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ABSTRACT

Essays on Information Economics

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This dissertation explores topics in information economics. A particular focus in this dissertation is how competition affects incentives for information acquisition and information sharing between competitors. The first chapter studies a principal-agent setting where two principals compete for the services of one agent. The second chapter studies how competition in the product market affects firms' incentives to conduct market research. The third chapter highlights how asymmetric information affects technology licensing and firms' incentives to conduct their research and development activities.

The first chapter studies a dynamic principal-agent model of adverse selection under competition among principals. Principals are ex-ante identical, but receive information about the agent independently which creates a setting of imperfect competition. I study how the agent's payoffs in this setting differ compared to the regular monopoly principal-agent case, and how that affects the agent's incentives to reveal information. The focus is on how the information structure affects the competition for the agent's services, and how the nature of competition in turn affects the agent's incentives. In a repeated setting with short term contracts and private observability of the agent's performance measure, the agent cannot

be incentivized to fully reveal his private information as the familiar ratchet effect persists. Finally, I show that allowing voluntary information sharing among principals can benefit principals and improve welfare in general.

In the second chapter, which is joint work with Colin Shopp, we apply the main result in Persico (2000), that decision-makers acquire more information when their payoffs are more risk-sensitive, to a duopoly model of Bertrand competition with uncertain demand following Vives (1984) in order to show how the amount of covert market research firms undertake depends on the level of competition. We decompose marginal returns to research into two effects, a competitive profit effect and a coordination effect, and show how each of these depends on competition. When the cost of market research is sufficiently high, the amount firms invest in market research is decreasing in the level of competition. In contrast, when the cost of market research is sufficiently low, firms perform the most market research at an intermediate level of competition. We partially extend this result to a public market research setting.

The third chapter studies a duopoly pricing game where firms may have asymmetric information about their production technology. I show that having asymmetric information may lead to firms being unable to engage in profitable technology licensing, and how this problem is mitigated when firms outsource their research activities as opposed to conducting research in-house.

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CHAPTER 1

Dynamic Screening with Differentially Informed Principals

1.1. Introduction

In an employment relationship, firms often acquire information over time about a worker's productivity, through various performance measures. This information is valuable for two reasons: first, it allows the firm to write more effective incentive contracts with their employees, and second, it gives them an informational advantage when competing with other firms to retain those employees. Nevertheless, many firms reveal at least some of this information to their potential rivals, either directly – through reference letters or outplacement services – or indirectly – through job titles or responsibilities.[1] Why might a firm reveal its private information to the rest of the labor market?

This paper argues that revealing information about past performance can effectively commit a firm to pay high wages to its high-ability workers. That is, without disclosing information, the firm would extract rents from its employees after they reveal that they are high ability. This manifestation of the classic ratchet effect means that firms would struggle to induce their high ability workers to stand out. By disclosing information and thereby inducing more severe competition for their employees, the firm can credibly promise to reward high-ability workers, which encourages those workers to reveal themselves.

[1]Top management consulting firms, for example, often have outplacement services that help their employees to find new jobs. They provide this help in the form of connecting the employee with prospective employers, and through providing credible information about the employee's productivity in the forms of references, evaluations, and performance reviews.

To make this point, I develop tools for understanding imperfect competition among differentially-informed principals who seek to hire an agent. I show that this problem can be formulated as a first-price common-value auction, where each principal's bid is a contract that maps output to wages, which in turn generates a schedule of rents given to agents with different abilities. While identifying closed-form equilibrium contracts in this setting is not tractable, I establish comparative statics results on the agent's equilibrium rent as a function of the principals' information. These comparative statics allow me to analyze the costs and benefits to the incumbent principal of revealing her private information.

Formally, I consider a two-period model with two principals and one agent. In each period, the agent's ability is perfectly known only to the agent; however, both principals independently receive informative signals[2] about the agent's ability, after which they offer contracts simulataneously. Higher signals correspond to more favorable beliefs regarding the agent's ability. The agent can only choose to work with one principal in each period. In the two-period model, I consider both the setting where principals can credibly publicize information, and the one where they cannot.

I show that in each period, the rent offered to the agent by a principal is increasing in the principal's signal; that is, a more favorable belief induces the principal to pay the agent more. This is true both when principals receive signals of the same level of informativeness and when the level of informativeness is different. However, between these two cases, a meaningful comparion of the agent's payoffs can be made, and it can be shown that when one principal is more accurately informed than the other, this reduces the agent's ex ante expected payoff from the setting where both principals have the lower accuracy.

[2]This can be thought of as an interview of the agent conducted by the pricipal, or another type of information acquisition.

In the two-period model presented in this paper, if a principal learns something about the agent's productivity in the first period, it gives her an informational advantage over her opponent in the second period. However, this informational advantage is detrimental for the agent's payoff, which makes it harder to induce the agent to reveal information. For this reason, even though ex post the principal would benefit from having superior information, ex ante it is beneficial for her to commit to share any information about the agent's productivity she learns in the first period. Publicizing information works as an incentive tool because the incremental benefits of public information is higher for more productive agents, which means more productive agents will have a stronger incentive to exert effort in order to publicize their productivity. Availability of more public information makes the competition for the agent's services stronger in the second period; however, because we have two principals competing for the agent, the agent's second period expected payoff is the higher of the two payoffs offered to him by the two principals, whereas from the principal's perspective, the added payoff she must offer due to stronger competition is only the expected increase in her own bid in the second period. Because of this, from the point of view of the principal, the lowered cost of providing incentives dominates the effect of added competition, so publicizing information is ex ante beneficial for the principal's payoff.

The formal model is introduced in section 1.2. In section 1.3, I characterize necessary conditions for incentive compatible contracts and list the relevant results for the monopolist principal benchmark.

Results for the one-period model is presented in section 1.4. Because equilibria in the two-period game crucially depends on the information structure in the second period, it will be very useful to compare equilibrium payoffs for the agent under different information structures for the principals, and we build on these results from the static game in the two-period model. The analysis of the two-period game is in section 1.5. Section 1.6 discusses possible extensions. Section 1.7 concludes.

1.1.1. Related Literature

Monopolistic screening has been studied in its various forms as a partial solution to this problem in early works such as Mussa & Rosen (1978), Baron & Myerson (1982), and Laffont & Tirole (1986). However, much less emphasis has been given to forms of incentive contracts for screening in a setting with competition. Imperfect competition has been mostly studied within the domain of IO theory where the "imperfection" arises from some form of differentiation among the competitors. Most of this literature has been focused on intrinsic differences among the competitors themselves. However, even though less studied, interesting insights can be generated from models where competitors only differ in terms of the information they are endowed with. Spulber (1988) studies a simple model of Bertrand competition where the marginal costs of the producers are realized from a distribution at the beginning of the game, and becomes private information of each producer. In this setting, the Bertrand paradox is mitigated, and this private information generates enough differentiation among competitors that they each make positive expected profits in equilibrium. The game analyzed there is a specialized version of an independent private values auction setting.

The "symmetric", or "mineral-rights" model of common value auctions has been studied in early papers such as Wilson (1967) and Milgrom & Weber (1982). Common-value auctions with asymmetrically informed bidders have received much less attention, although this setting plays an important role in this paper. Such settings have been analyzed in

Engelbrecht-Wiggans, Milgrom & Weber (1983) and Milgrom & Weber (1982b). Representing a screening contract as a form of auction has also been studied in Biglaiser & Mezzetti (1993, 2000) in the framework of the independent private values model.

The central theme of screening contracts with short-term commitment is the ratchet effect. Starting from Freixas, Guesnerie & Tirole (1985), the ratchet effect has been studied under a procurement setting in Laffont & Tirole (1988), as well as worker incentives in a firm in Gibbons (1987), Ickes & Samuelson (1987), and Carmichael & Macleod (2000), and in the economics of corruption (Choi & Thum 2003). Empirical analysis of the ratchet effect is not numerous, but recently there has been some work. For example, in Charness et al. (2011), they find that the ratchet effect is indeed a significant problem in the labor market when there is less competition between firms or between workers, however, competition in either side mediates the problem. They interpret this result as the parties' outside options playing an important role in solving the problem. Their work is partly inspired by the theoretical treatment of the ratchet effect in Kanemoto & Macleod (1992), which has a two period model of "second-hand workers" with short term commitment, but in the 2nd period the agent is free to choose offers from other competing principals, who crucially, does not observe the agent's performance in the 1st period. Competition among principals, therefore, mitigates the ratchet effect, and the first-best outcome is possible with perfect competition among principals.

1.2. The Model

There is one risk-neutral agent, A, and two risk-neutral principals P_1 and P_2, over two periods, $t = 1, 2$. The agent has private information about his type θ_t, which is drawn from a commonly known distribution $F(\cdot)$ at $t = 1$, and $F(\cdot|\theta_1)$ at $t = 2$, over the interval support $[\underline{\theta}, \bar{\theta}]$. We assume that $\underline{\theta} > 1$, and the associated density functions $f(\cdot)$ and $f(\cdot|\theta_1)$ are positive and atomless everywhere in the support. In each period, each principal P_i receives a private signal X_{it}, which is informative of the agent's type θ_t; the signals are distributed independently according to the signal-generating processes $S_{it}(\cdot|\theta_t)$ with the associated density functions $s_{it}(\cdot|\theta_t)$, which are positive and atomless everywhere in the support. This is also commonly known.

In each stage, if the agent chooses to work with one of the principals, he chooses a non-negative effort level $e_t \in [0, \bar{e}]$, and which produces output $y_t = e_t + \theta_t$. We assume that $\bar{e} > 1$. The agent's effort level is not observed by the principals. Output, however, is observed by the employing principal and it is contractible. Effort is costly for the agent, with $C(e_t) = e_t^2/2$. Each principal P_i can offer a contract $w_{it} : \mathbb{R}^+ \to \mathbb{R}$ which determines a payment $w_{it}(y_t)$ which the principal must pay the agent following the realization of output[3]. In the first period, as part of the contract, the principals can also commit to a public message $m_i(y_1)$ which the first period employing principal sends after realization of the output. We will look at two cases for the messages available to the principals. In the non-disclosure setting, the message space is $M_i^{ND} = \phi$, meaning principals cannot choose informative messages, whereas in the disclosure setting, the message space is $M_i^D = \left\{m_i : m_i \subseteq [\underline{\theta}, \bar{\theta}]\right\}$, so the principals can commit to reveal information about their beliefs regarding the agent's first-period type θ_1.

The agent's outside option is zero. Let's denote principal P_i's payoff as π_{it}, and the agent's payoff as U_t. The payoffs are therefore:

[3]Note that we restrict the contract to only be conditioned on the output produced. In particular, we do not allow one principal's contract to condition on another principal's contract

$$\pi_{it} = \begin{cases} y_t - w_{it}(y_t) & \text{if } A \text{ chooses } P_i\text{'s contract} \\ 0 & \text{otherwise} \end{cases}$$

$$U_t = \begin{cases} w_{it}(y_t) - {e_t^2}/{2} & \text{when } w_{it} \text{ is the chosen contract} \\ 0 & \text{if no contract is chosen} \end{cases}$$

The timing in the first period is:

(1) The agent's type θ_1 is realized and privately observed by the agent.

(2) Each principal P_i observes a signal x_{i1}, realized according the distribution $S_{i1}(\cdot|\theta_1)$.

(3) Each principal simultaneously and privately offers the agent a contract $w_{i1} : \mathbb{R}^+ \to \mathbb{R}$, which is a function that maps output to payment, and commits to a public message $m_i : \mathbb{R}^+ \to M_i$

(4) The agent accepts at most one contract; the agent's decision is $d_1 \in \{\phi, P_1, P_2\}$

(5) If no contract is accepted, the stage ends. If the agent accepts P_i's offer, he then exerts effort e_1, output y_1 is produced and only observed by P_i. The output accrues to P_i.

(6) P_i pays the agent $w_{i1}(y_1)$, and sends public message $m_i(y_1)$.

The timing in the second period is similar to that of the first period; however, principals don't choose any messages, and both principals use all of their information at the time of offering contracts. We denote P_i's available information at the time of offering second-period contracts as $\mathcal{I}_i = \{x_{i1}, x_{i2}, d_1, \mathcal{C}_i, m\}$, where $\mathcal{C}_i$ contains any information about θ_1 that P_i gained through a contractual relationship with the agent in the first period, and m is the public message sent in the first period.

I use the following assumptions throughout the paper. The first characterizes the informative nature of the principals' signals. The second is a technical assumption commonly used in screening models.

Assumption 1. *The signals X_{it} are affiliated with the agent's realized type,θ_t.*

Assumption 1 implies, for any $\theta_t^{'} > \theta_t$, the signal generating process $S_{it}(\cdot|\theta_t^{'})$ stochastically dominates $S_{it}(\cdot|\theta_t)$ according to the likelihood ratio order.

Assumption 2. *For any possible pair of signal realizations (x, y) for the two principals, each principal's posterior belief about the agent's type, $F_{it}(\cdot|x, y)$ satisfies the monotone hazard rate condition.*

Assumption 2 implies, for any pair of signal realizations for the two principals (x, y), $\frac{f_{it}(\theta_t|x,y)}{1-F_{it}(\theta_t|x,y)}$ is non-decreasing in θ_t.

1.3. The Monopoly Benchmark

It is instructive to start with the one-period monopoly benchmark where one principal is inactive throughout. This case is a straightforward analog of the framework analyzed in Mussa & Rosen (1978). An important observation in this setting is that the agent's payoff depends on the principal's beliefs about his type. By the revelation principle, the principal's maximization problem after receiving realized signal x is:

$$\max_{e(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [e(\theta) + \theta - w(\theta)] \, dF(\theta|x)$$

subject to:

$$U(\theta) \geq U(\hat{\theta}|\theta), \forall \theta, \hat{\theta} \quad \text{(IC)}$$

$$U(\theta) \geq 0, \forall \theta \qquad \text{(IR)}$$

Here, $U(\hat{\theta}|\theta)$ is the agent's payoff when his real type is θ and he chooses the allocation for type $\hat{\theta}$. $U(\theta) = U(\theta|\theta)$ is the agent's payoff under truth-telling.

Lemma 1. *For a smooth effort allocation $e(\cdot)$, The IC constraints are satisfied if and only if:*

(1) $e(\theta) + \theta$ is non-decreasing

(2) $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} e(q)dq$

Proof. See appendix. □

Throughout the paper, the first condition is referred to as the monotonicity constraint, and the second one as the envelope condition.

By Lemma 1, we can see that because $e(q)$ is non-negative, setting $U(\underline{\theta}) = 0$ satisfies the IC constraints for all types. The principal's relaxed maximization problem can therefore be written as:

$$\max_{e(\cdot|x)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x) + \theta - \frac{e(\theta|x)^2}{2} - \int_{\underline{\theta}}^{\theta} e(q|x)dq \right\} dF(\theta|x)$$

Let $e^M(\theta|x)$ be the solution to this problem, and $\pi^M(x)$ be the principal's expected payoff after receiving signal x.

Proposition 1 characterizes the e^M and π^M.

Proposition 1. *In the monopoly setting, the optimal contract in the stage game has the following properties:*

(1) The optimal effort level is $e^M(\theta|x) = 1 - \frac{1-F(\theta|x)}{f(\theta|x)}$, which is strictly decreasing in x, for all $\theta < \bar{\theta}$.

(2) $U(\theta|x)$ is strictly decreasing in x, for all θ.

(3) The principal's expected payoff, $\pi^M(x)$, is strictly increasing in x.

Proof. See appendix. □

The main idea behind these results is the tradeoff between efficiency and rent extraction. For an agent of type θ, total surplus equals $e + \theta - e^2/2$, which is maximized by choosing $e(\theta) = 1$, that is to say, in our setting, $e^{FB}(\theta) = 1$ for any θ. As we see from Lemma 1, the θ-type agent's payoff, which consists entirely of his information rent in the monopoly setting, is increasing in the proposed effort level the principal chooses for all types lower than θ. This leads the principal to choose an effort level that is less than first-best for all types other than the highest type, $\bar{\theta}$.In other words, the principal distorts the effort level from the efficient level of 1 in order to reduce rent for higher types. The amount of distortion depends on the principal's beliefs about the agent's type. When higher types are more likely, reducing rent for higher types becomes more important compared to efficiency for lower types, which leads to more distortion for the low types. This is why after receiving a higher signal, which makes higher types of agents more likely according to the principal's updated beliefs, the principal chooses a lower (more distorted) effort level for all agent types other than $\bar{\theta}$. This leads to a lower payoff for all types of the agent. However, this increases the principal's expected payoff, which means the principal's value from working with the agent goes up following a high signal.

1.4. The Static Game under Competition

In this section the analysis of the one-period game with two principals is presented. Here I use a subclass of perfect Bayesian equilibrium which I call *regular equilibrium.* I put conditions on the principals' equilibrium strategies so that the equilibrium involves each principal using a continuous strategy as a function of her signal.

Let us then analyze the principals' maximization problem. Suppose P_{-i} is playing the strategy $e_{-i}(\cdot|\cdot)$ with the associated payoff schedule $U_{-i}(\cdot|\cdot)$. Because the space of allowed contracts here is the same as in the monopoly setting, lemma 1 still pins down the necessary conditions for the agent's IC constraints.[4] Therefore, after receiving signal x, principal P_i's maximization problem is:

$$\max_{e_i(\cdot|x),U_i(\underline{\theta}|x)} \int_{-\infty}^{\infty} \left[\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left(e_i(\theta|x) + \theta - \frac{e_i(\theta|x)^2}{2} - U_i(\underline{\theta}|x) \right. \right. \right.$$
$$\left. \left. \left. - \int_{\underline{\theta}}^{\theta} e_i(t|x)dt \right) \mathbf{1}_{\{U_{-i}(\theta|y)<U_i(\theta|x)\}} \right\} dF_i(\theta|x,y) \right] dG_{-i}(y|x)$$

$$\text{Subject to:} \quad \begin{cases} e_i^{'}(\theta|x) \geq -1, \forall\theta & \text{monotonicity} \\ U_i(\underline{\theta}|x) \geq 0 & \text{IR} \end{cases}$$

Here, $G_{-i}(y|x)$ is the distribution of the opponent's signal, which is updated using Bayes' rule after receiving signal x. In other words, this is the principal's posterior belief about the signal of her opponent.

