}c.$$

The optimal solution would just set

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c= \frac{1}{\frac{1}{n}+\frac{1}{n-1}+...+\frac{1}{2}}c.$$

The expected payment to the bank decreases to c as $n\rightarrow\infty$.

If we can find a wage schedule which gives this value for the objective and is feasible under the original constraints, it will be the optimal contract under the original constraints. Since the aggregate constraint is satisfied with equality, a natural candidate is to look for a solution which satisfies each of the original constraints with equality and is feasible. Thus, the k^{th} equation gives

$$\frac{1}{n}\left[w(\frac{1}{n},1)+w(\frac{2}{n},1)+...+w(1,1)\right]-c=\frac{n-k+1}{n}w(\frac{k}{n},1)$$

B.2. The Optimal Wage Contract when Synergies Are Uniformly Distributed

The seller's problem is to minimize

$$\int_0^{1-\epsilon} w(V,1)dV+\epsilon w(1-\epsilon,1)$$

subject to

$$\int_0^{1-\epsilon} w(V,1)dV+\epsilon w(1-\epsilon,1)-c\geq(1-V)w(V,1) \text{ for } V\in[0,1-\epsilon)$$

$$\int_0^{1-\epsilon} w(V,1)dV+\epsilon w(1-\epsilon,1)-c\geq\epsilon w(1-\epsilon,1).$$

Denote the objective function $\int_0^{1-\epsilon} w(V,1)dV+\epsilon w(1-\epsilon)$ by k.

Divide first equation by $1-V$, integrate from 0 to $1-\epsilon$ and add the second equation to get an aggregate constraint, a weaker one

$$(k-c)\left[\int_0^{1-\epsilon} \frac{1}{1-V}dV + 1\right] \geq \int_0^{1-\epsilon} w(V,1)dV + \epsilon w(1-\epsilon).$$

Recognizing that the right hand side of the equation is k, the aggregate constraint can be rewritten as

$$(k-c)\left[\int_0^{1-\epsilon} \frac{1}{1-V}dV + 1\right] \geq k$$

which simplifies to

$$k \geq c + \frac{c}{ln(\frac{1}{\epsilon})}.$$

The seller's problem is to minimize k subject to the aggregate weaker constraint

$$k \geq c + \frac{c}{ln(\frac{1}{\epsilon})}.$$

The solution to this is simply to set

$$k = c + \frac{c}{ln(\frac{1}{\epsilon})}$$

which gives the minimum value of the objective function as $c + \frac{c}{ln(\frac{1}{\epsilon})}$.

The solution under the stronger family of constraints cannot be higher than under this weaker constraint. If we can find a wage schedule which gives this value for the objective and is feasible under the original constraints, it will be the optimal contract under the original constraints. Since the aggregate constraint is satisfied with equality. a natural candidate is to look for a solution which satisfies each of the original constraints with equality and is feasible. So, set

$$w(V,1) = \frac{1}{1-V}\frac{c}{ln(\frac{1}{\epsilon})}$$

which is a feasible wage schedule, satisfies each of the original constraints with equality and leads to the same minimum value of the objective function

$$k = c + \frac{c}{ln(\frac{1}{\epsilon})}$$

as under the weaker constraint. Hence, this is the optimal contract.

B.3. The Uninformed Seller's Strategy

The maximand is $Q((1-p_b)\mathbf{1}(Q \leq l) + p_b(1-Q))$.

First, consider $l \leq \frac{1}{2}$

If $Q \leq l$, this is equal to $Q\,(1 - Qp_b)$ which is increasing in $[0, \frac{1}{2p_b}]$ and hence maximised at $Q = l$. The maximum is equal to $l(1 - lp_b)$.

If $Q > l$, this is equal to $Q\,(p_b(1-Q))$ which is decreasing in the interval $(\frac{1}{2}, 1]$ since the first derivative $p_b(1-2Q)$ is negative in this interval. Quoting $\frac{1}{2}$ gives $\frac{1}{4}p_b$.

Hence he quotes l if $l(1-lp_b) > \frac{1}{4}p_b$ i.e. $p_b < \frac{l}{l^2+\frac{1}{4}}$ and gets utility $l(1-lp_b)$. He quotes $\frac{1}{2}$ if $l(1-lp_b) < \frac{1}{4}p_b$ i.e. $p_b > \frac{l}{l^2+\frac{1}{4}}$ and gets utility $\frac{1}{4}p_b$.

Now, consider $l \geq \frac{1}{2}$.

If $Q \leq l$, this is equal to $Q\,(1 - Qp_b)$ which is increasing in $[0, \frac{1}{2p_b}]$ and decreasing after that since the first derivative $(1 - 2Qp_b)$ is greater than 0 in this interval and less than 0 after that.