[4]Here, the rent offered to agent type θ by P_{-i} with realized signal y, which is $U_{-i}(\theta|y)$, is the θ type agent's outside option for P_i's offered contract. This outside option may be a random variable if P_{-i} is playing a strategy conditional on y. The fact that lemma 1 still pins down the agent's IC constraints for P_i's contract is an implication of lemma 2 in Rochet & Stole (2002), which studies screening with random outside options.

Before proving existence of equilibrium, it is helpful to define a few terms.

Let the pair $(e_i^*(\cdot|x), U_i^*(\underline{\theta}|x))$ be an optimal strategy in the above maximization problem. Let

$$U_i^*(\theta|x) = U_i^*(\underline{\theta}|x) + \int_{\underline{\theta}}^{\theta} e_i^*(t|x)dt$$

In competing with P_{-i}, P_i decides, for every agent type θ, what level of rent $U_i^*(\theta|x)$ to offer to the agent of type θ, as a function of P_i's realized signal x. This makes the game analogous to a first price common value auction, which we can then study as a bidding game, and analyze the two principals' bidding behaviors in terms of their information.

1.4.1. Regular Equilibrium

We will focus on a class of competitive equilibria where the principals' strategies are continuous functions of their signals. We will call an equilibrium in such strategies a regular equilibrium.

For any regular equilibrium, we define a "lowest contract" for that equilibrium, which is by construction the unique contract offered by both principals as their signal realizations approach the infimum of their signal supports. This lowest contract determines the initial value for the differential equations that govern how much rent P_i bids for each type of the agent, θ.

Definition. *The lowest contract, $(\underline{e}(\cdot), \underline{U}(\underline{\theta}))$ is defined as:*

$$\underline{e}(\cdot) := \lim_{x \to \underline{x}_i} e_i^*(\cdot|x); \textit{ for } i = 1, 2$$

$$\underline{U}(\underline{\theta}) = \lim_{x \to \underline{x}_i} U_i^*(\underline{\theta}|x); \textit{ for } i = 1, 2$$

where $\underline{x}_i$ is the infimum of P_i's signal space.

Throughout the rest of the paper, I put conditions that the lowest contract satisfy the monotonicity constraint, it allocates an inefficient effort level for all interior types of the agent, and the efficient level for the lowest type.

Condition 1. *$\underline{e}(\cdot)$ is continuous and differentiable over $[\underline{\theta}, \bar{\theta}]$, and $\underline{e}^{'}(\theta) > -1$, for all $\theta \in [\underline{\theta}, \bar{\theta}]$.*

Condition 2. *In any regular equilibrium, $\underline{e}(\underline{\theta}) = e^{FB}$, $\underline{e}(\theta) < e^{FB}$ for all $\theta \in (\underline{\theta}, \bar{\theta})$, and $\underline{U}(\underline{\theta}) = \frac{1}{2} + \underline{\theta}$.*

We will look at regular equilibria where in all equilibrium contracts, the effort allocation is efficient for the lowest type who gets the total surplus generated, and the effort level is inefficient for all interior types. If we start from a lowest contract where principals offer efficient effort allocations for all types, then the only equilibrium is where they both offer the same contract regardless of their realized signals. Condition 2 is required to have an equilibrium where principals choose non-constant strategies based on their signals.

Next, it is useful to formalize some tools that allow the contracting problem to be discussed in terms of a common values auction. I make use of these terms in proving results for the competitive screening game, through using results from auction theory.

Definition. *The actual value to principal P_i of obtaining agent of type θ, given P_i's realized signal x and $P'_{-i}s$ realized signal y, is*

$$\hat{R}_i(\theta|x,y) := \int_{\theta}^{\bar{\theta}} \left[\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} - \int_{\theta}^{q} e_i^*(t|x)dt \right\} \mathbf{1}_{\left\{U^*_{-i}(q|y) < U^*_i(q|x)\right\}} \right] dF_i(q|x,y)$$

This term is the expected payoff to the principal generated from all types of the agent above θ, under the optimal contract when the principal's own signal is x and the opponent's signal is y.

It will also be useful sometimes to define this function without the indicator. So let's define:

$$R_i(\theta|x,y) := \int_{\theta}^{\bar{\theta}} \left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} - \int_{\theta}^{q} e_i^*(t|x)dt \right\} dF_i(q|x,y)$$

Definition. *The interim value to principal P_i, conditional on winning, of obtaining agent of type θ, given her realized signal x, is*

$$V_i(\theta|x) := \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{ U^*_{-i}(\theta|y) < U^*_i(\theta|x) \right\}} \right) dG_{-i}(y|x)$$

The interim value is P_i's expected payoff given her realized signal in the equilibrium from all agent types above θ, where the expectation is taken over the agent's types and the opponent's signal realizations, conditional on P_i winning.

Definition. *The ex ante value to principal P_i, conditional on winning, of obtaining agent of type θ, is*

$$V_i(\theta) := \int_{-\infty}^{\infty} V_i(\theta|x)dS_i(x|\theta)$$

The ex ante value is the expected interim value for agent type θ, where the expectation is taken over all possible signal realizations for P_i given the agent's type is θ.

Now I present the main result for the static game with competition, that highlights the role of competition on the agent's equilibrium payoff. Unlike in the monopoly benchmark, in a regular equilibrium with competition, the agent's payoff is increasing in the principals' signals, which mean favorable beliefs are beneficial for the agent's payoff.

Proposition 2. *In any regular equilibrium of the static game, for any agent type* $\theta \in (\underline{\theta}, \bar{\theta})$*,* $U_i^*(\theta|x)$ *is strictly increasing in* x*, for* $i = 1, 2$.

Proof. First of all, suppose that in the equilibrium the opponent principal P_{-i} is playing a strictly increasing strategy in her signal for all $\theta \in (\underline{\theta}, \bar{\theta})$, that is, $U_{-i}^*(\theta|y)$ is strictly increasing in y for all $\theta \in (\underline{\theta}, \bar{\theta})$.

We consider principal P_i's equilibrium strategy after receiving signal x. Now pick an arbitrary $\theta \in (\underline{\theta}, \bar{\theta})$. Let $u = U_i^*(\theta|x)$.

We can separate out the principal's payoff coming from agent types below θ, from the ones above θ. This is useful because this allows us to focus on the effect of changing u only on the associated change in winning probability for types above θ. That is,

$$\pi_i(x) = \int_{-\infty}^{\infty} \int_{\underline{\theta}}^{\theta} \left(\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} \right\} \mathbf{1}_{\left\{U_{-i}^*(q|y) < U_i^*(q|x)\right\}} \right) dF_i(q|x,y) dG_{-i}(y|x) + \pi_i(\theta|x;u)$$
$$+ \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{U_{-i}^*(\theta|y) > U_i^*(\theta|x)\right\}} \right) dG_{-i}(y|x)$$

where

$$\pi_i(\theta|x;u) = \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{U_{-i}^*(\theta|y) < U_i^*(\theta|x)\right\}} - u \right) dG_{-i}(y|x)$$

Notice that the benefit of marginally increasing u in the form of increasing the probability of winning all agents of type θ or above, can be captured by the term $\pi_i(\theta|x;u)$, because as $u = U_i^*(\theta|x)$, and for any $\theta' \in [\theta, \bar{\theta}]$, $U_i^*(\theta'|x) = U_i^*(\theta|x) + \int_{\theta}^{\theta'} e(q)dq$, increasing u increases the rent offered to all types between θ and $\bar{\theta}$, and given P_{-i} is playing a strictly increasing strategy, increasing the rent for all types between θ and $\bar{\theta}$ increases the probability of winning all types between θ and $\bar{\theta}$.

Because P_{-i} is playing a strategy that is strictly increasing in her signal, we can write[5]

$$\pi_i(\theta|x;u) = \int\limits_{-\infty}^{U_{-i}^{*^{-1}}(u)} \left\{\hat{R}_i(\theta|x,y) - u\right\} dG_{-i}(y|x)$$

It is easy to see that in any equilibrium, no principal would ever offer a contract that specifies $e_i^*(\theta|x) > e^{FB} = 1$, because there are always profitable deviations that offer $e_i^*(\theta|x) = e^{FB}$, and adjusts the payment accordingly so that $U_i^*(\theta|x)$ remain unchanged and total surplus goes up, thereby increasing the principal's payoff without affecting the agent's incentive constraints or choice of contract. Now, for any $\theta > \underline{\theta}$, as long as $e_i^*(\theta|x) < e^{FB}$, because signals are affiliated, for any pair of signals (x,y), $R_i(\theta|x,y)$ is strictly increasing in x,by the same proof as that of part 3 in proposition 1. Because P_{-i} is playing a strictly increasing strategy, there are signals $y \in (-\infty, U_{-i}^{*^{-1}}(u)]$ of P_{-i} such that for $q \in (\theta, \bar{\theta})$ the indicator $\mathbf{1}_{\{U_{-i}(q|y)<U_i(q|x)\}}$ equals 1. Hence, $V_i(\theta|x) = \int\limits_{-\infty}^{U_{-i}^{*^{-1}}(u)} \hat{R}_i(\theta|x,y)dG_{-i}(y|x)$ is strictly increasing in x.

Now, we can rewrite $\pi_i(\theta|x;u) = G_{-i}\left[\left(U_{-i}^{*^{-1}}(u)\right)|x\right](V_i(\theta|x) - u)$.

By taking the cross-partial derivative in P_i's own signal x and bid u, we can see that

$$\frac{\partial^2 \pi_i}{\partial u \partial x} = g_{-i}\left[\left(U_{-i}^{*^{-1}}(u)\right)|x\right] \frac{1}{U_{-i}^{*'}\left(U_{-i}^{*^{-1}}(u)\right)} V_x$$

We assumed that the distribution of signals has strictly positive density everywhere within the domain, we assumed the opponent is playing a strictly increasing strategy in her signal,

[5]Here $U_{-i}^{*^{-1}}(u)$ refers to the signal of P_{-i} that induces her to offer rent u, for agent type θ.

and we showed $V_i(\theta|x)$ is strictly increasing in x. All of these together imply that that π_i is strictly supermodular in $(x; u)$. By Theorem 2.1 in Edlin & Shannon (1998) and the monotonicity theorem in Milgrom & Shannon (1994), we can say that $U_i^*(\theta|x)$ is strictly increasing in x. Because θ was arbitrarily chosen from $(\underline{\theta}, \bar{\theta})$, this holds for all $\theta \in (\underline{\theta}, \bar{\theta})$.

Because $V_i(\theta|x)$ is positive whenever $e_i^*(q|x) < e^{FB}$ for some $q \in [\theta, \bar{\theta}]$, there cannot be a regular equilibrium in which P_i plays constant bidding strategy $U_i^*(\theta|x)$ for some realizations x, because if the constant bidding is for some $e_i^*(\theta|x) < e^{FB}$, there is an atom at that effort level and P_{-i} can get a positive payoff by placing an atom at $e_i^*(\theta|x) + \epsilon$ and by making ϵ small enough, which makes P_i's strategy suboptimal. If there is constant bidding at some signal realization x with $e_i^*(q|x) = e^{FB}$ for all $q \in [\theta, \bar{\theta}]$, then a profitable deviation exists by choosing a smaller $e_i^*(\theta|x)$ because by condition 2, in a regular equilibrium $\underline{e}(\theta) < e^{FB}$, so for a positive measure of signal realizations, P_{-i} offers a less than first-best contract, therefore reducing $e_i^*(\theta|x)$ increases payoff because P_i can still win for some signal realizations of P_{-i}, which makes it a profitable deviation. Finally, because $V_i(\theta|x)$ is strictly increasing in x whenever $e_i^*(q|x) < e^{FB}$ for some $q \in [\theta, \bar{\theta}]$, both principals cannot play a decreasing strategy for any signal realizations, because P_i can increase her payoff by placing an atom at some $e_{-i}^*(\theta|y)$ where P_{-i} plays a decreasing strategy for y.

□

Unlike in the monopoly setting, where the principal has no benefit from offering rent to the agent, under competition the principals can increase the probability of winning the agent by offering more rent to the agent. Two factors pin down the increase in rents offered to the agent in terms of the principal's signal. First, just as in the monopolist principal benchmark, for any given interior agent type, a higher signal increases the principal's expected payoff from hiring the agent, so the agent becomes more valuable for the principal to hire. This leads to the principal wanting to increase the probability of winning the agent, which is done by increasing the rent offered to the agent. Second, because both principals' signals are affiliated with the agent's type, they are affiliated with each other, so a higher signal makes it more likely from the principal's perspective that the other principal also received a higher signal, which in this equilibrium means that the other principal is more likely to bid a higher rent. Therefore, in order to win the agent, the first principal must also offer higher rent to the agent. Both of these forces work in the same direction, therefore with a higher signal realization, principals offer more rent to all interior agent types.

Following is a couple of useful results that follow from proposition 2.

Corollary 1. *In any regular equilibrium, $U_i^*(\underline{\theta}|x)$ is nondecreasing in x, for $i = 1, 2$.*

Proof. Take any $x < x^{'}$. Suppose towards a contradiction that $U_i^*(\underline{\theta}|x) > U_i^*(\underline{\theta}|x^{'})$, and let the difference be $\delta = U_i^*(\underline{\theta}|x) - U_i^*(\underline{\theta}|x^{'}) > 0$. By proposition 2, for any $\theta > \underline{\theta}$, we must have $U_i^*(\theta|x) < U_i^*(\theta|x^{'})$, which means $U_i^*(\underline{\theta}|x) + \int\limits_{\underline{\theta}}^{\theta} e_i^*(q|x)dq < U_i^*(\underline{\theta}|x^{'}) + \int\limits_{\underline{\theta}}^{\theta} e_i^*(q|x^{'})dq$. So $\delta < \int\limits_{\underline{\theta}}^{\theta} e_i^*(q|x^{'})dq - \int\limits_{\underline{\theta}}^{\theta} e_i^*(q|x)dq$. But as $\theta \to \underline{\theta}$, this inequality cannot be satisfied for $\delta > 0$ because by condition 1, in a regular equilibrium contracts are continuous functions of θ and hence their integrals cannot have a discrete jump in value. Therefore $U_i^*(\underline{\theta}|x)$ must be nondecreasing in x. □

Corollary 2. *In any regular equilibrium, for all $\theta \in (\underline{\theta}, \bar{\theta})$, $e_i^*(\theta|x)$ is strictly increasing in x, for $i = 1, 2$.*

Proof. Notice that in any regular equilibrium we must have $U_i^*(\underline{\theta}|x) = \frac{1}{2} + \underline{\theta}$ for all x. Towards a contradiction, suppose not. By proposition 2, $U_i^*(\theta|x)$ is strictly increasing in x, and by condition 2 and corollary 1, $U_i^*(\underline{\theta}|x) \geq \frac{1}{2} + \underline{\theta}$. If $U_i^*(\underline{\theta}|x) > \frac{1}{2} + \underline{\theta}$, and $e_i^*(\theta|x) = e^{FB}$ for all θ, then P_i is making a negative payoff so can benefit by decreasing $U_i^*(\underline{\theta}|x)$. If $e_i^*(\theta|x) < e^{FB}$ for some θ, then it is profitable for P_i to decrease $U_i^*(\underline{\theta}|x)$ and increase $e_i^*(\theta|x)$ as to keep $U_i^*(\theta|x)$ unchanged, but this increases total surplus so increases P_i's payoff. So we must have $U_i^*(\underline{\theta}|x) = \frac{1}{2} + \underline{\theta}$ for all x. We know that for any $\theta > \underline{\theta}$, $U_i^*(\theta|x) = U_i^*(\underline{\theta}|x) + \int\limits_{\underline{\theta}}^{\theta} e_i^*(q|x)dq$. By proposition 2, $\frac{\partial}{\partial x}\left(U_i^*(\theta|x)\right) > 0$, and $U_i^*(\underline{\theta}|x)$ is constant in x, so $\frac{\partial}{\partial x}\left(U_i^*(\theta|x)\right) = \int\limits_{\underline{\theta}}^{\theta} \frac{\partial}{\partial x}\left(e_i^*(q|x)\right) dq > 0$, and because this holds for all $\theta > \underline{\theta}$, it follows that in a regular equilibrium the allocated effort level, $e_i^*(\theta|x)$ is strictly increasing in x. $\square$

As proposition 2 shows, a higher signal induces principals to offer more rent to the agent. The way principals offer higher rent in a regular equilibrium is through offering higher effort allocations to the agent. The alternative is paying higher wages without increasing efficiency, which is less profitable, because increasing effort to a more efficient level increases total surplus, so for the same increase in rent for the agent, the principal can benefit more in terms of capturing the added surplus.

Having proven in proposition 2 that in any regular equilibrium the bids for all interior types of the agent are strictly increasing in the principal's signal, and because both principals have a common lowest contract, it is now possible to define a correspondence between realized signals of the two principals, based on their optimal bidding strategies. For some given $\theta > \underline{\theta}$, and for any realized signal x of P_i, we can find the corresponding realization y of P_{-i} that makes her bid the same amount. We will call this the *tying function*. The tying function $Q_i(\cdot)$ maps P_i's realized signals to P_{-i}'s corresponding realized signal that induces P_{-i} to bid the same amount. We also define the inverse bidding functions. Both of these are well-defined by proposition 2. The following definitions are used to formalize this.

Definition. *For an arbitrary $\theta > \underline{\theta}$, the inverse bidding function $\phi_i(\cdot)$ is defined as*

$$\phi_i(u) \coloneqq U_i^{*^{-1}}(u)$$

That is, if $U_i^*(\theta|x) = u$, then $\phi_i(u) = x$.

Definition. *For an arbitrary $\theta > \underline{\theta}$, for any signal x of P_i, define the tying function $Q_i(x)$ as*

$$Q_i(x) \coloneqq \phi_{-i}\left(U_i^*(\theta|x)\right)$$

Definition. *For an arbitrary $\theta > \underline{\theta}$, for any signal x of P_i and signal y of P_{-i}, define the total value of obtaining agent of type θ as*

$$\tilde{R}_i(\theta|x,y) \coloneqq \int\limits_{\underline{\theta}}^{\theta} \left(\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} \right\} \mathbf{1}_{\left\{U_{-i}^*(q|y) < U_i^*(q|x)\right\}} \right) dF_i(q|x,y) + \hat{R}_i(\theta|x,y)$$

The following result formalizes how each principal's bidding strategy is determined based on her signals as well as the other principal's bidding strategy. It is also useful in establishing the existence of equilibrium in the static game.

Proposition 3. *Given a common lowest contract $\underline{e}(\cdot)$ for both principals, the tying function for an arbitrary $\theta > \underline{\theta}$ is the solution to the following differential equation:*[6]

$$\frac{dQ_i(x)}{dx} = \left\{ \frac{\tilde{R}_{-i}\left(\theta|Q_i(x),x\right) - U_{-i}^*(\theta|Q_i(x))}{\tilde{R}_i\left(\theta|x,Q_i(x)\right) - U_i^*(\theta|x)} \right\} \frac{s_i(x)}{s_{-i}(Q(x))} \frac{G_{-i}(Q_i(x)|x)}{G_i(x|Q_i(x))}$$

[6]where $G_{-i}(\cdot|x)$ is the cumulative distribution of P_{-i}'s signal given P_i's signal x, and $s_i(\cdot)$ is the prior unconditional density function of P_i's signal, that is, $s_i(x) = \int\limits_{\underline{\theta}}^{\bar{\theta}} s_i(x|\theta)dF(\theta)$

With the associated initial condition $Q_i(\underline{x}_i) = \underline{x}_{-i}$

And the associated equilibrium bid profile is characterized by:

$$\begin{aligned} U_i^*(\theta|x) =& \underline{U}(\theta) + \int\limits_{-\infty}^{x} \tilde{R}_i(\theta|t, Q_i(t))dL(t|x) \\ U_{-i}^*(\theta|y) =& U_i^*\left(\theta|Q_i^{-1}(y)\right) \end{aligned}$$

where $L(t|x) \coloneqq \exp\left(-\int\limits_t^x \frac{g_i(s|Q_i(s))}{G_i(s|Q_i(s))}ds\right)$*, and* $\underline{U}(\theta)$ *is the type-*θ *agent's payoff under a lowest contract satisfying conditionss 1 and 2.*

Proof. See appendix. □

1.4.2. Existence of Regular Equilibria

Taking any lowest contract that satisfies conditions 1 and 2 as the boundary condition, we can write down a differential equation for each principal using the tying function from proposition 3, which pins down, for any given θ, how $U_i(\theta|x)$ must be increasing in x. This differential equation takes $\tilde{V}_i(\theta|x) \coloneqq \int\limits_{-\infty}^{Q_i(x)} \tilde{R}_i(\theta|x, y)dG_{-i}(y|x)$ as given, and using it we can write down an expression for $U_i(\theta|x)$ in terms of $\tilde{V}_i(\theta|x)$. For some given signal x, we can then write down the optimal bid for two interior agent types $\hat{\hat{\theta}}$ and $\hat{\theta}$, then by taking the limit as $\hat{\hat{\theta}} \to \hat{\theta}$, we can set it up as a calculus of variations problem using the observation that $U_i^{'}(\theta|x) = e_i(\theta|x)$ where the derivative is taken with respect to θ, along with the boundary conditions $e_i(\underline{\theta}) = e_i(\bar{\theta}) = e^{FB}$. Because the integrand in the principal's maximization problem is concave in $e_i(\cdot|x)$ and $U_i(\cdot|x)$, a solution exists to the maximization problem.

1.4.3. Other Equilibria

Apart from regular equilibria, where principals offer contracts based on their signals, there is an equilibrium in constant strategies. In this equilibrium, both principals offer the efficient effort allocation to all types of the agent, and all of the surplus generated to the agent as rent. However, unlike the regular equilibria, this equilibrium is in weakly dominated strategies, because both principals receive a payoff of zero, principal P_i can deviate by choosing a distorted contract and still receive the same payoff. However, in that case, it is no longer a best response for P_{-i} to offer the first-best contract. Therefore, this equilibrium is not robust to perturbations in principals' strategies.

Proposition 4. *(Price war equilibrium) Both principals offering the first-best contract to all agent types, and offering all the surplus to the agent is an equilibrium in the static game.*

Proof. See appendix. □

1.4.4. Accuracy of Principals' Information

We now look at how the informativeness of the principals' signals affects the payoff of the agent. In the terminology used in auction theory, this is analyzing the expected revenue under different signal structures. Note that we are still in a pure common value environment, which keeps the analysis more tractable than with a general interdependent values setting.

As shown previously, a principal's expected payoff is supermodular in her signal and bid, and higher signals result in higher optimal bids. The principal's decision problem is therefore a monotone decision problem. The most common approach to modeling quality of information is Blackwell's "sufficiency" criterion, whereby one signal is more informative than another if the less informative signal is constructed by "garbling" the more informative one, meaning the better informed principal cannot learn anything from the less informative signal. Not only is this a very restrictive setting which does not allow ranking a wide range of signal structures where intuitively some signals seem more informative than others, it is also not very tractable in an affiliated information setting. A more general (and more convenient) notion of informativeness is what's called "accuracy" in Persico (2000), which first appeared in Lehmann (1988). In terms of notation of the signal structures, here we will drop the subscripts for the principals and replace them with superscripts as accuracy levels $\{\alpha_i\}_{i=1.2}$

Definition. *Given two signal structures $S^{\alpha_1}(\cdot|\theta)$ and $S^{\alpha_2}(\cdot|\theta)$, both of which are affiliated with the parameter θ, we say that $S^{\alpha_1}(\cdot|\theta)$ is more accurate than $S^{\alpha_2}(\cdot|\theta)$ if*

$$T_{\alpha_1,\alpha_2,\theta}(x) := S^{\alpha_1^{-1}}(S^{\alpha_2}(x|\theta)|\theta)$$

is strictly increasing in θ, for all signals x.[7]

Here, $T_{\alpha_1,\alpha_2,\theta}(x)$ is the signal y with accuracy α_1 which is the corresponding signal to signal x with accuracy α_2 under the parameter θ in the sense that the probability of getting a signal no higher than y under structure S^{α_1} is the same as the probability of getting a signal no higher than x under structure S^{α_2}. Suppose we take any signal x from S^{α_2}, find the corresponding signal y from S^{α_1} when the underlying parameter is θ. For a higher parameter $\theta' > \theta$, for the same signal x from S^{α_2}, the new corresponding y' from S^{α_1} will be to the right of y if S^{α_1} is more accurate than S^{α_2}. One way to understand this notion of

[7]Here we impose "strictly increasing" as opposed to "non-decreasing", this is without loss because we have an unbounded signal space in our setup.

informativeness is that given both signals are affiliated with θ, both of them will respond to an increase in θ by redistributing probabilities and by putting more probability mass to the right. The more accurate signal structure responds "more", in the sense that for an equal increase in the parameter θ, the more accurate signal structure shifts more probability to the right compared to the less accurate one. As explained in Persico (2000), this can also be understood by noticing that the transformation $T_{\alpha_1,\alpha_2,\theta}(x)$ varies together with θ, meaning by plugging in the same signal x, for a low θ, the transformation gives us a lower signal y compared to a higher signal y' when θ is high. In this way, the transformed signal is more correlated with the parameter, hence "more accurate".

Now consider a symmetric setting with $\alpha_1 = \alpha_2 = \alpha_S$. Contrast that with an asymmetric setting where $\alpha_1 > \alpha_2 = \alpha_S$. For an arbitrary type of the agent $\theta > \underline{\theta}$, let $\left\{U_i^S(\theta|\cdot)\right\}_{i=1,2}$ be the payoffs offered to the agent under equilibrium contracts in the symmetric setting, and $\left\{U_i^D(\theta|\cdot)\right\}_{i=1,2}$ be the payoffs offered in the setting with different accuracy levels. Let $U_i^S(\theta)$ be the expected payoff offered to agent type θ in the symmetric equilibrium by P_i, where the expectation is taken over all realizations of the signal. That is,

$$U_i^S(\theta) = \int_{-\infty}^{\infty} U_i^S(\theta|x)dS_i(x|\theta)$$

And similarly define $U_i^D(\theta)$ for the asymmetric case.

Just as in section 4, for an arbitrary $\theta > \underline{\theta}$, when P_{-i} is using an increasing strategy $U_{-i}(\cdot)$, we can write down P_i's payoff from bidding u after receiving signal x as:

$$\pi_i(x,u) = \int_{-\infty}^{U_{-i}^{-1}(u)} \left\{\hat{R}_i(\theta|x,y) - u\right\} dG(y|x)$$

Let's define $\Pi_i(\alpha) := \max_u \int_{-\infty}^{\infty} \pi_i(x,u) dS_i(x|\theta)$ under the signal X^α where α denotes the accuracy level.

By proposition 2, in a regular equilibrium P_{-i} indeed does use an increasing strategy, and as shown in the proof of proposition 2, this implies that this payoff is supermodular in (x,u), which implies that it has the single-crossing property in (x,u). We now restate an important result from Lehmann (1988) that links the accuracy of signals with payoffs having the single-crossing property.

Lemma 2. *Suppose signals X^{α_1}, X^{α_2} are affiliated with θ. Then, X^{α_1} is more accurate than X^{α_2} if and only if for all payoffs $\pi(x,u)$ having the single-crossing property,* $\Pi(\alpha_1) > \Pi(\alpha_2)$

Proof. See Lehmann (1988). □

Proposition 5. *In any regular equilibrium, $U_i^D(\theta) < U_i^S(\theta)$ for any $\theta > \underline{\theta}$, for i=1,2.*

Proof. See appendix. □

The intuition here is the following. In both settings, we are looking at a competition analogous to a common-value auction. Therefore there will be a winner's curse effect active in either situation. However, in the symmetric information setting, after winning, the winning principal will only know that her opponent's signal was lower than that of hers. This could be because she herself received an unusually high signal and the agent's type is actually quite low (winner's curse), or it could be that her opponent got an unlikely low signal and the agent's type is actually quite high. Because both possibilities exist in the symmetric information setting, the winner's curse is weaker compared to the setting with asymmetrically informed principals. In the asymmetric setting, after winning, the less informed principal will induce that it's more likely that her signal was unusually high and the agent's realized type is more likely to be low, because the other principal received a more accurate signal. Therefore the less informed principal must bid pessimistically enough to account for this stronger winner's curse, and knowing this, the informed principal will also lower her bid. This leads to overall lower utility (revenue) for the agent regardless of his type.

1.5. The Two-period Game

Suppose now that the game is repeated in a second period, where in period 1, the agent's type, θ_1, is realized from the distribution $F(\cdot)$, whereas in period 2, his type, θ_2, is realized from the distribution $F(\cdot|\theta_1)$. In the two-period setting with only short-term contracts, we will assume that all players maximize the undiscounted sum of their payoffs over the two periods.

Assumption 3. *θ_1 and θ_2 are affiliated.*

Assumption 3 says that for any $\theta_1^{'} > \theta_1$, $F(\cdot|\theta_1^{'})$ stochastically dominates $F(\cdot|\theta_1)$ in the likelihood ratio sense. This.captures the connection between the agent's productivity in period 1 and his productivity in period 2. This assumption means that a higher ability agent in period 1 is also more likely to be higher ability in period 2, and thus any information learned by the principals in period 1 about θ_1 is useful in period 2 as well.

In the repeated game, payoffs are the same for each period as in the static game. For the two-period setting, as described in section 2, we denote P_i's information structure at the end of period 1 as $\mathcal{I}_i$.

Let $U^*_{i2}(\theta_2|m)$ be the expected equilibrium payoff offered by P_i in the second period to the agent of realized type θ_2, when the public message m was sent in the first period. That is,

$$U^*_{i2}(\theta_2|m) = \int\limits_{-\infty}^{\infty} U^*_{i2}(\theta_2|m,x)dS_i(x|\theta_2)$$

Let $U_2(\theta_2|m)$ denote the expected payoff of type θ_2 when message m was sent; that is, $U_2(\theta_2|m)$ is the expected value of the higher of the two payoffs offered by the principals.

When in the first period, P_i offers a contract that specifies for a given θ_1 its allocated effort level $e_{i1}(\theta_1)$ together with $w_{i1}(\theta_1)$ and $m_i(y_1(\theta_1))$, let $U_i^{TP}(\hat{\theta}_1|\theta_1)$ denote the agent's two-period expected payoff when he chooses to work with P_i in the first period, and mimics type $\hat{\theta}_1$. Therefore,

$$\begin{aligned}
U_i^{TP}(\hat{\theta}_1|\theta_1) &= w_{i1}(\hat{\theta}_1) - C\left(e_{i1}(\hat{\theta}_1|\theta_1)\right) + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\} dF(\theta_2|\theta_1) \\
&= w_{i1}(\hat{\theta}_1) + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\} dF(\theta_2|\theta_1) - C\left(e_{i1}(\hat{\theta}_1|\theta_1)\right) \\
\frac{d}{d\hat{\theta}_1}\left(U_i^{TP}(\hat{\theta}_1|\theta_1)\right) &= w'_{i1}(\hat{\theta}_1) - \left(C'\left(e_{i1}(\hat{\theta}_1|\theta_1)\right)\right)\left(e_{i1}(\hat{\theta}_1)+1\right) \\
&\quad + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left[\frac{d}{d\hat{\theta}_1}\left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\}\right] dF(\theta_2|\theta_1)
\end{aligned}$$

Similar to Lemma 1, by applying the requirement for first period local incentive compatibility, we get

$$w_{i1}^{'}(\theta_1) = \left(C^{'}\left(e_{i1}^{*}(\theta_1)\right)\right) e_{i1}^{*'}(\theta_1) + C^{'}\left(e_{i1}^{*}(\theta_1)\right) - \int\limits_{\underline{\theta}}^{\bar{\theta}} \left[\frac{d}{d\theta_1}\left\{U_2\left(\theta_2|m_i(y_1(\theta_1))\right)\right\}\right] dF(\theta_2|\theta_1)$$

Which means,

$$U_{i1}^{'}(\theta_1) = e_{i1}^{'}(\theta_1) - \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{\frac{d}{d\theta_1}\left(U_2\left(\theta_2|m_i(y_1(\theta_1))\right)\right)\right\} dF(\theta_2|\theta_1)$$

This leads to the necessary envelope condition for first-period incentive compatibility:

$$U_{i1}(\theta_1) = \int\limits_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int\limits_{\underline{\theta}}^{\bar{\theta}} \left(U_2(\theta_2|m_i(y_1(q))\right) dF(\theta_2|\theta_1) \right\} dq$$

However, without knowing the shape of $m_i(\cdot)$, we cannot say whether this envelope condition is sufficient for incentive compatibility.

1.5.1. Non-Disclosure Policy

Now consider a second-period situation where P_i employed the agent in the first period. In the second period, at the time of offering contracts, P_i may have learned some information about θ_1 through her contractual relationship with the agent in the first period. Because θ_1 and θ_2 are affiliated, any such information is also informative of the agent's second period type θ_2. This is an informational advantage that the first period employing principal may have over the outsider principal, in case the informational learned through the first period contractual relationship, $\mathcal{C}_i$, is nonempty. We model this by assuming that under a non-disclosure policy (where $M_i = \phi$), P_i's second-period information has accuracy level $\alpha_{i_2} > \alpha_S$, where α_S is the accuracy level of the outsider principal's information, and the accuracy level of both principals' information in period 1.

Proposition 5 illustrates that we cannot have a first period separating equilibrium in the setting with non-disclosure, because the required high-powered incentive in the first-period will attract lower types to mimic as higher types, and to "take the money and run". This is the Ratchet effect as described in Laffont & Tirole (1988).

Assumption 4. *Under a non-disclosure policy, if $\mathcal{C}_i$ is nonempty, then in the second period, the accuracy levels of signals satisfy $\alpha_{i_2} > \alpha_{-i_2} = \alpha_S$.*

We now focus on what this implies about possible equlibria in the two-period game.

Proposition 6. *In the two-period game under non-disclosure, there does not exist an equilibrium where the agent fully reveals his type to the employing principal in the first period.*

Proof. See appendix. □

This is simply an instance of the ratchet effect as in Laffont & Tirole (1988). Even though unlike that paper (which has a monopolist principal offering a spot contract to the agent), we have competition in our setting, as long as we do not have second period competition between symmetrically informed principals, the ratchet effect persists. Because as we see in proposition 6, the agent's second-period payoff will be lower if he completely reveals his type to the employing principal in the first period, so he has an extra incentive to not reveal his type. Only monetary incentives are ineffective in overcoming this ratchet effect problem, because without long-term commitment, nothing stops lower types to mimic higher types and take the extra money in the first period.

1.5.2. Disclosure Policy

Under the disclosure policy, the employing principal in the first period can commit to sending a public message containing any information learned in period 1. We will assume that the principals' second period signals are of symmetric accuracy, therefore using a public message, the first period incumbent principal can give away any informational advantage.