Quote $Q = l$ if $l \leq \frac{1}{2p_b}$ i.e. $p_b \in [0, \frac{1}{2l}]$. The maximum is equal to $l(1 - lp_b)$. Quote $Q = \frac{1}{2p_b}$ if $l \geq \frac{1}{2p_b}$ i.e. $p_b \in [\frac{1}{2l}, 1]$. The maximum is equal to $\frac{1}{4p_b}$.

If $Q > l$, this is equal to $Q\,(p_b(1 - Q))$ which is decreasing in the interval $(l, 1]$. Quoting l is optimal irrespective of p_b.

B.4. The Informed Seller's Strategy

The seller quotes Q to maximize $Q\,((1 - p_b)\mathbf{1}(Q \leq l) + p_b\mathbf{1}(Q \leq V))$.

If $V \leq l$, then this function is increasing in Q in $[0, V]$ and $(V, l]$. This means that the optimal quote has to be either l or V. l is accepted with probability $1 - p_b$ and V is always accepted.

So, quote l if $(1 - p_b)l \geq V$ i.e. $V \in [0, (1 - p_b)l]$ and quote V if $V \in [(1 - p_b)l, l]$.

If $V \geq l$, then this function is increasing in Q in $[0, l]$ and increasing from $(l, V]$.

Again, the optimal quote has to be either l or V. l is always accepted whereas V is accepted with probability p_b. So, quote l if $l \geq p_bV$ i.e. $V \in [l, \frac{l}{p_b}]$ and quote V if $V \in [\frac{l}{p_b}, 1]$.

If $p_b \leq l$, $l \geq p_bV$ can never be satisfied. So quote l if $V \in [l, 1]$

B.5. Additional Constraints

Subgame perfection implies that the payoff for the buyer has to be 0 if he is uninformed and if the offer is l. Also, the payoff must be greater than 0 if the offer is less than l. First, consider the following case for the uninformed seller

$$Q_u = \begin{cases} l & if\ p_b \in [0, \frac{1}{2l}] \\ & \\ \frac{1}{2p_b} & if\ p_b \in [\frac{1}{2l}, 1] \end{cases} \quad if\ l > \frac{1}{2}$$

The case where the uninformed seller quotes $\frac{1}{2p_b}$ $if\ p_b \in [\frac{1}{2l}, 1]$ is impossible. The buyer would know that only an uninformed seller would quote $\frac{1}{2p_b}$ and would be paying $\frac{1}{2p_b}$ for something worth $\frac{1}{2}$ in expectation. He would thus refuse to accept the offer, even though it is less than l in this range of p_b, which is inconsitent with his strategy of accepting anything less than l. So, we eliminate this interval and only need to consider

$$Q_u = l\ \ if\ p_b \in [0, \frac{1}{2l}]\ \ if\ l > \frac{1}{2}.$$

"lVl" Equilibria

Consider equilibria of the form

$$Q_i = \begin{cases} l & if\ V \in [0, (1 - p_b)l] \\ & \\ V & if\ V \in [(1 - p_b)l, l] \\ & \\ l & if\ V \in [l, 1] \end{cases}$$

which occur when $p_b \leq l$.

Probability of being offered l by informed seller is $(1 - p_b)l + 1 - l$ i.e. $1 - p_b l$.

Expected value conditional on being offered l by informed seller is

$$\frac{(1-p_b)l}{1-p_bl}\frac{(1-p_b)l}{2}+\frac{1-l}{1-p_bl}\frac{1+l}{2}=\frac{1}{2(1-p_bl)}\left(l^2p_b(p_b-2)+1\right).$$

Expected payoff conditional on being offered l by the informed seller is

$$\frac{1}{2(1-p_bl)}\left(l^2p_b(p_b-2)+1\right)-l=\frac{1}{2(1-p_bl)}\left(l^2p_b^2+1-2l\right).$$

If $l \leq \frac{1}{2}$. In this case, the uninformed seller offering l leads to a payoff greater than 0 since the buyer pays $l \leq \frac{1}{2}$ for something worth $\frac{1}{2}$ in expectation.

The payoff from the informed seller offering l, i.e. $\frac{1}{2(1-p_bl)}\left(l^2p_b^2+1-2l\right)$, is greater than 0.

So there is no way in which the expected payoff conditional on an offer of l can be zero. Hence, there is no such equilibrium for $l \leq \frac{1}{2}$ whether the uninformed seller offers l or not.

If $l \geq \frac{1}{2}$. In this case, the uninformed seller offering l leads to a payoff less than 0 since the buyer pays $l \geq \frac{1}{2}$ for something worth $\frac{1}{2}$ in expectation.

The payoff from the informed seller offering l, i.e. $\frac{1}{2(1-p_bl)}\left(l^2p_b^2+1-2l\right)$, has to be greater than 0.