Assumption 5. *Under the disclosure policy, principals in the second period receive symmetric signals.*

Proposition 7. *In the repeated game under disclosure, there exists a regular equilibrium where the agent fully reveals his type to the employing principal in the first period, and the employing principal chooses to publicly reveal the agent's first-period type.*

Proof. When both principals and the agent play the strategies under this separating equilibrium, in the first period, P_i would choose $m_i(y) = \theta_1$ such that $y = e^*_{i1}(\theta_1) + \theta_1$. In this case, $m_i(y)$ is a sufficient statistic for $\mathcal{I}_i^D$ because in the second period, the realization of the agent's first-period type is the only piece of information from $\mathcal{I}_i^D$ which is payoff-relevant. When choosing P_i's contract in the first period and exerting effort $e^*_{i1}(\theta_1)$, an agent of type θ_1 can get a two-period expected payoff of

$$U_i^{TP}(\theta_1) = U^*_{i1}(\theta_1) + \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|\theta_1)dF(\theta_2|\theta_1)$$

where $U^S(\theta_2|\theta_1)$ is the θ_2 type agent's expected payoff in the second period when both principals receive symmetric signals and update their beliefs to $F_i(\cdot|\theta_1, x_i)$, where x_i is the realization of P_i's signal in the second period. Because principals update their beliefs using Bayes' rule, and by assumption 3, for any $\theta_1^{'} > \theta_1$, $F(\cdot|\theta_1^{'})$ stochastically dominates $F(\cdot|\theta_1)$

according to the monotone likelihood ratio, applying proposition 2, we get that $U^S(\theta_2|\theta_1') > U^S(\theta_2|\theta_1)$ for all $\theta_2 \in (\underline{\theta}, \bar{\theta}]$. Moreover, for any $\theta_2' > \theta_2$, because

$$\frac{d}{d\theta_1}U^S(\theta_2'|\theta_1) = \int_{\underline{\theta}}^{\theta_2'} \frac{d}{d\theta_1}e^S(q|\theta_1)dq > \int_{\underline{\theta}}^{\theta_2} \frac{d}{d\theta_1}e^S(q|\theta_1)dq = \frac{d}{d\theta_1}U^S(\theta_2|\theta_1)$$

and because $F(\cdot|\theta_1')$ first order stochastically dominates $F(\cdot|\theta_1)$, we have that

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta_1'}U^S(\theta_2|\theta_1')dF(\theta_2|\theta_1') > \int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta_1}U^S(\theta_2|\theta_1)dF(\theta_2|\theta_1)$$

which means that the marginal benefit of disclosure is higher for higher first period types.

We can thus say that incentive-compatibility in the two-period game requires that for any $\theta_1' > \theta_1$,

$$U_{i1}(\theta_1') = U_{i1}(\theta_1) + \int_{\theta_1}^{\theta_1'} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q)dF(\theta_2|\theta_1') \right\} dq$$

In particular, we can thus write down the first period rent that needs to be paid for any type θ_1 as

$$U_i(\theta_1) = U_i(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q)dF(\theta_2|\theta_1) \right\} dq$$

Notice that this is smaller than the required rent in the static game. For any choice of first-period effort allocation $e_{i1}(\cdot)$, the principal can adjust the payment accordingly so that this IC requirement is satisfied.

We still need to show that the principal cannot do better by choosing a first-period contract that involves pooling, and a message rule that does not completely reveal the agent's type. Based on the linkage principle in Milgrom & Weber (1982)[8] we know that the agent's second period rent will be highest when the maximum possible information is publicly available, and as can be seen from the IC requirement above, the first period principal can extract the incremental rent the agent can get in period 2 from improving public information about his type in the first period. In the first period, suppose $U_{i1}^*(\theta_1|x)$ is the two-period payoff-maximizing bid offered by P_i. Because

$$U^S(\theta_2|\theta_1) = \mathbb{E}_{X_j}\left[\max_{j=1,2}\left\{U_{j2}^*(\theta_2|\theta_1)\right\}\right] \geq \mathbb{E}_{X_i}\left[U_{i2}^*(\theta_2|\theta_1)\right]$$

Lastly, given this is a symmetric contract, if P_{-i} offers a separating contract, and by condition 2, the lowest contract satifies the monotonicity constraint, it is optimal for P_i to also offer a separating contract.

P_i can maximize her two-period payoff by choosing a separating equilibrium and revealing all information learned in the first period. □

Committing to disclosure of information has two effects on both total surplus and the incumbent principal's payoff. First, by committing to disclose information to her opponent, the principal implicitly commits to bid aggressively for the agent in period 2. That is, the principal commits to pay the agent more rent in period 2, relative to the case with no disclosure. But by corollary 2 of proposition 2, the principal optimally promises rent to the agent by asking that agent to exert more effort. Higher effort increases total surplus, so more aggressive bidding in period 2 implies higher total surplus. The incumbent principal can extract some of this future surplus by paying lower wages in period 1. Consequently, both principals earn higher expected ex ante payoffs in an equilibrium with disclosure. Note

[8]Especially Theorem 17 in Milgrom &Weber (1982) and Theorem 7 in Milgrom &Weber (1982b).

that principals cannot fully extract the additional surplus created from disclosure, since they compete with one another. However, they earn at least part of the additional surplus whenever that competition is imperfect, as it is when each of them has private information about the agent's ability.

Second, committing to disclosure increases the slope of the agent's expected second period rent as a function of his first period type. Consequently, with disclosure, it is cheaper to incentivise higher ability agents to separate from the lower ability agents, this second effect mitigates the ratchet effect problem, and allows principals to offer screening contracts in the first period.

1.6. Extensions

1.6.1. More Than Two Principals

It is natural to think about implications of having stronger competition for the agent when there are $N > 2$ principals. As in auction theory, the analysis of this situation is very similar to the two principals case. From each principal's perspective, the relevant belief about the opponents' bidding behavior is only the distribution of the highest of the $N-1$ other principals' bid. The inverse bidding function $\phi_{-i}(u)$ that is used to map the opponent's bid to her signal needs to be modified to be $\phi_{Y_1}(u)$, which is the inverse bidding function that maps the highest of the $N-1$ bids to a random variable Y_1 which is the signal associated the highest bid. That is, $U^*_{Y_1}(\cdot|\cdot)$ is the bidding strategy P_i bids against, where for a given agent type θ, and for signal realization y,

$$U^*_{Y_1}(\theta|y) = \max_{j \neq i} U^*_j(\theta|y)$$

The effect of increased competition in this way is straightforward. As seen earlier, with 2 principals, the best response to a more aggressive bidding strategy from the opponent is to become more aggressive. When there are more than two principals, the highest of the other principal's bids is higher as we are now considering the first order statistic of $N-1$ other bids. The outcome is that each principal bids more aggressively, and as $N \to \infty$ the contract offered by P_i approaches the first-best contract given any realization of her signal, and the agent gets all of the surplus he generates.

For this part we will again assume that all signals are affiliated with the agent's type θ_t.

Assumption 6. *X_{it} is affiliated with θ_t, for $i = 1, ..., N$*

Here we will show that the analog of proposition 2 for $N > 2$ principals holds; that is, with more than 2 principals, the utility offered to the agent by each principal is strictly increasing in the principal's signal realization. Before establishing this result, we state a useful lemma which is part of theorem 2 in Milgrom & Weber (1982).

Lemma 3. *X_{it} and Y_{1t} are affiliated.*

Proof. See Milgrom & Weber (1982). □

Because in the one-period game with $N > 2$ principals, from P_i's perspective, the maximization problem is the same as in the two principal case with Y_1 being the relevant signal, and because X_i and Y_1 are affiliated, the following analog for proposition 2 holds.

Proposition 8. *In the one-period game with competition between N principals, for any agent type $\theta \in (\underline{\theta}, \bar{\theta})$, $U_i^*(\theta|x)$ is strictly increasing in x, for $i = 1, ..., N$.*

1.6.2. Short-lived Principals

Consider a two-period setup where there are $N_1 \geq 1$ principals active in the first period, and $N_2 \geq 2$ principals active in the second period. This means there is competition among principals in the second period. We can have a set of active principals in period 1, $\mathcal{P}_1$, and a set of principals active in period 2, $\mathcal{P}_2$. It may be that some of the principals belonging to $\mathcal{P}_1$ are also in $\mathcal{P}_2$, while some are not, and $\mathcal{P}_2$ can have principals that are not in $\mathcal{P}_1$. So some principals may be short-lived, while others may be long-lived, and there may be some who are only active in period 2. However, the set of active principals in each period is fixed at the beginning of the game, so there are no entry or exit decisions made by principals. In this setup, in a disclosure setting, that is, in the first period, letting the message space be M_i^D, there is a fully separating equillibrium in the two-period game.

Proposition 9. *In the repeated game under disclosure with sets of principals $\mathcal{P}_1$ and $\mathcal{P}_2$, there exists a regular equilibrium where the agent fully reveals his type to the employing principal in the first period, and the employing principal chooses to publicly reveal the agent's first-period type.*

Proof. Under the strategies described, if $P_i \in \mathcal{P}_1$ employs the agent of type θ_1 in the first period and chooses message $m_i(y) = \theta_1$ such that $y = e^*_{i1}(\theta_1) + \theta_1$. Because θ_1 and θ_2 are affiliated, and $N_2 \geq 2$, $U^S(\theta_2|\theta_1)$ is strictly increasing in θ_1, by propositions 2 and 8. Similar to proposition 7, under the revealing public message, the agent's first period IC constraints under P_i's contract can be written as

$$U_i(\theta_1) = U_i(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q) dF(\theta_2|\theta_1) \right\} dq$$

This means P_i can extract the incremental rent the agent gets in the second period by adjusting the payment accordingly in the first period. If $P_i \notin \mathcal{P}_2$, then it is clearly beneficial for P_i to maximize the agent's second period payoff by revealing all information and extracting it in the first period. If $P_i \in \mathcal{P}_2$, because $U^S(\theta_2|\theta_1) = \mathbb{E}_{X_j}\left[\max_{j \in \mathcal{P}_2}\left\{U^*_{j2}(\theta_2|\theta_1)\right\}\right] \geq \mathbb{E}_{X_i}\left[U^*_{i2}(\theta_2|\theta_1)\right]$, it is still profitable for P_i to reveal the agent's type. □

1.6.3. Public Output

When the agent works with P_i in the first period, generates output y which is publicly observable, but the contract he was offered is not publicly observed, this may lead to a setting where P_i still has an informational advantage over the other principal because she may infer the agent's first-period type more accurately as she knows what contract the agent chose, while P_{-i} can only use a probability distribution over P_i's signal and the subsequent contract offered by P_i. As we saw in proposition 4, such a setting does not allow for separating contracts, as the ratchet effect is still present. When both output and contract offers are publicly observed, this makes information structure the same as under the disclosure policy. So similar to the disclosure setting, the first period employing principal can still capture some of the incremental rent the agent receives in the second period due to improved public information, because it is still the first period employing principal who is generating this value by giving the agent to credibly signal his type.

1.7. Conclusion

The implications of public disclosure of some performance measure can be seen through an increase in the degree of competition for the agent's services in the future. This creates value for the agent in the future through a reduction in the firms' uncertainty regarding the agent's worth (winner's curse), which leads to firms offering more efficient contracts, generating more surplus. However, this value is being created by the principals through their ability to credibly reveal information about the agent's performance, and as such, the principals will appropriate this additional surplus upfront by offering lower payment to the agent in the first period, utilizing the agent's incentive to work hard in the first period for the rent in the future. As the incremental rent from better public information is higher for higher types, this incentive is also stronger for higher types, which makes screening higher types from lower types easier for the principal. However, because both principals can generate this value, they compete away some of these rents to the agent, and how much of these rents the agent gets depends on the initial level of competition.

CHAPTER 2

Market Research and Differentiated Bertrand Competition (joint with Colin Shopp)

2.1. Introduction

Firms learn demand in order to optimally set prices. In competitive settings, market research not only directly informs a firm about demand for its own good, but indirectly informs the firm about how its competitor will price in the face of uncertain demand for its good. Firms will only perform market research to the extent that the returns from doing so exceed the costs, and these returns may vary with the level of differentiation between one firm's product and its competitor's product.

We explore this phenomenon in the context of a standard differentiated duopoly Bertrand model with uncertain linear demand, in the style of Vives (1984). Rather than assume exogenous signals of the demand intercept, we instead allow firms to covertly choose the accuracy of their signals at some cost. We compare the level of market research in (symmetric) equilibrium across different levels of competition, as measured by how differentiated the goods are. We give sufficient conditions such that endogenous market research monotonically decreases in the level of competition, as well as sufficient conditions such that endogenous market research is non-monotonic in the level of competition.

In this model, fixing some exogenous level of market research, a firm optimally prices by setting an average price plus a linear function of its signal. The more accurate a firm's signal, the more it will condition its price on its signal. Its average price will not change, fixing the other firm's behavior. As the goods become less differentiated, competition sharpens: both firms' prices will go down for any given signal, which lowers overall profits.

Fixing the level of competition, as one firm's accuracy increases, its expected profits increase through two channels. First, it is better able to match its price to demand. Second, it is better able to coordinate its price with the other firm. Fixing average prices, one firm would rather price high when the other firm prices high, and low when the other firm prices low. A more accurate signal of demand is also a more accurate signal of the other firm's price. Because of this, if either firm's accuracy exogenously increases, both firms will condition their prices more on their signals. Otherwise, they will price conservatively in order to coordinate better. At any level of differentiation (other than perfectly homogenous goods), profits for both firms increase when either firm's accuracy increases.

The size of the marginal return to increasing accuracy varies with the amount of competition and can be broken down into two effects, which we call the *competitive profit effect* and the *coordination effect*. Both of these effects are weighted by the sensitivity of the firm's price to its signal; prices compress towards marginal cost as competition increases, so that the accuracy of a signal becomes less important fixing the other firm's behavior. The competitive profit effect is that as goods become less differentiated, so that both firms not only set prices lower on average but also condition prices less on the state, the firm cannot improve profits as much by setting high prices when the state is high and low prices when the state is low. If a firm is a monopoly, it can better align its prices with the state by increasing the accuracy of its signal. However, when the firm is forced to price conservatively because of increased competition, it cannot fully take advantage of a more accurate signal to match its price to the state.

The coordination effect has two components in addition to the sensitivity of the firm's price to its signal: the *substitution effect* and the *competitor pricing effect*. The substitution effect is that as goods become less differentiated, demand for one firm's good is more sensitive to the difference between the firms' prices. It becomes more important for a firm to coordinate its price with the other firm's price. The competitor pricing effect moves in the other direction. As competition intensifies, the firm's competitor not only lowers its price after any signal, but also compresses those prices towards marginal cost. This makes it easier to coordinate pricing, since the firms' prices are close even if their signals are very different. In the extreme case of homogenous goods, prices equal marginal cost and the competitor pricing effect is zero. At the other extreme, when goods are completely differentiated and firms function as monopolies, the substitution effect is zero. The total coordination effect is inverted U-shaped, so that it is highest at some intermediate level of competition.

We examine the competitive profit and coordination effects together. Marginally increasing accuracy always helps firms match the state better and coordinate better. However, the amount that it allows one firm to better coordinate with the other depends on the other's accuracy level. When both firms have very low accuracy, one firm marginally increasing its accuracy does not help it coordinate much with the other firm, whose price is not very correlated with demand. When both firms have high accuracy, one firm increasing its accuracy also allows it to better coordinate its price with the other firm. Thus, the relative importance of the competitive profit effect and the coordination effect depends on accuracy levels. We show that the competitive profit effect dominates when research costs are sufficiently high, so that equilibrium market research is monotonically decreasing in the level of competition. We also show that when research costs are sufficiently low, the coordination effect is large enough that equilibrium research is highest at an intermediate level of competition.

This paper is related to a wider literature on market research and competition. Building on the differentiated duopoly models of Singh & Vives (1984), Vives (1984) examines whether firms would prefer to commit to making their endogenous research public. He shows that firms prefer to pool their information in a Bertrand setting but not in a Cournot setting. Other models have endogenized market research, although they have tended to focus on Cournot rather than Bertrand competition, overt rather than covert research, and on different measures of competition than we do. For example, Hwang (1993) studies overt research in Cournot duopolies when goods are homogenous, but firms face different costs of acquiring information. Hwang (1995) also studies overt research in a Cournot setting with homogenous goods, but measures competition as the number of firms as well as a somewhat idiosyncratic "conjectural variation" model of competition. That paper finds a result qualitatively similar to ours: firms perform the least amount of research when competition is perfect, and perform the most amount of research either in an oligopoly or in a monopoly, depending on the parameters. Hauk & Hurkens (2000) study covert research in a Cournot setting, where competition is measured as the number of firms and goods are homogenous. Vives (2000) is an excellent overview of competition more broadly, and addresses some models of market research.

We utilize the central result of Persico (2000) in order to compare equilibrium market research at different levels of competition. That paper shows that when signals are ordered by accuracy, a concept first presented by Lehmann (1988), marginal returns to accuracy can be ranked according to a relatively straightforward single crossing condition. The paper then applies that ranking to compare information acquisition in first and second price auctions, building on the work of Milgrom and Weber (1982). To our knowledge, this is the first direct application of the theorem to a duopoly setting.

The paper shares some similarities to the literature on innovation, though in our setting market research hurts rather than helps consumers, since firms use the information to extract more surplus rather than to create better products.[1] Questions about the effects of competition on innovation have been raised and debated since seminal works by Schumpeter (1912, 1942). We do not address this debate, except to note that Aghion et al. (2005) find evidence of an inverted-U shape in equilibrium innovation that is qualitatively similar to our coordination effect. Goettler & Gordon (2014) also find an inverted-U shape between innovation and competition in their model of dynamic oligopoly with endogenous market structure.

The rest of the paper is organized as follows. Section 2.2 contains the model. Section 2.3 applies Persico's theorem to identify the two effects of competitiveness on returns to market research and gives the main results. Section 2.4 concludes.