Since this payoff is increasing in p_b, we need $l^2p_b^2+1-2l>0$, which gives $p_b^2 > \frac{2l-1}{l^2}$, so there is a p_b above which this expression is > 0.

However, remember that $p_b \leq l$, so we must also have $\frac{2l-1}{l^2} \leq l^2$. This gives $l \leq 0.54$. Also, note that $p_b \leq \frac{1}{2l}$ is redundant since $p_b \leq l$ and $l \leq \frac{1}{2l}$ in this range.

p_s can be obtained by solving $(1-p_s)(\frac{1}{2}-l)+p_s\frac{1}{2(1-p_bl)}\left(l^2p_b^2+1-2l\right)=0$.

p_s always exists since it is the weight that makes the average of a positive and negative number equal to zero. Also, p_s is decreasing in p_b since the second term is increasing in p_b and less weight is required on the second term for the average to be 0.

Putting it all together, there is an"lVl" equilibrium if

$$\begin{aligned} l &\in [\frac{1}{2}, 0.54] \\ p_b &\in [\sqrt{\frac{2l-1}{l^2}}, l] \ \mathit{and} \\ p_s &= \frac{(2l-1)(1-p_b l)}{p_b l(p_b l + 1 - 2l)} \end{aligned}$$

"lVlV" Equilibria

Consider equilibria of the form

$$Q_i = \begin{cases} l & \mathit{if}\ V \in [0, (1-p_b)l] \\ V & \mathit{if}\ V \in [(1-p_b)l, l] \\ l & \mathit{if}\ V \in [l, \frac{l}{p_b}] \\ V & \mathit{if}\ V \in [\frac{l}{p_b}, 1]. \end{cases}$$

which occur when $p_b > l$

Probability of being offered l by informed seller is $l(1-p_b) + \frac{l}{p_b} - l$ i.e. $(\frac{1-p_b^2}{p_b})l$.

Probability that $V \in [0, (1-p_b)l]$ conditional on being offered l by the informed seller is

$$\frac{(1-p_b)l}{(\frac{1-p_b^2}{p_b})l} = \frac{p_b}{1+p_b}.$$

Probability that $V \in [l, \frac{l}{p_b}]$ conditional on being offered l by the informed seller is

$$\frac{\frac{l}{p_b} - l}{(\frac{1-p_b^2}{p_b})l} = \frac{1}{1+p_b}.$$

Expected value conditional on being offered l by informed seller is

$$\left(\frac{p_b}{1+p_b} \frac{(1-p_b)l}{2} + \frac{1}{1+p_b} \frac{l+\frac{l}{p_b}}{2} \right) - l = \frac{1}{2} l \left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2 \right).$$

The function $\left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2 \right)$ is decreasing in p_b since its derivative $-\frac{p(b)^4+2p(b)^3+2p(b)+1}{p(b)^2((1+p(b))^2}$ is less than 0. The function itself is less than 0 if $p_b \geq 0.54$.

If $l \leq \frac{1}{2}$. In this case, the uninformed seller offering l leads to a payoff greater than 0 since the buyer pays $l \leq \frac{1}{2}$ for something worth $\frac{1}{2}$ in expectation.

The payoff from the informed seller offering l, i.e. $\frac{1}{2} l \left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2 \right)$, has to be less than 0 or $p_b \geq 0.54$

But note that for this kind of an equilibrium, $p_b \geq l$ which is automatically satisfied since $l \leq \frac{1}{2}$ and $p_b \geq 0.54$.

Also, for the uninformed seller to be offering l, we need $p_b < \frac{l}{l^2+\frac{1}{4}}$.

So really, we will have such an equlibrium for any $p_b \in [0.54, \frac{l}{l^2+\frac{1}{4}}]$.

If $0.54 > \frac{l}{l^2+\frac{1}{4}}$, then there is no solution. This corresponds to $l \in [0, 0.15]$.

p_s can be obtained by solving $(1-p_s)(\frac{1}{2} - l) + p_s \frac{1}{2} l \left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2 \right) = 0$.

p_s always exists since it is the weight that makes the average of a positive and negative number equal to zero.

Also, p_s is decreasing in p_b since the second term is decreasing in p_b and hence less weight is required on the second term for the average to be 0.

If $l \geq \frac{1}{2}$. In this case, the uninformed seller offering l leads to a payoff less than 0 since the buyer pays $l \geq \frac{1}{2}$ for something worth $\frac{1}{2}$ in expectation.

The payoff from the informed seller offering l, i.e. $\frac{1}{2}l\left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2\right)$, has to be greater than 0 or $p_b \leq 0.54$.

However, $p_b \geq l$, so then $l \in [0.5, 0.54]$ and $p_b \in [l, 0.54]$. Note that $p_b \leq \frac{1}{2l}$, a condition for the uninformed seller to offer l, is automatically satisfied in this range.

p_s can be obtained by solving $(1-p_s)(\frac{1}{2} - l) + p_s\frac{1}{2}l\left(\frac{p_b(1-p_b)}{1+p_b} + \frac{1}{p_b} - 2\right) = 0$.

p_s always exists since it is the weight that makes the average of a positive and negative number equal to zero.

Also, p_s is increasing in p_b since the second term is decreasing in p_b and hence less weight is required on thesecond term for the average to be 0.

Note that ranges of p_b for which $Q = \frac{1}{2p_b}$ need not be considered. The uninformed buyer will know that only the uninformed seller will make this offer, and will not accept it since he is paying more than $\frac{1}{2}$ for something worth $\frac{1}{2}$, even though it satisfies the condition that the quote is less than l. This is inconsistent with his strategy.

Putting it all together, there is an"lVlV" equilibrium if

$$
\begin{aligned}
&l \in [0.15, \frac{1}{2}] \\
&p_b \in [0.54, \frac{l}{l^2 + \frac{1}{4}}] \\
&p_s = \frac{(2l-1)p_b(1+p_b)}{(1-p_b^3)l - (p_b^2 + p_b)(1-l)}
\end{aligned}
$$

or if

$$l \in [\frac{1}{2}, 0.54]$$

$$p_b \in [l, 0.54]$$

$$p_s = \frac{(2l-1)p_b(1+p_b)}{(1-p_b^3)l - (p_b^2+p_b)(1-l)}$$

B.6. The Seller's Cost of Hiring the Advisor

If the seller doesn't hire the advisor, he quotes l. This offer is accepted either if the buyer is uninformed or if he is his informed and V is less than l. So, the probability of the offer being accepted is $1 - p_b + p_b(1-l)$ i.e. $1 - lp_b$ and the utility of the uninformed seller is $l(1 - lp_b)$.

Since the seller mixes, this has to be equal to the utility of the informed seller. The latter depends on the form of the equlibria.

Consider equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1]. \end{cases}$$

If $V \in [0, (1-p_b)l]$, which happens with probability $(1-p_b)l$, the seller quotes l. This is accepted only if the buyer is uninformed. So, the utility conditional on the value being in this range is $(1-p_b)l$.

If $V \in [(1-p_b)l, l]$, which happens with probability $p_b l$, the seller quotes V, which is always accepted. So, the utility conditional on the value being in this range is the expected value in this range, $\frac{l(2-p_b)}{2}$.

If $V \in [l, l]$, which happens with probability $1-l$, the seller quotes l. This is always accepted. So, the utility conditional on the value being in this range is l.

So, the expected utility of the informed seller is

$$(1-p_b)l(1-p_b)l + \frac{l(2-p_b)}{2} p_b l + l\,(1-l) - c_s.$$

Equating this with the utility of the uninformed seller $l(1-lp_b)$ gives

$$l(1-lp_b) = (1-p_b)l(1-p_b)l + \frac{l(2-p_b)}{2} p_b l + l\,(1-l) - c_s$$

which simplifies to

$$c_s = \frac{1}{2} l^2 p_b^2.$$

If the equilibria are of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

we have two new ranges of value $V \in [l, \frac{l}{p_b}]$ and $V \in [\frac{l}{p_b}, 1]$.

If $V \in [l, \frac{l}{p_b}]$, which happens with probability $\frac{l}{p_b} - l$, the seller quotes l. This is always accepted. So, the utility conditional on the value being in this range is l.

If $V \in [\frac{l}{p_b}, 1]$, which happens with probability $1 - \frac{l}{p_b}$, the seller quotes V, which is accepted only if the buyer is informed. The expected value in this range is $\frac{l+p_b}{2p_b}$. So, the utility conditional on the value being in this range is $p_b \frac{l+p_b}{2p_b}$.

So, the expected utility of the informed seller is

$$(1-p_b)l(1-p_b)l + \frac{l(2-p_b)}{2}p_b l + l\left(\frac{l}{p_b} - l\right) + p_b \frac{l+p_b}{2p_b}\left(1 - \frac{l}{p_b}\right) - c_s.$$

Equating this with the utility of the uninformed seller $l(1-lp_b)$ gives

$$l(1-lp_b) = (1-p_b)l(1-p_b)l + \frac{l(2-p_b)}{2}p_b l + l\left(\frac{l}{p_b} - l\right) + p_b \frac{l+p_b}{2p_b}\left(1 - \frac{l}{p_b}\right) - c_s$$

which simplifies to

$$c_s = \frac{1}{2}l^2 p_b^2 + \frac{1}{2}(\sqrt{p_b} - \frac{l}{\sqrt{p_b}})^2.$$

B.7. The Buyer's Cost of Hiring the Advisor

The expected payoff of the informed buyer from hiring the bank or not hiring the bank must be zero.