2.2. The Model

We give the timing and payoffs and review the relevant result of Vives (1984). Two symmetric firms indexed by i each privately choose a signal distribution indexed by $v_i \in [0, \infty)$ at differentiable cost $K(v_i)$. The state $\alpha \sim \mathscr{N}(\bar{\alpha}, V_\alpha)$ is realized. The cdf of this disribution, $G(\alpha)$, is commonly known to the firms when they choose v_i. Each firm receives a private signal realization $s_i = \alpha + \epsilon_i$, where $\epsilon_i \sim \mathscr{N}(0, v_i)$, and ϵ_1 and ϵ_2 are independent. Define $t_i = \frac{V_\alpha}{V_\alpha + v_i} \in (0, 1]$. Since for any V_α there is a one-to-one, continuous relationship between v_i and t_i, we consider firm i to be choosing t_i at cost $C(t_i)$. We assume that $C(t_i) \geq 0$ and $C'(t_i) \geq 0$.

[1]We address this further in Section 2.4.

We write the conditional distribution on α after seeing signal realization s_i as $G^{t_i}(\alpha|s_i)$. For a given α' and t_i we write the conditional distribution on all signals s_i as $F^{t_i}(s_i|\alpha')$. For a given t_i, we write the prior distribution on all signals s_i as $F^{t_i}(s_i)$.

After privately receiving signals, firms simultaneously set prices p_1 and p_2. Following Vives (1984), firm i faces the following linear inverse demand:[2]

$$p_i = \alpha - q_i - \gamma q_{-i}.$$

Direct demand is

$$q_i = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}p_{-i}.$$

Goods are substitutes, i.e. $\gamma \in [0,1)$.[3] The state α, the demand intercept, captures the level of demand, while increasing γ decreases the level of differentiation between firms. When $\gamma = 0$ the firms are monopolies, while as $\gamma \to 1$ demand approaches perfect competition. We normalize the marginal cost of production to be 0 for simplicity. After privately observing a signal, each firm chooses price. Firm i earns profits $p_i q_i$.

We consider Perfect Bayesian Equilibrium of this game, with firm i's equilibrium strategy written $\{t_i^*, p_i^*(s_i|t_i)\}$. In the Bertrand competition stage, firms maximize their expected profits given their conjecture of the other firm's pricing strategy as a function of their signal. Firm i's maximization problem after receiving signal s_i when their signal structure is indexed by t_i and the conjectured signal structure of firm $-i$ is indexed by t_-i, is

$$\max_{p_i} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dG^{t_i}(\alpha|s_i).$$

[2]This is a special case of Vives (1984) with β normalized to 1, so that $\gamma \in [0,1]$ fully characterizes the level of substitutability across firms, and with independent signals to simplify the firm's choice of t.
[3]Direct demand is undefined at $\gamma = 1$, where profits are discontinuous in price.

Equilibrium prices must be as in Vives (1984):

$$p_i^*(s_i|\gamma, t_i) = A + B_i t_i \left(s_i - \frac{\bar{\alpha}}{1+\gamma} \right)$$

Where

$$A = \frac{\bar{\alpha}(1-\gamma)}{2-\gamma}$$

$$B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}.$$

Anticipating this, firm i chooses t_i to maximize $R(t_i) - C(t_i)$, with

$$R(t_i) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_i^*(s_i|\gamma, t_i) q_i(p_i^*(s_i|\gamma, t_i), p_{-i}(s_{-i}|\gamma, t_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dF^{t_i}(s_i|\alpha) dG(\alpha).$$

We call this the *market research problem* and we call t_i firm i's *accuracy level*.

Following Persico (2000), let *asymmetric marginal revenue* $AMR_\gamma(t, t')$ be firm i's marginal returns from increasing t_i from $t_i = t$ when the level of differentiation is γ and firm $-i$ plays pricing strategy $p_{-i}^*(s_i|\gamma, t_{-i} = t', t_i = t')$, i.e. when firm $-i$ has accuracy level t' and prices as if firm i also has accuracy level t'. Define marginal revenue of accuracy at level of differentiation γ as as $MR_\gamma(t) \equiv AMR_\gamma(t, t)$. Define the marginal cost of accuracy as $MC(t) \equiv C'(t)$.

We focus on symmetric equilibrium in which $t_i^* = t_{-i}^* = t^*(\gamma)$ and $p_i^*(s_i|\gamma, t^*(\gamma)) = p_{-i}^*(s_{-i}|\gamma, t^*(\gamma))\ \forall s_i = s_{-i}$. At such an equilibrium it must be that $MR_\gamma(t^*(\gamma)) = MC(t^*(\gamma))$.

2.3. Returns to Market Research

This section contains the main results of the paper. We state the relevant result from Persico (2000) in the framework of our model. Without directly solving for marginal returns to accuracy, we are able to apply the result in order to rank marginal returns to accuracy across different levels of differentiation. We decompose relative marginal revenue from accuracy into two components, the competitive profits component and the coordination component. We then give two main results: (1) when the cost of accuracy, $C(\cdot)$, is sufficiently high, market research in the unique symmetric equilibrium is decreasing in the level of competition, and (2) when $C(\cdot)$ is sufficiently low, equilibrium market research is higher at an intermediate level of competition than in either the monopoly or perfect competition setting. Finally, we show that the second result extends to a setting in which the both firms' choice of accuracy is publicly observed.[4]

Let $u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i})) \equiv \int\limits_{s_{-i}=-\infty}^{\infty} p_i^*(s_i) q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha)$. When $t_1 = t_2 = t$, denote this as $u_\gamma(\alpha, p^*(s,t))$. Given two payoff functions $u_{\gamma'}(\alpha, p_i^*(s,t))$ and $u_{\gamma''}(\alpha, p_i^*(s,t))$, we write $u_{\gamma'} \succeq u_{\gamma''}$ if $u_{\gamma'} - u_{\gamma''}$ has the single-crossing property, i.e. if $\frac{\partial u_{\gamma'}(\alpha,p)}{\partial p}$ crosses $\frac{\partial u_{\gamma''}(\alpha,p)}{\partial p}$ at most once, and from below, as α increases. We write $u_{\gamma'} \succ u_{\gamma''}$ if $u_{\gamma'} \succeq u_{\gamma''}$ and $u_{\gamma''} \not\succeq u_{\gamma'}$.

Lemma 4. *For γ' and γ'', if $u_{\gamma'}(\alpha, p^*(s,t)) \succ u_{\gamma''}(\alpha, p^*(s,t))$, then $MR_{\gamma'}(t) > MR_{\gamma''}(t)$.*

Proof. See Appendix B.1. □

The lemma states that in order to compare the marginal returns of accuracy at two different competition levels, it suffices to show that their difference satisfies single-crossing.[5]

Note that $p_i^*(s_i)$ is non-decreasing in s_i. In order to show for a given pair γ',γ'' that $MR_{\gamma''}(t_i) > MR_{\gamma'}(t_i)$, it suffices to show that

$$\frac{\partial}{\partial s_i}\left[u_{\gamma''}(\alpha, p_i^*(s_i|\gamma'', t)) - u_{\gamma'}(\alpha, p_i^*(s_i|\gamma', t))\right]$$

[4]We do not extend the first result to the public setting.
[5]See Persico (2000) for a detailed discussion.

is increasing in α. To that end, we first examine $\frac{\partial^2}{\partial s_i \partial \alpha}\left[u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i}))\right]$ for fixed $\gamma \in [0, 1)$, which satisfies the following.[6]

$$\frac{\partial^2}{\partial s_i \partial \alpha}\left[u_\gamma(\alpha, p_i^*(s_i|, t_i))\right] = \frac{\partial q_\infty}{\partial \alpha}\frac{\partial p_i^*}{\partial s_i} + \left(\frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}\right). \tag{2.1}$$

Where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$.

From the equation we see that firm i's marginal return to accuracy has two components. Both components are weighted by the sensitivity of the firm's optimal price to their signal, and they are smaller if the firm's optimal price is not very sensitive to the signal.

The first component is the *competitive profit effect*, $CMP(t, \gamma) \equiv \frac{\partial q_\infty}{\partial \alpha}\frac{\partial p_i^*}{\partial s_i}$. This depends on the change in expected profit as the state changes evaluated when firm $-i$ sets its price at ∞. When the state increases, the quantity demanded at any price also increases. As the firm's accuracy increases, it is better able to tailor its demand to the state, α. However, if the firm's are very insensitive to their signal due to either low accuracy or high competition, then the firm cannot benefit as much from a high state. Even though this effect is evaluated when the competing firm chooses a fixed high price, we call it the "competitive" profit effect because it is dependent on the firm's ability to price high and condition its price on the state, which is determined by the level of competition.

The second component is the *coordination effect*, $CRD(t, \gamma) \equiv \frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}$. As the firm's accuracy increases, it not only learns more about the state, but also learns more about the other firm's pricing. It is able to better coordinate its pricing with the competing firm.

[6]See Appendix B.2 for a derivation of this equation, which depends on our distributional and linear demand assumptions. Arguments are suppressed for neatness.

Fixing an average price, the firm is better off pricing high when its competitor prices high, and low when its competitor prices low. The coordination effect measures this benefit.

The coordination effect has two components in addition the sensitivity of firm i's price to its signal: the sensitivity of firm i's demand to firm $-i$'s price, $\frac{\partial q_i}{\partial p_{-i}}$, i.e. the substitution effect, and the sensitivity of firm $-i$'s price to its signal, $\frac{\partial p_{-i}}{\partial s_{-i}}$, i.e. the competitor pricing effect. The substitution effect reflects that if demand is more sensitive to firm $-i$'s price, it is more important that firm i prices accordingly. As goods become less differentiated, then the quantity a firm sells is highly dependent on the difference between the two firms' prices. This is magnified by the competitor pricing effect. If firm $-i$'s price is more sensitive to its signal, then it is more important for firm i to coordinate signals with firm $-i$. A small difference in signals leads to a large difference in prices when firm $-i$'s price is very sensitive to its signal.

We now plug in equilibrium prices to Equation 2.1. For given accuracy levels t_i, t_{-i}, the equation is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i))}{\partial\alpha\partial s_i} = \frac{1}{1+\gamma}B_i t_i + B_i t_i \frac{\gamma}{1-\gamma^2}B_{-i}t_{-i}. \tag{2.2}$$

Recall that $B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}$. Since we are interested in symmetric equilibrium, suppose $t_i = t_{-i} = t$, in which case $B_i = B_{-i}$. Then Equation 2.2 is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t))}{\partial\alpha\partial s_i} = \frac{1-\gamma}{2-\gamma t}t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2 \tag{2.3}$$

We can now examine how both CMP and CRD depend on the level of competition γ.

Proposition 10. *For any $t \in (0, 1]$, the competitive profit effect $CMP(t, \gamma)$ is strictly decreasing in γ.*

Proof. For $t \in (0,1]$, $\frac{\partial}{\partial\gamma}\left[\frac{1-\gamma}{2-\gamma t}t\right] = \frac{-t(2-t)}{(2-\gamma t)^2} < 0$. □

As the environment becomes more competitive and firms price more aggressively, not only does the size of the pie effectively shrink, but the firms are less able to maximize their profits by tailoring prices to demand. The more a firm is forced to compete, the less it is able to condition its price on its signal and better match its price to the state. Accuracy becomes marginally less valuable.

Proposition 11. *For any $t \in (0,1]$, the coordination effect $CRD(t,\gamma)$ is single-peaked in γ, and $CRD(t,0) = \lim_{\gamma\to 1} CRD(t,\gamma) = 0$.*

Proof. First note that at $\gamma = 0$ and at $\gamma = 1$ the coordination effect is $\frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2 = 0$. The derivative of the coordination effect w.r.t. γ is

$$\frac{\partial}{\partial\gamma}\left[\frac{(1-\gamma^2)\,\gamma}{(2-\gamma t)^2}t^2\right] \quad = \quad \frac{-6\gamma^2 + (\gamma^3+\gamma)t + 2}{(2-\gamma t)^3}t^2.$$

This is continuous, positive at $\gamma = 0$, and negative at $\gamma = 1$. Setting it equal to zero, there is only one real-valued solution in γ, which must be interior by the intermediate value theorem. It must be the global maximum in γ on $\gamma \in [0,1)$. □

Changes in competition change the relative size of the coordination effect in two ways. First, as γ increases, firm i's profits are more dependent on firm $-i$'s price. Thus, it becomes more important to learn the state in order to learn more about firm $-i$'s price. Second, the size of this effect depends on how sensitive firm $-i$'s price is to the signal s_{-i}. Since these effects are multiplicative, the coordination effect is highest at intermediate levels of competition, where prices are sensitive enough to signals that coordinating prices requires high accuracy, and goods are similar enough that price coordination is important.

Examples of the competitive profit effect and the coordination effect as a function of γ are shown in Figure 2.1 for $t = 0.5$.

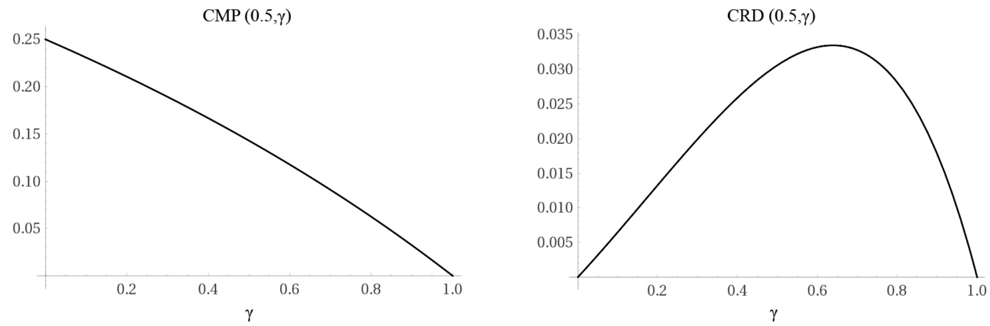

Figure 2.1. Competitive Profit Effect and Coordination Effect

Corollary 3. *For any* $t \in (0, 1]$, $MR_0(t) > \lim_{\gamma \to 1} MR_\gamma(t)$.

The competitive profit effect is positive in the monopoly case, i.e. $\gamma = 0$, where firms' profits when they price optimally are very sensitive to the state. The coordination effect is 0 in the monopoly case, since one firm's price has no impact on the other firm's demand or optimal price. In the (almost) perfect competition case, i.e. as γ approaches 1, both the competitive profit effect and the coordination effect approach 0. Each firm's equilibrium price approaches marginal cost at all signals, so there are minimal returns to better information.

The change in the total effect across competition levels, $\frac{\partial}{\partial \gamma}\left[CMP(t, \gamma) + CRD(t, \gamma)\right]$, depends on the level of accuracy, t. If both firms' signals are not very accurate, then one firm getting better accuracy does not help coordination very much, but it does help that firm better match the state. Thus, when t is low enough, the competitive profit effect is relatively more important than the coordination effect. The marginal return to accuracy is monotonically decreasing in γ in that case. When t is high enough, the coordination effect becomes relevant so that the marginal return to accuracy is no longer monotonically decreasing in the level of competition, but instead is highest at some intermediate level of competition. Examples of $CMP(t,\gamma)+CRD(t,\gamma)$ are shown in Figure 2.2 for $t = 0.5$ on the left and $t = 0.98$ on the right. In the right-hand graph, $CMP(t,\gamma)+CRD(t,\gamma)$ is maximized at an interior value of γ.

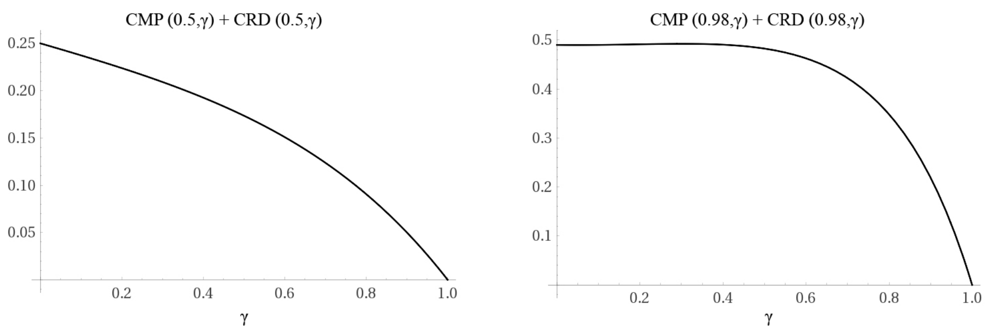

Figure 2.2. Total Effect at Low Accuracy and High Accuracy

The following lemmas formally state that the marginal return to accuracy is monotonically decreasing in γ when t is low, and that it is maximized at some interior γ when t is high.

Lemma 5. *$\exists \bar{t}$ such that for any $t \in (0, \bar{t})$, $\frac{\partial}{\partial \gamma}\left[MR_\gamma(t)\right] < 0$.*

Proof. See Appendix B.3. $\square$

Lemma 6. *$\exists \underline{t}$ s.t. for any $t > \underline{t}$ $\exists \gamma' > 0$ s.t. $MR_{\gamma'}(t) > MR_0(t)$.*[7]

Proof. See Appendix B.4. $\square$

We can compare equilibrium levels of market research across levels of competition as long as there exists a symmetric equilibrium in market research. For any pair t, γ both

[7]The lowest such $\underline{t}$ is approximately 0.96778. Note that the notation is somewhat idiosyncratic in that the minimum $\underline{t}$ satisfying Lemma 3 is larger than the maximum $\bar{t}$ satisfying Lemma 2.

the competitive profit and coordination effects are weakly positive. This is true even if $t_i \neq t_{-i}$, as in Equation 2.2. It is immediate by inspection that for any tuple $\{\gamma, t_i, t_{-i}\}$, $AMR_\gamma(t_i, t_{-t}) > 0$, i.e. firm i always benefits from more accuracy.[8]

This implies that we can find a cost function $C(t)$ such that when this is the cost of accuracy for both firms, at any γ there exists a unique symmetric equilibrium in accuracy $t^*(\gamma)$. Furthermore, we can find a cost function such that for some $\bar{t}$, $t^*(\gamma) \in (0, \bar{t})$ $\forall \gamma \in [0, 1]$. Call such a cost function $C^{\bar{t}}(t)$. We can also find a cost function such that for some $\underline{t} > \bar{t}$, $t^*(\gamma) \in (\underline{t}, 1)$ $\forall \gamma \in [0, 1]$. Call such a cost function $C_{\underline{t}}(t)$.