The informed buyer may face either the uninformed seller, with probability $1 - p_s$, or the informed seller with probability p_s.

If the seller is uninformed, he quotes l. The informed buyer accepts it only if $V > l$, which happens with probability $1 - l$. His expected utility conditional on accepting the offer is the expected difference between the value V and the payment l conditional on

$V \in [l, 1]$, $\frac{1-l}{2}$. So, his ex ante expected utility conditional on the seller being uninformed is $(1-l)\frac{1-l}{2}$.

What if the seller is informed? First consider equilibria of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, 1] \end{cases}$$

If $V \in [0, (1-p_b)l]$, which happens with probability $(1-p_b)l$, the seller quotes l. The informed buyer does not accept this offer. So, the utility conditional on the value being in this range is 0

If $V \in [(1-p_b)l, l]$, which happens with probability $p_b l$, the seller quotes V. The informed buyer accepts this offer, but gets no utility from it since he pays V for an object worth V. So, the utility conditional on the value being in this range is again 0.

If $V \in [l, l]$, which happens with probability $1-l$, the seller quotes l. The informed buyer accepts this offer. So, the utility conditional on the value being in this range is the difference between the expected value in this range and the payment l, $\frac{1-l}{2}$

So, the expected utility of the informed buyer is

$$(1-p_s)(1-l)\frac{1-l}{2} + p_s(1-l)\frac{1-l}{2} - c_b.$$

Equating this to 0 yields

$$c_b = \frac{1}{2}(1-l)^2.$$

Now, consider equilibria which are of the form

$$Q_i = \begin{cases} l & \textit{if } V \in [0,(1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

We have two new ranges of value $V \in [l, \frac{l}{p_b}]$ and $V \in [\frac{l}{p_b}, 1]$.

If $V \in [l, \frac{l}{p_b}]$, which happens with probability $\frac{l}{p_b} - l$, the seller quotes l. The informed buyer accepts this offer. So, the utility conditional on the value being in this range is the difference between the expected value in this range and the payment l, $\frac{l}{2}(\frac{1}{p_b} - 1)$.

If $V \in [\frac{l}{p_b}, 1]$, which happens with probability $1-\frac{l}{p_b}$, the seller quotes V. The informed buyer accepts this offer, but gets no utility from it since he pays V for an object worth V. So, the utility conditional on the value being in this range is again 0.

So, the expected utility of the informed buyer is

$$(1-p_s)(1-l)\frac{1-l}{2} + p_s\left(\frac{l}{2}(\frac{1}{p_b} - 1)(\frac{l}{p_b} - l)\right) - c_b.$$

Equating this to 0 yields

$$c_b = \frac{1}{2}(1-l)^2 + \frac{1}{2}p_s\left(\frac{l^2}{p_b}(\frac{1}{p_b} - 2) + 2l - 1\right).$$

B.8. Seller's Contract with the Advisor

Minimise the expected payment to the bank

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\, dV + \int\limits_{\frac{l}{p_b}}^{1} w(V)\, dV - c$$

subject to the constraints

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\, dV + \int\limits_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq (1 - p_b l)w(l)$$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\, dV + \int\limits_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq (1 - Vp_b)w(V)$$

if $V \in [(1 - p_b)l, l]$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\, dV + \int\limits_{\frac{l}{p_b}}^{1} w(V)\, dV - c \geq p_b(1 - V)w(V)$$

if $V \in [\frac{l}{p_b}, 1]$

We solve this for the modified uniform distribution U_ϵ introduced in Section 2.4.3. Under this distribution, the problem is modified to

Minimise the expected payment to the bank

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon) - c$$

subject to the constraints

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon) - c \geq (1 - p_b l)w(l)$$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon) - c \geq (1 - V p_b)w(V)$$

if $V \in [(1-p_b)l, l]$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon) - c \geq p_b(1 - V)w(V)$$

if $V \in [\frac{l}{p_b}, 1]$

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon) - c \geq p_b \epsilon w(1-\epsilon)$$

The proof is similar to Appendix B.2. Denote the objective function

$$\left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l) + \int\limits_{(1-p_b)l}^{l} w(V)\,dV + p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\,dV + p_b \epsilon w(1-\epsilon)$$

by k. The problem reduces to minimizing k subject to the constraints

$$k - c \geq (1 - p_b l)w(l)$$

$$k - c \geq (1 - V p_b)w(V) \;\; if\; V \in [(1 - p_b)l, l]$$

$$k - c \geq p_b(1 - V)w(V) \;\; if\; V \in [\frac{l}{p_b}, 1 - \epsilon]$$

$$k - c \geq p_b \epsilon w(1 - \epsilon)$$

We now consider minimizing k subject to the weaker constraints

$$\left(\frac{p_b^2 - 2p_b + \frac{1}{p_b}}{1 - p_b l}\right) l(k - c) \geq \left(p_b^2 - 2p_b + \frac{1}{p_b}\right) lw(l)$$

$$\int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b}(k - c)\, dV \geq \int\limits_{(1-p_b)l}^{l} w(V)\, dV$$

$$\int\limits_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1 - V}(k - c)\, dV \geq p_b \int\limits_{\frac{l}{p_b}}^{1-\epsilon} w(V)\, dV$$

$$k - c \geq p_b \epsilon w(1 - \epsilon)$$

Now make the constraint even weaker by replacing it with an aggregate constraint which is the sum of all these constraints. If these constraints are added up, the right hand side of the aggregate constraint reduces to k. The aggregate constraint is

$$\left(\frac{p_b^2 - 2p_b + \frac{1}{p_b}}{1 - p_b l} l + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b}\, dV + \int\limits_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1 - V}\, dV + 1\right)(k - c) \geq k$$

which simplifies to

$$k \geq c + \frac{1}{\left(\frac{p_b^2 - 2p_b + \frac{1}{p_b}}{1 - p_b l} l + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b} \, dV + \int\limits_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1-V} \, dV \right)} c$$

The original problem is thus to minimize k subject to this constraint, which is accomplished by setting

$$k = c + \frac{1}{\left(\frac{p_b^2 - 2p_b + \frac{1}{p_b}}{1 - p_b l} l + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b} \, dV + \int\limits_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1-V} \, dV \right)} c$$

The solution to the weaker constraint must be the solution to the stronger constraint if this value is feasible. It is easy to see that the way to make this feasible is to satisfy each of the constraints with equality, thus giving us the contract as

$$w(V) = \begin{cases} k' c \frac{1}{1 - p_b V} & \textit{if } V \in [0, (1 - p_b) l] \\ k' c \frac{1}{p_b (1 - V)} & \textit{if } V \in [\frac{l}{p_b}, 1 - \epsilon] \end{cases}$$

where

$$k' = \frac{1}{\left(\frac{p_b^2 - 2p_b + \frac{1}{p_b}}{1 - p_b l} l + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b} \, dV + \int\limits_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1-V} \, dV \right)}$$

$k^{'}$ vanishes as $\epsilon \to 0$ since the denominator contains the term

$$\int_{\frac{l}{p_b}}^{1-\epsilon} \frac{1}{1-V}\,dV = log(1-\frac{l}{p_b}) + log(\frac{1}{\epsilon})$$

which $\to \infty$ as $\epsilon \to 0$.

The expected payment to the bank is given by $k = c(1+k^{'})$ which $\to c$ as $\epsilon \to 0$.

B.9. Misreporting-proofness

Each row in the table below corresponds to the value discovered by the bank falling in the interval in column 1. The second column is the offer made by the seller. The third column gives the bank's utility if he reports truthfully and the following columns the utility if he misreports to any of the other intervals.

Value lies in	Offer	Bank's Utility	$[0,(1-p_b)l]$	$[(1-p_b)l,l]$	$[l,\frac{l}{p_b}]$	$[\frac{l}{p_b},1]$
$V \in [(1-p_b)l,l]$	V	$w(V)$	$(1-p_b)w(l)$	–	$(1-p_b)w(l)$	0
$V \in [l,\frac{l}{p_b}]$	l	$w(l)$	$w(l)$	$w(V^{'})$	–	0
$V \in [\frac{l}{p_b},1]$	V	$p_b w(V)$	$w(l)$	$w(V^{'})$	$w(l)$	–

From row 1, to avoid misrpeorting, $w(V) \geq (1-p_b)w(l)$ has to hold whenever $V \in [(1-p_b)l,l]$.

$$\begin{aligned}(1-p_b)w(l) &= (1-p_b)\frac{w(V)(1-p_bV)}{1-p_bl} \text{ since, in this range, } w(V)(1-p_bV) = w(l)(1-p_bl)\\ &= w(V)(1-p_bV)\frac{(1-p_b)}{1-p_bl}\end{aligned}$$

which is $< w(V)$ since other terms are less than 1.

From row 2, to avoid misrpeorting, $w(l) \geq w(V^{'})$ which is satisfied for $V^{'} \in [(1-p_b)l, l]$ since $w(V) = k^{'}c\frac{1}{1-p_bV}$ if $V \in [0, (1-p_b)l]$.

From row 3, which is the last interval to be checked, misreporting is avoided if $p_b w(V) \geq w(l)$. But

$$\begin{aligned} p_b w(V) &= \frac{k^{'}c}{1-V} \text{ if } V \in [\frac{l}{p_b}, 1] \\ &\geq \frac{k^{'}c}{1-p_bV} \\ &\geq \frac{k^{'}c}{1-p_bl} \end{aligned}$$

Hence, misreporting is avoided in all intervals.