Finally, in order to state the main result we must formally define "higher costs" and "lower costs" of accuracy. For a given cost function $\hat{C}(t)$, let $\{\hat{C}(t)\}_L$ be the set of all cost functions $C(t)$ such that $\forall \gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0, 1]$, $C(t') \leq \hat{C}(t')$ and $C'(t') \leq \hat{C}'(t')$. Similarly, let $\{\hat{C}(t)\}_H$ be the set of all cost functions $C(t)$ such that $\forall \gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0, 1]$, $C(t') \geq \hat{C}(t')$ and $C'(t') \geq \hat{C}'(t')$.

Theorem 1. *There exist* $\{\bar{t}, \underline{t}\}$ *with* $1 > \underline{t} > \bar{t} > 0$ *such that:*
(1) $\exists C^{\bar{t}}(t)$ *such that for any cost function* $C(t) \in \{C^{\bar{t}}(t)\}_H$*, at every* $\gamma \in [0, 1)$ *there exists a unique symmetric equilibrium with market research* $t^*(\gamma)$ *s.t.* $\frac{\partial}{\partial \gamma}[t^*(\gamma)] < 0$*, and*
(2) $\exists C_{\underline{t}}(t)$ *such that for any cost function* $C(t) \in \{C_{\underline{t}}(t)\}_L$*, at every* $\gamma \in [0, 1)$ *there exists a unique symmetric equilibrium with market research* $t^*(\gamma)$ *s.t.* $t^*(\gamma') > t^*(0) > \lim_{\gamma \to 1} t^*(\gamma)$ *for some* $\gamma' \in (0, 1)$.

Proof. By Lemma 4, single crossing is sufficient for ranking marginal returns to accuracy. Existence of symmetric equilibrium is immediate from Lemmas 5 and 6. $t^*(\gamma)$ is continuous

[8] $AMR_\gamma(t_i, t_{-t})$ approaches 0 as $\gamma \to 1$.

in γ by the continuity of equilibrium prices and equilibrium payoffs in all arguments. For (1), by Lemma 5 there exist some $\bar{t}$ and $C^{\bar{t}}(t)$ such that $\frac{\partial}{\partial\gamma}[MR_\gamma(t)] \leq 0\ \forall t \in [0,\bar{t}]\ \forall\gamma$ and $t^*(\gamma) < \bar{t}\ \forall\gamma$. This is true for all higher cost functions such that there exists a unique equilibrium at every γ. For (2), by Lemma 6 there exist some $\underline{t}$ and $C_{\underline{t}}(t)$ such that $\forall t > \underline{t}\ \exists\gamma' \in (0,1)$ s.t. $MR_{\gamma'}(t) > MR_0(t)$ and $t^*(\gamma) > \underline{t}\ \forall\gamma$. In particular, for $t^*(0)\ \exists\gamma'$ s.t. $MR_{\gamma'}(t^*(0)) > MR_0(t^*(0))$. Therefore it must be that $t^*(\gamma') > t^*(0)$. This is true for all lower cost functions such that there exists a unique equilibrium at every γ. $\square$

The theorem states that, for cost functions such that there exists a unique equilibrium at all levels of competition, equilibrium private market research is decreasing in competition when accuracy costs are sufficiently high, and is maximized at some intermediate level of competition when accuracy costs are sufficiently low.

The second part of the result readily extends to the case of public market research. Suppose that after firms choose accuracy levels v_i and v_{-i}, both firms observe v_i and v_{-i} prior to choosing prices. The game is otherwise as in Section 2.2. Call this the overt game. In this setting, both accuracy and prices are strategic complements.[9] Thus, firms have weakly higher marginal returns to accuracy compared to the private market research game. However, in the monopoly case there is no strategic effect from increasing accuracy, so marginal returns are the same in both settings. Let $t^*_O(\gamma)$ denote market research in a symmetric equilibrium of the overt market research game. For any cost function such that there exists a unique symmetric equilibrium in both the private research game and the overt game at some γ, it must be that $t^*_O(\gamma) \geq t^*(\gamma)$. In the monopoly case ($\gamma = 0$), $t^*_O(0) = t^*(0)$. Furthermore,

[9]See Chapter 8 in Vives (2000) for a more thorough discussion.

returns to market research approach zero in both settings as competition approaches perfect competition: $\lim_{\gamma \to 1} t^*_O(\gamma) = \lim_{\gamma \to 1} t^*(\gamma) = 0$.

As in the private market research setting, in the overt game for any $\underline{t}$ one can find a cost function $C_{\underline{t}}(t)$ such that at every γ there exists a symmetric equilibrium in the overt game with $1 > t^*_O(\gamma) > \underline{t}$. Define $\{\hat{C}(t)\}^O_L$ in the overt game analogously to $\{\hat{C}(t)\}_L$ in the private market research game. Corollary 4 immediately follows.

Corollary 4. *$\exists \underline{t}, C_{\underline{t}}(t)$ such that for any cost function $C(t) \in \{C_{\underline{t}}(t)\}^O_L$, at every $\gamma \in [0, 1)$ there exists a unique symmetric equilibrium with market research $t^*_O(\gamma)$ s.t. $t^*_O(\gamma') > t^*_O(0) > \lim_{\gamma \to 1} t^*_O(\gamma)$ for some $\gamma' \in (0, 1)$.*

As in the private market research game, in the overt game when accuracy costs are sufficiently low, firms facing some intermediate level of competition invest more in market research than monopolistic firms.

2.4. Conclusion

This paper examines how differentiation affects equilibrium market research in a Bertrand duopoly. We conjecture that in symmetric Bertrand oligopolies with $n > 2$ firms, the results hold qualitatively, meaning there exist parameters such that firms with partially differentated goods invest more in market research than firms with completely differentiated goods.

We do not explicitly analyze consumer welfare across differentiation levels, as to do so would require finding equilibrium market research in closed form, but we can say something about it. Increased accuracy has competing effects on consumer welfare. When firms increase their accuracy, they condition their prices more on their signals and thus better align their prices with the state. This is partially beneficial for consumers, since fixing the average price, they would prefer to pay a high price when the state is high and a low price when the state is low, rather than the same price in all states. However, consumers also prefer for firms to have different prices from each other, as it allows them to substitute the cheaper good for the more expensive good. When firms increase their accuracy, their prices tend to be closer. This harms consumers. The net effect in our model is that consumer surplus decreases in the firms' accuracy.[10]

Fixing the accuracy of both firms, consumer welfare increases as goods become closer substitutes. However, as we have shown accuracy is sometimes non-monotonic in the level of differentiation. This highlights a challenge in regulating either market research or pricing behavior when market research is endogenous. For a given market research cost function, it may be that consumer welfare is sometimes higher when goods are less differentiated than when goods are more differentiated.

[10] See Proposition 6 in Vives (1984).

CHAPTER 3

Eating Your Cake and Selling it too: Adverse Selection in Technology Licensing

3.1. Introduction

Traditionally much of the analysis of marketing an invention has focused on the assumption that the technology-holder acts as a monopolist in marketing the invention. Unsurprisingly, a lot of the analysis of licensing technology has also focused on a monopoly inventor. Katz & Schapiro (1986) develops a model where a single inventor, who does not compete in the product market, licenses to firms who competes downstream, and determine the optimal strategy to license. Since then it has been noted that the presence of substitute technologies is a salient feature in R&D intensive industries. For example, Arora (1997) finds that in the chemical industry, leading firms compete with each other in selling polypropylene licenses. In an era of rapid technological growth and high-frequency R&D activity, it seems unreasonable to think that most inventions are unique and without substitutes. Hence, for most inventions, the useful lifetime of the invention is more accurately measured by the time it takes for a newer, better invention to come along, rather than simply by the legal patent length. Scotchmer et al (1998) takes this view, and defines this time as the "effective patent life"[1]. Mansfield (1984) finds from survey evidence that in some industries, 60% of the patents effectively terminate within 4 years, and Schankerman and Pakes (1986) find that European patents lose about 20% of their value each year. In this paper, I focus more

[1] Their focus is on how the breadth of a patent influences effective patent life.

on the implications this substitutability has on licensing, what this means for the inventing firm, and for a potential licensee. Perhaps it behooves us to see that in a bilateral trade framework, the inventor resembles a seller, the licensee resembles a buyer, and the effective life is a property of the product (the invention) about which the inventor may have better information than the licensee, provided that licensing periods cannot be arbitrarily small.

Before going into the main idea, we should justify the strategy of licensing to competitors in the first place. The traditional idea is that licensing creates dissipation in rents because of increased competition and significant transaction costs, and so the inventor can reap the most benefit from an invention by excluding competitors from using it. Explanations for licensing include a lower ability of the inventor to utilize the gains from the invention compared to rivals, and licensing as a means to set an owned technology as the industry standard. Arora and Fosfuri (2003) explain licensing through a mechanism that creates negative pecuniary externality on rival firms, and obtain that even though licensing is inefficient, in that it decreases industry profits, the patent-holder chooses to license because it gets a larger share of a smaller pie through licensing. This result stems from the "efficiency effect" (Tirole (1988)) which says there is a negative correlation between aggregate industry profits and the number of firms in the market. However, this presupposes strong Bertrand competition, and may not hold with substantial product differentiation. The ideas in what follows, are quite different from those in Arora and Fosfuri (2003) in that the focus is more on the inefficiencies of *failing* to license, rather than on the inefficient aspects of licensing. We create a simple model of differentiated Bertrand competition with a firm holding the rights to a valuable invention and a potential licensee, where after a round of licensing (or a lack of it), the two compete by choosing prices. In a rather Coasian setting, ignoring inefficient transaction costs, we show that successful licensing is more profitable for the inventor, but may be infeasible due to an asymmetric information problem and lack of a commitment device. The problem arises mainly because the inventor firm itself is a consumer of the invention, and as such may have better information regarding the usefulness of the invention. Hence, even though we will assume that the licensee can directly observe the quality of the technology, its true value is its quality relative to that of substitute technologies, and that may not be perfectly observable. We then extend the model to include a specialized R&D firm (outsourcing option) and see how this can affect the profitability of the inventor firm and also the aggregate industry profits.

Using a simple model, we illustrate a problem in licensing that an inventor firm faces when inventions are strong substitutes. This is an adverse selection problem, and stems from the fact that an inventor firm that is also a user of its own product; namely, the invention; always has an incentive to protect its most valuable inventions and license out the less valuable ones. One implication of the model in this paper is that firms can overcome the commitment problem by outsourcing R&D to an external partner. The intuition here is that while the development cost of an in-house R&D project is sunk/unobservable, trade with an external partner creates an endogenous cost of "bluffing" or selling an inferior product, thereby making it possible to credibly signal the quality of the technology.[2] Therefore, a firm planning to license a future invention would develop it by contracting with an external partner, and only develop in-house the technologies it plans to exclude rivals from using. Even in the absence of comparative cost advantages of specialized R&D firms, this suggests a possible incentive competitive firms may have to outsource R&D.

In section 3.2 we introduce the model and provide a numerical example to illustrate the key ideas of the paper. In section 3.3 we explain the asymmetric information problem

[2] Signaling quality by burning money.

stemming from firms conducting in-house R&D, and how that problem may be mitigated by outsourcing R&D activities. Section 3.4 concludes.

3.2. The Model

Two firms indexed by $i \in \{I, A\}$ engage in price competition with differentiated products. The demand firm i faces is given by $D_i(p_i, p_{-i})$, where $\frac{\partial D_i}{\partial p_i} < 0$, $\frac{\partial^2 D_i}{\partial p_i^2} \leq 0$, $\frac{\partial D_i}{\partial p_{-i}} > 0$ and $\left|\frac{\partial D_i}{\partial p_i}\right| > \left|\frac{\partial D_i}{\partial p_{-i}}\right|$, and p_i and p_{-i} represents prices chosen by firm i and firm $-i$, respectively. We assume that demand is invertible, and the inverse demand function firm i faces is given by $P_i(q_i, q_{-i})$, where q_i and q_{-i} are the quantities produced by firm i and firm $-i$, respectively.

Each firm i produces its product using technology c_i which is its constant marginal cost of production. For simplicity, we assume there are no fixed costs of production. There are three possible technologies that firms can use, which are $c_2 > c_1 > c_0$, where c_0 is the best technology and c_2 is the worst. Only firm I may access technology c_0, and the ex ante probability of firm I being able to access c_0 is $q \in [0, 1]$. Technology c_1 may be licensed at a fee L, and the licensing agreement takes place before firms choose prices.

Firm i's maximization problem is therefore:

$$\max_{p_i} (p_i - c_i) D_i(p_i, p_{-i})$$

The conditions on the demand functions guarantee that the maximands are strictly concave in p_i, therefore a unique solution exists. Let us denote firm i's optimal price as $p_i^B(c_i, c_{-i})$, where the first argument stands for the firm's own marginal cost and the second argument stands for its opponent's marginal cost. Let us denote by $D_i^M(p_i)$ the demand firm

i faces when firm $-i$ chooses to not produce any output (monopoly demand).[3]Let us denote by $p_i^M(c_i)$ the optimal price in the monopoly case. Similarly denote profits by $\pi_i^B(c_i, c_{-i})$ and $\pi_i^M(c_i)$.

3.2.1. Illustrative Numerical Example

Let us consider a pricing game where each of two firms, indexed by $i \in \{I, A\}$, produce differentiated products and face demand $D_i(p_i, p_{-i}) = 10 - 2p_i + p_{-i}$, where p_i is the price of the firm's product and p_{-i} is the price of its competitor's product. We can also derive from this demand specification the monopoly demand firm i would face if firm $-i$ chooses to not enter the market, which is $D_i^M(p_i) = 15 - \frac{3}{2}p_i$. There are three possible technologies (modeled as marginal costs of production) in this world, which are $c_2 = \frac{19}{2}$, $c_1 = 8$, and $c_0 = 0$. There are no fixed costs of production.

Firm I currently possesses technology c_1, whereas firm A possesses c_2. It is easy to see that if both firms engage in price competition, the equilibrium price for firm A will be $p_A = \frac{142}{15} < \frac{19}{2} = c_2$. Which means firm A would earn negative profits from competing, so it would choose not to enter the market. Firm I can therefore enjoy monopoly profits $\pi_I^M(c_1) = \frac{3}{2}$.

However, if firm I instead chooses to license its technology c_1 to firm A at a licencing fee L and induce firm A to enter the market, each firm will make a profit of $\frac{8}{9}$, so the joint competitive profit would be $\pi_I^B + \pi_A^B = \frac{16}{9} > \frac{3}{2} = \pi_I^M$. So it is more profitable for firm I to license by choosing some L satisfying $\frac{11}{18} \leq L \leq \frac{8}{9}$ than to enjoy the monopoly profits. Let

[3]This is derived from $D_i(p_i, p_{-i})$ by first deriving the inverse demand function $P_i(q_i, q_{-i})$ for firm i, deriving $P_i(q_i, 0)$ by plugging in $q_{-i} = 0$, and inverting $P_i(q_i, 0)$ again to get $D_i^M(p_i)$.

$\underline{\text{L}} = \frac{11}{18} = \pi_I^M - \pi_I^B$ denote the minimum licencing fee firm I would be willing to accept when it knows c_1 is the best technology it can use.

Now suppose firm I knows whether it has access to technology c_0, whereas firm A believes that with probability $q = \frac{3}{5}$, firm I has access to technology c_0. If firm I uses technology c_0 and firm A only has c_1, then the equilibrium price in that case for firm A will be $p_A = \frac{38}{5} < 8 = c_1$, which means firm A in that case will not enter the market and make a profit of 0. Given this, the maximum licensing fee firm A would be willing to pay is $\bar{L} = \frac{2}{5}(\frac{8}{9}) + \frac{3}{5}(0) = \frac{16}{45}$. Notice that $\bar{L} = \frac{16}{45} < \frac{11}{18} = \underline{\text{L}}$, which means in this case there is no licensing fee which firm A is willing to pay and firm I is willing to accept when c_1 is the best technology firm I can use. This illustrates a breakdown in the market for technology licensing very similar to the lemons problem in Akerlof (1970).

Instead, if both firms believe with some probability $q \in [0, 1]$ that firm I has access to c_0, then licensing is optimal if the following holds:

$$q\left(\frac{75}{2}\right) + (1-q)\left(\frac{8}{9}\right) + L \geq q\left(\frac{75}{2}\right) + (1-q)\left(\frac{3}{2}\right)$$

This tells us that the minimum licensing fee firm I would accept is $\underline{\text{L}} = (1-q)\,\frac{11}{18}$. Notice now that $\bar{L} = (1-q)\,\frac{8}{9} > (1-q)\,\frac{11}{18} = \underline{\text{L}}$, which means there are licensing fees that are mutually agreeable for any value of q.

Let us now consider another situation, where firm I knows if it has access to c_0 , while firm A believes with probability $q \in [0, 1]$ that firm I has access to c_0. Both firms have technology $c_2 = \frac{19}{2}$, and firm I may choose to acquire technology $c_1 = 8$ at a commonly known price $W = 1$. Now in the case where firm I has access to c_0, it knows its market profit will be

$\pi_I^M(c_0) = \frac{75}{2}$ regardless of whether it licenses c_1 to firm A. Given $\bar{L} = (1-q)\,\frac{8}{9} < W$, firm I will not acquire c_1 in the case it has access to c_0.

Because of this, if firm A observes that firm I chose to acquire c_1 at price $W = 1$, it will infer that firm I does not have access to c_0, and would be willing to pay a licensing fee up to $\frac{8}{9}$. When it does not have access to c_0, it is optimal for firm I to acquire c_1 and license it if:

$$\frac{8}{9} + L - 1 \geq \frac{3}{2} - 1$$

Which means for any $L \geq \frac{11}{18}$, it is profitable to acquire c_1 and license it. For any $q \in [0,1]$, there are licensing fees L satisfying $\frac{11}{18} \leq L \leq \frac{8}{9}$, that are agreeable to both firms I and A. So by acquiring c_1 at a high enough price firm I can credibly signal that it does not have access to c_0, which is another way of mitigating the asymmetric information problem.