B.10. Seller's Contract with the Advisor for "lVl" Equilibria

The proof is similar to that in Appendix B.8 above.

The significant difference is that there are only 3 intervals to consider now. If the intermediary reports l without putting in effort, the transaction goes through with probability $(1-p_b)(1-p_b)l + (1-l)$ or $1 + lp_b(p_b - 2)$

Denote the objective function

$$(1 + p_b l(p_b - 2))\, w(l) + \int\limits_{(1-p_b)l}^{l} w(V)dV$$

by k.

The problem reduces to minimizing k subject to the constraints

$$k - c \geq (1 - p_b l) w(l)$$

$$k - c \geq (1 - V p_b) w(V) \ \textit{if}\ V \in [(1 - p_b) l, l]$$

We now consider minimizing k subject to the weaker constraints

$$\frac{1 + p_b l (p_b - 2)}{1 - p_b l}(k - c) \geq (1 + p_b l (p_b - 2))\, w(l)$$

$$\int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b}(k - c)\, dV \geq \int\limits_{(1-p_b)l}^{l} w(V)\, dV$$

Now make the constraint even weaker by replacing it with an aggregate constraint which is the sum of all these constraints. If these constraints are added up, the right hand side of the aggregate constraint reduces to k. The aggregate constraint is

$$\left(\frac{1 + p_b l (p_b - 2)}{(1 - p_b l)} + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b} dV \right) (k - c) \geq k$$

which simplifies to

$$k \geq c + \frac{1}{\left(\frac{1 + p_b l (p_b - 2)}{(1 - p_b l)} + \int\limits_{(1-p_b)l}^{l} \frac{1}{1 - V p_b} dV - 1 \right)} c$$

The original problem is thus to minimize k subject to this constraint, which is accomplished by setting

$$k = c + \frac{1}{\left(\frac{1+p_b l(p_b-2)}{(1-p_b l)} + \int\limits_{(1-p_b)l}^{l} \frac{1}{1-Vp_b} dV - 1 \right)} c.$$

Note that

$$\begin{aligned}
\frac{1+p_b l(p_b-2)}{(1-p_b l)} + \int\limits_{(1-p_b)l}^{l} \frac{1}{1-Vp_b} dV &\geq \frac{1+p_b l(p_b-2)}{(1-p_b l)} + \int\limits_{(1-p_b)l}^{l} \frac{1}{1-lp_b} dV \\
&= \frac{1+p_b l(p_b-2)}{(1-p_b l)} + \frac{p_b l}{(1-lp_b)} \\
&= \frac{1-p_b l(1-p_b)}{1-p_b l} \\
&\geq 1
\end{aligned}$$

so that the denominator is ≥ 0.

Simplifying the integral yields

$$k = c + \frac{1}{\left(\frac{1+p_b l(p_b-2)}{(1-p_b l)} + \frac{1}{p_b} log \left(\frac{1-p_b l(1-p_b)}{1-lp_b} \right) - 1 \right)} c.$$

The solution to the weaker constraint must be the solution to the stronger constraint if this value is feasible. It is easy to see that the way to make this feasible is to satisfy each of the constraints with equality, thus giving us the contract as

$$w(V) = k^{''} c \frac{1}{1 - p_b V} \ \textit{if}\ V \in [0, (1 - p_b)l]$$

where $k^{''}$ is the constant

$$\frac{1}{\left(\frac{1+p_b l(p_b-2)}{(1-p_b l)} + \frac{1}{p_b} log\left(\frac{1-p_b l(1-p_b)}{1-l p_b}\right) - 1\right)}.$$

The expected payment to the bank is $c(1 + k^{''})$. The misreporting-proofness obtains as a simpler case of Appendix B.9 above with one less interval to check for misreporting.

B.11. Probability of the Transaction Being a Success

Informed seller and informed buyer

Seller offers V, buyer accepts V, so the transaction always goes through.

Uninformed seller and uninformed buyer

Seller offers l, buyer accepts l, so the transaction always goes through.

Informed Seller and Uninformed Buyer

Seller offers

$$Q_i = \begin{cases} l & \textit{if } V \in [0, (1-p_b)l] \\ V & \textit{if } V \in [(1-p_b)l, l] \\ l & \textit{if } V \in [l, \frac{l}{p_b}] \\ V & \textit{if } V \in [\frac{l}{p_b}, 1] \end{cases}$$

Buyer accepts upto l.

The transaction goes through in all cases except $V \in [\frac{l}{p_b}, 1]$ i.e. with probability $1-(1-\frac{l}{p_b})$ or $\frac{l}{p_b}$.

Unformed Seller and Uninformed Buyer

Seller offers l, buyer accepts till V.

Transaction goes through if $V \geq l$ i.e. with probability $1-l$.

So, considering all the cases, the transaction goes through with probability

$$p_s p_b(1) + (1-p_s)(1-p_b)(1) + p_s(1-p_b)(\frac{l}{p_b}) + (1-p_s)(p_b)(1-l)$$

APPENDIX C

Appendix to Chapter 3 - Variable Definitions

Panel A: Main Variables	
Merger	Dummy Variable: one if the year is the same as the merger year, zero otherwise.
PMR	Product Market Relatedness is a measure that lies in the interval (0,1) and is based on the degree to which two firms use the same words in their 10-K product descriptions.
Technology Overlap	It is calculated (based on Jaffe (1986)) as : $\frac{S_{acq}S'_{tar}}{\sqrt{S_{acq}S'_{acq}}\sqrt{S_{tar}S'_{tar}}}$ where the vector $S_{acq} = (S_{acq,1}, ...S_{acq,J})$ captures the innovation activity of the acquirer firm and vector $S_{tar} = (S_{tar,1}, ...S_{tar,J})$ captures the innovation activity of the target firm, and J denotes the technology class. S_{acq} is calculated as the ratio of number of patents awarded in technology class J till date to the total number of patents awarded till date in all technology classes to the acquirer. S_{tar} is defined similarly.
HHI	Text-based Network Industry Classifications Herfindahl index in Hoberg and Phillips (2016), calculated using a dynamic industry classification based on each firm's product descriptions from annual 10-K filings.

Panel B: Deal-Year Variables	
Acquirer (Target) Total Assets	Total assets of the acquirer (target) in billions of dollars.
Acquirer (Target) Return on Assets	Earnings before interest, taxes, depreciation, and amortization scaled by total assets of the acquirer (target).
Acquirer (Target) Leverage	Total financial debt (long-term debt plus debt in current liabilities) divided by the book value of total assets of the acquirer (target).
Acquirer (Target) Cash to Assets	Cash and short term investments scaled by total assets of the acquirer (target).
Acquirer (Target) Book to Market	The book value of common equity scaled by the market value of common equity of the acquirer (target).
Acquirer (Target) Stock Runup	Market-adjusted buy-and-hold return of the bidding firm's stock over the period beginning 205 days and ending 6 days prior to the announcement date from CRSP of the acquirer (target).
Acquirer (Target) Volatility	Standard deviation of the firm's market-adjusted daily returns from CRSP over the period beginning 205 and ending 6 days before deal announcement date of the acquirer (target).

Panel C: Deal Variables	
Acquirer (Target) CAR	Cumulative abnormal return of the acquirer (target) firm's stock in the 11-day event window [-10,0] where 0 is the announcement day. The abnormal returns are calculated using the market adjusted model where CRSP value-weighted index return is the market return.
Synergies	Sum of bidder and target dollar denominated gains, computed as the sum of acquirer market value of equity 10 days prior to the announcement from CRSP in US$ Million times acquirer's CAR [-10,0] and target market value of equity 10 days prior to the announcement from CRSP in US$ Million times target's CAR [-10,0]) scaled by the sum of acquirer and target market value of equity 10 days prior to the announcement.
TSOS	Target share of synergy defined as target dollar denominated gain (market value of equity 10 days prior to the announcement from CRSP times target's CAR [-10,0]) divided by sum of bidder and target dollar denominated gains (computed as the sum of acquirer market value of equity 10 days prior to the announcement from CRSP in US$ Million times acquirer's CAR [-10,0] and target market value of equity 10 days prior to the announcement from CRSP in US$ Million times target's CAR [-10,0]) if total dollar denominated gain is positive and 1 minus (target dollar denominated gain/total dollar denominated gain) if total dollar denominated gain is negative (Bradley et al., 1988).

Panel C: Deal Variables *Continued*	
Deal Value	Value of transaction from Thomson Financial SDC in US$ Billion.
Premium	Takeover premium computed as Deal Value divided by the market value of target's equity 10 days before the acquisition announcement.
Hostile Takeover Indicator	Dummy variable: one for deals defined as hostile or unsolicited by Thomson Financial SDC, zero otherwise.
All Cash Deal Indicator	Dummy variable: one for deals in which the sole consideration is cash, zero otherwise.
Mixed Deal Indicator	Dummy variable: one for deals in which consideration is neither all-cash nor all-stock, zero otherwise.
Tender Offer Indicator	Dummy variable: one for tender offers , zero otherwise.
Reverse Takeover Indicator	One for deals defined as reverse takover by Thomson Financial SDC, zero otherwise.
Acquirer (Target) Market Capitalization	Market value of equity 10 trading days prior to the acquisition announcement from CRSP in US$ Billion.