3.3. Asymmetric Information in Licensing

Here we consider the possible differences in information structures that may arise from firm I conducting its R&D activities in-house as opposed to outsourcing them. Then we consider the impacts different information structures may have on licensing outcomes and what effect that has on profits of the two firms.

To concisely reflect on the problem of asymmetric information in this licensing setting, we make a few simplifying assumptions that considerably reduces the complexity of analysis.

Assumption 7. $p_i^B(c_2, c_1) < c_2$ *and* $p_i^B(c_1, c_0) < c_1$

This assumption states that each of the three technologies is a "drastic" improvement over its next-best technology. This means that a firm would make negative profits from competing against an opponent that has a superior technology.

Assumption 8. $\pi_I^B(c_1, c_1) + \pi_A^B(c_1, c_1) > \pi_I^M(c_1)$

This assumption is about the product variety the two firms bring to the market, and this assumption says that when c_1 is the best technology firm I possesses, by licensing this technology, the profitability firm I can achieve by utilizing the product variety of the two differentiated goods is greater than the profitability resulting from monopoly market power that firm I can achieve.

Assumption 8 guarantees that when firm I does not have access to c_0, the profit-maximizing strategy is to license c_1 to firm A at some licensing fee L satisfying $\pi_I^M(c_1) - \pi_I^B(c_1, c_1) \leq L \leq \pi_A^B(c_1, c_1)$. This is because $\pi_A^B(c_1, c_1)$ is the maximum fee firm A would ever be willing to pay,[4] and $\pi_I^M(c_1) - \pi_I^B(c_1, c_1)$ is the loss in profits firm I faces from inducing competition. The exact value of L would depend on the relative bargaining power of the two firms.

3.3.1. In-house R&D

In this section we highlight the problem of asymmetric information a firm may face when it conducts its R&D in-house. For this section, suppose that firm I currently has technology c_1 whereas firm A only has technology c_2, and both of these are commonly known between the two firms. However, firm I also knows if it can also access technology c_0,[5] whereas firm A only knows the prior probability q of firm I having access to technology c_0.

Proposition 12. *If* $q > \frac{\pi_I^B(c_1,c_1)+\pi_A^B(c_1,c_1)-\pi_I^M(c_1)}{\pi_A^B(c_1,c_1)}$*, then there is no licensing fee* $L > 0$ *which both firms can agree on.*

[4]This depends on the value of q, for example firm A is only willing to pay $\pi_A^B(c_1, c_1)$ when $q = 0$.
[5]One may think of this as firm I's insider information about whether the new technology is in the pipeline and will soon be available. Firm I possesses this insider information when it conducts its R&D in-house.

Proof. By assumption 7, $p_i^B(c_2, c_1) < c_2$, which means if licensing of technology c_1 does not occur, firm A's payoff will be 0, as it will choose to not enter the market. In case firm A obtains a license to use technology c_1, with probability q firm I can access technology c_0 and the competitive equilibrium price firm A will be able to charge is $p_A^B(c_1, c_0)$, which, by assumption 1, is less than c_1. This means in that case firm A will decide not to produce and earn a market profit of 0. With probability $1-q$ firm I will have technology c_1, and firm A's profit will be $\pi_A^B(c_1, c_1)$. So following licensing of technology c_1 at fee L, firm A's expected payoff is $q * 0 + (1-q)\pi_A^B(c_1, c_1) - L = (1-q)\pi_A^B(c_1, c_1) - L$. This must be weakly greater than 0, firm A's payoff from not licensing. So to satisfy firm A's incentives, the licensing fee L must satisfy $L \leq (1-q)\pi_A^B(c_1, c_1)$.

Now consider firm I's situation. Consider first the case when c_1 is the best technology it can access. By not licensing, firm I can achieve monopoly profits $\pi_I^M(c_1)$, which follows from assumption 7. By licensing technology c_1 at a fee L, firm I can make a total payoff of $\pi_I^B(c_1, c_1) + L$. In order for licensing to be optimal, this payoff must be weakly higher than the monopoly profit firm I could make, meaning $\pi_I^B(c_1, c_1) + L \geq \pi_I^M(c_1)$, or $L \geq \pi_I^M(c_1) - \pi_I^B(c_1, c_1)$.

In case firm I has access to technology c_0, it would be willing to charge a fee lower than $\pi_I^M(c_1) - \pi_I^B(c_1, c_1)$, however, any fee $L < \pi_I^M(c_1) - \pi_I^B(c_1, c_1)$ reveals to firm A that firm I has access to technology c_0, which means licensing will yield a payoff of 0 for firm A, which means firm A will not pay any positive licensing fee L. This means there is no licensing fee L satisfying $0 < L < \pi_I^M(c_1) - \pi_I^B(c_1, c_1)$.

Now consider fees L satisfying $\pi_I^M(c_1) - \pi_I^B(c_1, c_1) \leq L \leq (1-q)\pi_A^B(c_1, c_1)$. When $q > \frac{\pi_I^B(c_1,c_1)+\pi_A^B(c_1,c_1)-\pi_I^M(c_1)}{\pi_A^B(c_1,c_1)}$, this implies $\pi_I^M(c_1) - \pi_I^B(c_1, c_1) > (1-q)\pi_A^B(c_1, c_1)$, which means when $q > \frac{\pi_I^B(c_1,c_1)+\pi_A^B(c_1,c_1)-\pi_I^M(c_1)}{\pi_A^B(c_1,c_1)}$ there exists no L which satisfies both firms' incentives. $\square$

This illustrates the problem with asymmetric information in technology licensing. It is in part a problem of firm I not being able to credibly reveal whether it has access to the superior technology c_0. It is also a result of firm I not being able to commit to not using c_0 in case it has access.[6]When the prior probability of firm I having technology c_0 is high enough, the asymmetric information problem is strong enough to deter any possible licensing agreement.

3.3.2. Outsourced R&D

Now consider the situation when firm I outsources its R&D activity to a third-party firm R, which is a research firm and does not compete with either of the two firms in the product market. Here we will consider two scenarios that illustrate mechanisms firm I may utilize to alleviate the asymmetric information problem using firm R.

3.3.2.1. Symmetric Information About c_1. In this subsection, suppose that all of firm I's R&D activities are undertaken by firm R, and only firm R knows whether firm I will have access to technology c_0 or not. Therefore both firms I and A only knows that with probability q that firm I's marginal cost will be c_0, and with probability $1-q$ the marginal cost will be c_1.

Proposition 13. *When firms I and A have symmetric information about technology c_0, licensing always takes place.*

[6]Formal agreements to not using technology c_0 amounts to price-fixing.

Proof. As before, the maximum licensing fee firm A would be willing to pay is $(1-q)\pi_A^B(c_1, c_1)$. If firm I chooses to license technology c_1 at fee L, its expected payoff is $q\pi_I^M(c_0)+(1-q)\pi_I^B(c_1, c_1)+L$, otherwise its expected payoff is $q\pi_I^M(c_0)+(1-q)\pi_I^M(c_1)$. So in order for licensing to be optimal, the fee L must satisfy

$$q\pi_I^M(c_0)+(1-q)\pi_I^B(c_1, c_1)+L \geq q\pi_I^M(c_0)+(1-q)\pi_I^M(c_1)$$

This simplifies to

$$L \geq (1-q)\pi_I^M(c_1)-(1-q)\pi_I^B(c_1, c_1)$$

By assumption 8, $(1-q)\pi_A^B(c_1, c_1) > (1-q)\pi_I^M(c_1)-(1-q)\pi_I^B(c_1, c_1)$, so for any value of q, there are always licensing fees L that firm A is willing to pay and firm I is willing to accept. Which means under symmetric information, licensing always takes place. □

3.3.2.2. Public Procurement of Technology. In this subsection, suppose that firm R develops the technology c_1 at production cost F, and both firms I and A initially possess technology c_2 only. As in the previous subsection, firm I may access technology c_0 with the prior probability q. Importantly, we assume that firm R has an exclusive dealing contract with firm I in selling technology c_1, so firm R cannot sell c_1 to firm A. Firm I may procure technology c_1 from firm R at a publicly observed price W. For simplicity, let us assume here that firm I has all the bargaining power in choosing the licensing fee L.

Proposition 14. *If $W > \pi_A^B(c_1, c_1)$, whenever firm I acquires technology c_1 from firm R, it successfully licenses the technology to firm A.*

Proof. Let us consider the situation when firm I publicly acquires technology c_1 at a price $W > \pi_A^B(c_1, c_1)$. The maximum licensing fee L that firm A would ever be willing to pay is $\pi_A^B(c_1, c_1)$, so in the case where firm I has access to c_0, it makes a loss from acquiring technology c_1 because it is always the case that $L < W$, and firm I's market profit is $\pi_I^M(c_0)$ regardless of W and L. So after observing a price $W > \pi_A^B(c_1, c_1)$, firm A believes with probability 1 that firm I does not have access to technology c_0. Therefore it is willing to pay $L = \pi_A^B(c_1, c_1)$.

It follows from the previous argument that firm I would only obtain c_1 if it does not have access to c_0. For acquiring c_1 to be worthwhile, it must be the case that $\pi_I^B(c_1, c_1) + \pi_A^B(c_1, c_1) - W \geq \pi_I^B(c_2, c_2)$, which means $W \leq \pi_I^B(c_1, c_1) + \pi_A^B(c_1, c_1) - \pi_I^B(c_2, c_2)$. So firm I would choose to obtain c_1 and license it whenever W satisfies the following condition:

$$\pi_A^B(c_1, c_1) < W \leq \pi_I^B(c_1, c_1) + \pi_A^B(c_1, c_1) - \pi_I^B(c_2, c_2)$$

If firm I cannot choose a W that satisfies the above condition, then firm I would choose to not acquire c_1. Whenever the above condition is satisfied, firm I chooses to acquire technology c_1 from firm R, and successfully licenses the technology to firm A. $\square$

3.4. Conclusion

The simple model in this paper illustrates an adverse selection problem a firm faces in licensing its technologies when it conducts its R&D in-house. The problem is well-understood in information economics, where inefficient outcomes arise because of a friction caused by asymmetric information. We proposed two ways in which outsourced R&D development might mitigate this problem. One of these solves the problem by creating a symmetric information structure, and the other way allows the licensor firm to signal the value of the technology by credibly investing enough to acquire it (burning money).

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APPENDIX A

Omitted Proofs: Chapter 1

A.1. Proof of Lemma 1

Given an allocation rule $e(\theta)$, define $e(\hat{\theta}|\theta)$ by the equation $e(\hat{\theta}|\theta) + \theta = e(\hat{\theta}) + \hat{\theta}$; that is, it is the level of effort an agent of type θ has to exert in order to mimic type $\hat{\theta}$. Therefore, $e^{'}(\hat{\theta}|\theta) = e^{'}(\hat{\theta}) + 1$, where the derivative on LHS is taken with respect to $\hat{\theta}$. For an agent of type θ, his payoff if he mimics type $\hat{\theta}$ is:

$$U(\hat{\theta}|\theta) = w(\hat{\theta}) - C\left(e(\hat{\theta}|\theta)\right);$$

where $C(\cdot)$ is the cost of effort function. Therefore,

$$U^{'}(\hat{\theta}|\theta) = w^{'}(\hat{\theta}) - C^{'}\left(e(\hat{\theta}|\theta)\right)e^{'}(\hat{\theta}|\theta) = w^{'}(\hat{\theta}) - C^{'}\left(e(\hat{\theta}|\theta)\right)\left(e^{'}(\hat{\theta}) + 1\right)$$

Local incentive compatibility[1] requires that

$$U^{'}(\hat{\theta}|\theta)\Big|_{\hat{\theta}=\theta} = w^{'}(\theta) - C^{'}\left(e(\theta)\right)e^{'}(\theta) - C^{'}\left(e(\theta)\right) = 0$$

So, $w^{'}(\theta) = C^{'}(e(\theta))e^{'}(\theta) + C^{'}(e(\theta))$

Now, as $U(\theta) = w(\theta) - C(e(\theta))$, $U^{'}(\theta) = w^{'}(\theta) - C^{'}(e(\theta))e^{'}(\theta) = C^{'}(e(\theta))$

[1]Because the agent's utility function $U(e, \theta)$ satisfies the single-crossing condition, local IC implies global IC.

Because we are using the quadratic cost of effort function $C(e) = {}^{e^2}\!/_2$, $C^{'}(e(\theta)) = e(\theta)$, therefore $U^{'}(\theta) = e(\theta)$, which gives us the envelope condition

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} e(t)dt$$

For the monotonicity part, let $\theta_1 > \theta_2$ be arbitrary. IC requires

$$U(\theta_2|\theta_1) \leq U(\theta_1)$$

Now, when IC is satisfied,

$$\begin{aligned}
U(\theta_2|\theta_1) &= w(\theta_2) - \frac{e(\theta_2|\theta_1)^2}{2} \\
&= w(\theta_2) - \frac{1}{2}\left[e(\theta_2) + (\theta_2 - \theta_1)\right]^2 \\
&= w(\theta_2) - \frac{1}{2}e(\theta_2)^2 + e(\theta_2)(\theta_1 - \theta_2) - \frac{1}{2}(\theta_1 - \theta_2)^2 \\
&= U(\theta_2) + (\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_1) \\
\therefore U(\theta_1) - U(\theta_2) &\geq (\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right]
\end{aligned}$$

Similarly, we need $U(\theta_1|\theta_2) \leq U(\theta_2)$. Now,

$$\begin{aligned} U(\theta_1|\theta_2) &= w(\theta_1) - \frac{e(\theta_1|\theta_2)^2}{2} \\ &= w(\theta_1) - \frac{1}{2}\left[e(\theta_1) + (\theta_1 - \theta_2)\right]^2 \\ &= w(\theta_1) - \frac{1}{2}e(\theta_1)^2 - e(\theta_1)(\theta_1 - \theta_2) - \frac{1}{2}(\theta_1 - \theta_2)^2 \\ &= U(\theta_1) - (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_2) \\ \therefore U(\theta_1) - U(\theta_2) &\leq (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right] \end{aligned}$$

Combining the two, we get:

$$(\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_1) - U(\theta_2) \leq (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right]$$

Dividing by $\theta_1 - \theta_2$ and rearranging gives us $e(\theta_2) + \theta_2 \leq e(\theta_1) + \theta_1$, which establishes the monotonicity requirement.

A.2. Proof of Proposition 1

Using integration by parts, the monopolist principal's maximization problem can be written as

$$\max_{e(\cdot|x)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x) + \theta - \frac{e(\theta|x)^2}{2} - \frac{1 - F(\theta|x)}{f(\theta|x)} e(\theta|x) dq \right\} dF(\theta|x)$$

Let $H(\theta|x) = \frac{f(\theta|x)}{1-F(\theta|x)}$, then by taking the first order condition, the principals maximization problem can be pointwise solved and the optimal effort allocation $e^M(\theta|x)$ can be written out as

$$e^M(\theta|x) = 1 - \frac{1}{H(\theta|x)}$$

By assumption 1, for any pair of signals $x_1 > x_2$, $F(\cdot|x_1)$ stochastically dominates $F(\cdot|x_2)$ in the likelihood-ratio sense (LRD). LRD implies hazard-rate dominance (HRD), therefore $F(\cdot|x_1)$ hazard-rate dominates $F(\cdot|x_2)$, that is, $H(\theta|x_1) < H(\theta|x_2)$ for any θ. Therefore, $1 - \frac{1}{H(\theta|x_1)} < 1 - \frac{1}{H(\theta|x_2)}$, that is, $e^M(\theta|x)$ is decreasing in x for any θ.

The type-θ agent's payoff is simply

$$U(\theta|x_1) = \int_{\underline{\theta}}^{\theta} e^M(t|x_1)dt < \int_{\underline{\theta}}^{\theta} e^M(t|x_2)dt = U(\theta|x_2)$$

This proves the second part of proposition 1.

Finally, consider the principal's expected payoffs under the signals x_1 and x_2. LRD implies first-order stochastic dominance (FOSD), therefore $F(\cdot|x_1)$ FOSD $F(\cdot|x_2)$.

$$\pi(\theta|x_1) = \max_{e(\theta|x_1)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x_1) + \theta - \frac{e(\theta|x_1)^2}{2} - \frac{1}{H(\theta|x_1)} e(\theta|x_1) \right\} dF(\theta|x_1)$$

Consider the Principal's payoff even if she (suboptimally) choses the schedule $e^M(\theta|x_2)$ after receiving signal x_1. Her payoff in this case is:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2) \right\} dF(\theta|x_1)$$

Now, because $e^M(\theta|x_2)$ is increasing in θ, and $e^M(\theta|x_2) < 1$ for all θ other than $\bar{\theta}$,

$$\frac{d}{d\theta}\left(e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2} \right) = \left(1 - e^M(\theta|x_2)\right) \frac{d}{d\theta}\left(e^M(\theta|x_2)\right) > 0$$

Hence, $e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2}$ is an increasing function of θ. Therefore,

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2} \right\} dF(\theta|x_1) > \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2} \right\} dF(\theta|x_2)$$

It can be showed using integration by parts that

$$E\left[\theta|x_1\right] = \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_1) d\theta = \theta F(\theta|x_1)|_{\underline{\theta}}^{\bar{\theta}} = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_1) d\theta$$

And similarly, $E\left[\theta|x_2\right] = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_2) d\theta$

So, $E\left[\theta|x_1\right] - E\left[\theta|x_2\right] = \left(\bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_1) d\theta \right) - \left(\bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_2) d\theta \right) = \left(\int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_2) d\theta \right) - \left(\int_{\underline{\theta}}^{\bar{\theta}} F(\theta|x_1) d\theta \right) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ F(\theta|x_2) - F(\theta|x_1) \right\} d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ (1 - F(\theta|x_1)) - (1 - F(\theta|x_2)) \right\} d\theta$

Because $F(\theta|x_1)$ FOSD $F(\theta|x_2)$, $E\left[\theta|x_1\right] > E\left[\theta|x_2\right]$

$$\begin{aligned}
\text{So, } \int_{\underline{\theta}}^{\bar{\theta}} \{(1-F(\theta|x_1)) - (1-F(\theta|x_2))\}\, d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\left(1-e^M(\theta|x_2)\right)\right\} d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \left\{(F(\theta|x_2) - F(\theta|x_1)) - ((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0 \\
E[\theta|x_1] - E[\theta|x_2] - \int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_1) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_2) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0
\end{aligned}$$

$$\begin{aligned}
\text{therefore,} \quad & \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_1) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1-F(\theta|x_1))\, e^M(\theta|x_2)\right\} d\theta \\
> \quad & \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_2) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1-F(\theta|x_2))\, e^M(\theta|x_2)\right\} d\theta
\end{aligned}$$

which means,

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{\theta - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2)\right\} dF(\theta|x_1) > \int_{\underline{\theta}}^{\bar{\theta}} \left\{\theta - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2)\right\} dF(\theta|x_2)$$

Combining the two inequalities, we can say that

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2) \right\} dF(\theta|x_1)$$

$$> \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2) \right\} dF(\theta|x_2)$$

So even if after receiving signal x_1, the principal suboptimally chooses $e^M(\theta|x_2)$, her expected payoff is higher than $\pi(\theta|x_2)$. This means,

$$\begin{aligned} \pi(x_1) &= \max_{e(\theta|x_1)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x_1) + \theta - \frac{e(\theta|x_1)^2}{2} - \frac{1}{H(\theta|x_1)} e(\theta|x_1) \right\} dF(\theta|x_1) \\ &\geq \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2) \right\} dF(\theta|x_1) \\ &> \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2) \right\} dF(\theta|x_2) \quad = \quad \pi(x_2) \end{aligned}$$

That is, $\pi(x_1) > \pi(x_2)$, which concludes the proof.

A.3. Proof of Proposition 3

Take an arbitrary $\theta > \underline{\theta}$. For the ease of exposition, we shall henceforth omit the argument θ from this proof and the proof of existence of equilibrium, and just write $\hat{R}(x, y)$ as the actual value for winning the agent of type θ given signals (x, y).

Suppose P_{-i} is playing the strategy $U^*_{-i}(\cdot)$ as described in proposition 3. P_i's expected payoff from bidding u after receiving signal x is therefore

$$\pi_i(x;u) = \int_{-\infty}^{\phi_{-i}(u)} \left\{ \hat{R}_i(x,y) - u \right\} dG_{-i}(y|x)$$

Differentiating with respect to u, we get

$$\frac{\partial \pi_i(x;u)}{\partial u} = \left[\left\{ \hat{R}(x, \phi_{-i}(u)) - u \right\} g_{-i}\left((\phi_{-i}(u)|x) \right] \phi'_{-i}(u) - G_{-i}\left(\phi_{-i}(u)|x \right)$$

The first order condition is derived by equating this derivative to 0. Therefore, in equilibrium we have

$$\frac{1}{\phi'_{-i}(u)} = \left\{ \hat{R}_i(x, \phi_{-i}(u)) - u \right\} \frac{g_{-i}\left(\phi_{-i}(u)|x \right)}{G_{-i}\left(\phi_{-i}(u)|x \right)}$$

Because in equilibrium $x = \phi_i(u)$, and analogously for signal y of P_{-i}, $y = \phi_{-i}(u)$, we can write down the first order conditions for each principal as

$$\frac{1}{\phi'_{-i}(u)} = \left\{ \hat{R}_i(\phi_i(u), \phi_{-i}(u)) - u \right\} \frac{g_{-i}\left(\phi_{-i}(u)|\phi_i(u) \right)}{G_{-i}\left(\phi_{-i}(u)|\phi_i(u) \right)} \textit{for i=1,2}$$

These, together with the common boundary conditions

$$\lim_{x \to -\infty} U_i^*(\theta|x) = \underline{\mathrm{U}}^*(\theta) = \lim_{y \to -\infty} U_{-i}^*(\theta|y)$$

characterizes the equilibrium bids. That the equilibrium bids characterized in the exposition of proposition 3 form a solution to this system is the same as in Milgrom & Weber (1982).

Using the aforementioned first order conditions, one can write

$$\frac{\phi'_{-i}(u)}{\phi'_i(u)} = \frac{\left\{\hat{R}_{-i}(\phi_{-i}(u),\phi_i(u)) - u\right\}\frac{g_i(\phi_i(u)|\phi_{-i}(u))}{G_i(\phi_i(u)|\phi_{-i}(u))}}{\left\{\hat{R}_i(\phi_i(u),\phi_{-i}(u)) - u\right\}\frac{g_{-i}(\phi_{-i}(u)|\phi_i(u))}{G_{-i}(\phi_{-i}(u)|\phi_i(u))}}$$
$$= \left\{\frac{\hat{R}_{-i}(\phi_{-i}(u),\phi_i(u)) - u}{\hat{R}_i(\phi_i(u),\phi_{-i}(u)) - u}\right\}\frac{s_i(\phi_i(u))}{s_{-i}(\phi_i(u))}\frac{G_{-i}\left(\phi_{-i}(u)|\phi_i(u)\right)}{G_i\left(\phi_i(u)|\phi_{-i}(u)\right)}$$

Where the second line follows using Bayes' rule.

Now, by definition of the tying function,

$$Q_i(\phi_i(u)) = \phi_{-i}(u)$$

Taking derivative with respect to u and using the chain rule, we get

$$Q'_i(\phi_i(u)) = \frac{\phi'_{-i}(u)}{\phi'_i(u)}$$

Which then means

$$Q'_i(x) = \left\{\frac{\hat{R}_{-i}(\phi_{-i}(u),\phi_i(u)) - u}{\hat{R}_i(\phi_i(u),\phi_{-i}(u)) - u}\right\}\frac{s_i(\phi_i(u))}{s_{-i}(\phi_i(u))}\frac{G-_i\left(\phi_{-i}(u)|\phi_i(u)\right)}{G_i\left(\phi_i(u)|\phi_{-i}(u)\right)}$$
$$\text{or, } \frac{dQ_i(x)}{dx} = \left\{\frac{\hat{R}_{-i}\left(\theta|Q_i(x),x\right) - U^*_{-i}(\theta|Q_i(x))}{\hat{R}_i\left(\theta|x,Q_i(x)\right) - U^*_i(\theta|x)}\right\}\frac{s_i(x)}{s_{-i}(Q(x))}\frac{G_{-i}(Q_i(x)|x)}{G_i(x|Q_i(x))}$$

Now going back to the first order conditions, and rewriting them in terms of bids instead of inverse bids, we get

$$U^{*'}_i(x) = \left[\hat{R}_i(x,Q_i(x)) - U^*_i(x)\right]\frac{g_{-i}(Q_i(x)|x)}{G_{-i}(Q_i(x)|x)}$$

One can check that the equilibrium bidding strategy $U_i^*(x)$ in the exposition of proposition 3 satisfies this differential equation. We can also show that for the symmetric case, that is, when $S_i(\cdot|\theta) \equiv S_{-i}(\cdot|\theta)$, the equilibrium bidding strategy is unique up to the choice of the lowest contract. Suppose, for a contradiction, that $U_i(\cdot)$ and $\hat{U}_i(\cdot)$ are both solutions to this equation, and for some x, $U_i(x) < \hat{U}_i(x)$. Because in the symmetric case, $Q_i(x) = x$, this implies, from the differential equation, that $U_i'(x) > \hat{U}_i'(x)$, which means that they cannot both be solutions to the first order condition. Hence, the equilibrium bidding strategies in the exposition of proposition 3 form the unique regular equilibrium of the bidding game for the agent of type θ, taking $\hat{R}_i(\theta|x, y)$ as given.

A.4. Proof of Proposition 4

In this equilibrium both principals get a payoff of 0. Given P_{-i} is offering the first-best contract for all types and giving the agent all the surplus, any contract offered by P_i that is distorted downwards will be rejected by the agent, hence P_i cannot do better by offering a distorted contract. Offering a contract that distorts upwards from the first-best contract reduces total surplus, and incentive compatibility requires that the agent has to be offered more surplus than under P_{-i}'s contract. In this case the agent will accept P_i's contract but because total surplus decreases and the agent's rent increases, this means P_i is strictly worse off, and hence no deviation from offering the first-best contract is strictly beneficial for P_i. This proves the existence of the price war equilibrium.

A.5. Proof of Proposition 5

Let us first denote the bidding strategies in the symmetric equilibrium (where $\alpha_1 = \alpha_2 = \alpha_S$). Let's say for the particular θ, $U^S(\cdot)$ is the symmetric bidding function, which maps the principal's signal to the bid. We proceed by analyzing how the best reaction functions change from the symmetric setting as one principal (P_1) gets a signal with a higher accuracy level $\alpha_1 > \alpha_S$.

When P_2 has accuracy level α_S, and bidding with the symmetric equilibrium strategy $U^S(\cdot)$, P_1's maximization problem is the same when she has accuracy $\alpha_1 > \alpha_S$ as opposed to α_S as in both cases the payoff function to be maximized is $\pi_1(x, u)$. By lemma 3, we can say that against the symmetric bidding strategy by P_2, P_1's expected payoff will be higher under accuracy α_1 compared to α_S[2]. However, because we start with a common prior distribution of the agent's type, $F(\cdot)$ is the same no matter the accuracy levels of the signals. Hence, taking the expectation over all realizations of signals, the expected posterior for any α_i must equal the common prior for both principals. That is,

$$\int_{-\infty}^{\infty} f_i(\theta|x) dS_i^{\alpha_i}(x) = f(\theta)$$

Now, because the less informed principal's accuracy is the same as in the symmetric case, her signal distribution conditional on any type is also the same. That is, $s_2(y|\theta)$ is the same in both cases, for any y and θ.

Which means that for P_1, ex ante the distribution of her opponent's signal is the same under both cases. That is,

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \int_{-\infty}^{\infty} g_2(y|x) dS_1^{\alpha_1}(x|\theta) \right\} dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} s_2(y|\theta) dF(\theta) = s_2(y)$$

Now we analyze the bidding behavior of the less informed principal. Consider P_2's best response, after seeing signal y. Suppose that she is facing the same bids from her opponent

[2]This is presented as Fact1 in Persico (2000).

as in the symmetric case, that is, $U_1(\theta|\cdot) = U^S(\theta|\cdot)$. In the asymmetric case, after received the realized signal y, the less informed principal's calculation of the interim expected value conditional on the opponent's signal being less than or equal to y, is

$$V_2^D(\theta|y) = \int_{-\infty}^{y} \hat{R}_2(\theta|y,x)dG_1^{\alpha_1}(x|y)$$

Because in the asymmetric case the opponent gets more accurate signals, and calculating $\hat{R}_2(\theta|y,x)$ involves putting probabilities over $[\theta, \bar{\theta}]$, and $V_2^D(\theta|y)$ is the expected value of $\hat{R}_2(\theta|y,x)$ taken only over signals of the opponent smaller than y, in calculating $V_2^D(\theta|y)$ the less informed principal puts higher probabilities on lower values of the agent's type compared to the symmetric case. Therefore, using the same calculation as in the proof of part 3 of proposition 1, we get

$$V_2^D(\theta|y) < V_2^S(\theta|y)$$

Where $V_2^S(\theta|y)$ is P_2's expected value under the symmetric information structure.

Because this is true for all θ, it must be that when the opponent plays the symmetric strategy $U^S(\theta|\cdot)$, in any regular equilibrium, for any type $\theta > \underline{\theta}$, based on the bidding strategies formulated in proposition 3, P_2's best response is to bid lower in the asymmetric case. That is, for any y, $U_2^D(\theta|y) < U^S(\theta|y)$.

Now from the better informed principal's perspective, the ex ante distribution of her opponent's signal, $s_2(\cdot)$, is the same as in the symmetric case. By lemma 2, she must have an ex ante higher payoff in the asymmetric case, which can only happen if either her ex ante expected value conditional on winning is higher, that is, $\int_{\underline{\theta}}^{\bar{\theta}} V_1^D(\theta)dF(\theta) > \int_{\underline{\theta}}^{\bar{\theta}} V_1^S(\theta)dF(\theta)$, or her ex ante expected bid is lower, that is, $\int_{\underline{\theta}}^{\bar{\theta}} U_1^D(\theta)dF(\theta) < \int_{\underline{\theta}}^{\bar{\theta}} U_1^S(\theta)dF(\theta)$; or both.

Suppose towards a contradiction that her ex ante expected bid is higher in the asymmetric case, so $\int_{\underline{\theta}}^{\bar{\theta}} U_1^D(\theta)dF(\theta) > \int_{\underline{\theta}}^{\bar{\theta}} U_1^S(\theta)dF(\theta)$. Then her ex ante expected value must be higher, so at least for some signal realizations her interim value must be higher. Because under the more accurate signal structure, higher signal realizations are more correlated with higher types of the agent, this means that these signal realizations are the highest possible realizations. So we can find some x such that $\int_{x}^{\infty} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} V_1^D(\theta|z)dF_1^{\alpha_1}(\theta|z) \right\} dS_1^{\alpha_1}(z) > \int_{x}^{\infty} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} V_1^S(\theta|z)dF_1^{\alpha_S}(\theta|z) \right\} dS_1^{\alpha_S}(z)$. In order for her ex ante expected value to be higher, these signal realizations in $[x, \infty)$ must have a sufficiently higher probability under X^{α_1} compared to X^{α_S}. However, because X^{α_1} is more accurate, it is more correlated with θ, and a higher probability of realizations $[x, \infty)$ implies ex ante some subset of of highest types $[\theta, \bar{\theta}]$ has greater probability under X^{α_1} compared to X^{α_S}. This violates the fact that the type distribution $F(\cdot)$ has a common prior distribution, as we must have $\int_{-\infty}^{\infty} f_i(\theta|x)dS_i^{\alpha_i}(x) = f(\theta)$, for all θ, as shown before. Hence the better informed principal's ex ante expected bid cannot be higher under the more accurate signal X^{α_1}.

For any type of the agent $\theta > \underline{\theta}$, because the less informed principal's reaction function moves towards lower bids when $\alpha_1 > \alpha_S$, and the more informed principal's average bid does not increase, both principals' expected bid in the asymmetric equilibrium must be lower than in the symmetric equilibrium, therefore the agent's ex ante expected payoff in the asymmetric equilibrium must be lower.

A.6. Proof of Proposition 6

Consider two first-period types of the agent $\theta_1^{'} > \theta_1$. Suppose, towards a contradiction, that a fully separating equilibrium exists. Denote by $U_i^{TP}(\theta_1|\theta_1^{'})$ as the sum of expected two-period payoff type $\theta_1^{'}$ could get by taking the allocated effort for type θ_1 at period 1, under the separating contract. As described in part 3, incentive compatibility requires that

$$U_i^{TP}(\theta_1^{'}) = U_i^{TP}(\theta_1) + \int_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \int_{\underline{\theta}}^{\bar{\theta}} U_2(\theta_2) dF(\theta_2|\theta_1^{'}) \right\} dq$$

where the agent's second-period expected contract is $U_2(\cdot)$. By proposition 6, for any second period realization of type $\theta_2 > \underline{\theta}$, $U_2^D(\theta_2) < U_2^S(\theta_2)$, so in order to compensate the agent for the loss of payoff in the second period, the first period contract must give the higher type agent

$$\begin{aligned} U_1(\theta_1^{'}) &= U_1(\theta_1) + \int_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \mathbb{E}_{(\theta_2|\theta_1^{'})} \left[U_2^S(\theta_2) - U_2^D(\theta_2) \right] \right\} dq \\ &= U_1(\theta_1) + \int_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \mathbb{E}_{(\theta_2|\theta_1^{'})} \left[U_2^S(\theta_2) - U_2^D(\theta_2) \right] \right\} dq \\ \therefore U_1(\theta_1^{'}) - U_1(\theta_1) &= \int_{\theta_1}^{\theta_1^{'}} e_{i1}(q) dq + \mathbb{E}_{(\theta_2|\theta_1^{'})} \left[U_2^S(\theta_2) - U_2^D(\theta_2) \right] \end{aligned}$$

Here, $\mathbb{E}_{(\theta_2|\theta_1^{'})} \left[U_2^S(\theta_2) - U_2^D(\theta_2) \right]$ refers to the expected value of $\left[U_2^S(\theta_2) - U_2^D(\theta_2) \right]$, where the expectation is taken over all possible second period realizations of type θ_2, given the agent's first period type is θ_1.

Because $U_2^S(\theta_2) - U_2^D(\theta_2) > 0$ for any θ_2, we can say $\mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] > 0$. By picking a small enough $\epsilon > 0$, we can find type $\hat{\theta}_1 = \theta_1^{'} - \epsilon$ such that

$$\begin{aligned}
C\left(e_{i1}(\theta_1^{'}|\hat{\theta}_1)\right) - C\left(e_{i1}(\theta_1^{'})\right) &= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{C^{'}\left(e_{i1}(q)\right)\left(1 + e_{i1}^{'}(q)\right)\right\} dq \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q) + e_{i1}(q)e_{i1}^{'}(q)\right\} dq \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} e_{i1}(q)dq + \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q)e_{i1}^{'}(q)\right\} dq \\
&< \int_{\hat{\theta}_1}^{\theta_1^{'}} e_{i1}(q)dq + \mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q) + \mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right]\right\} dq
\end{aligned}$$

where the inequality follows from the fact that $\mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] > 0$, and the integral $\int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q)e_{i1}^{'}(q)\right\} dq$ can be made small enough by picking a small enough ϵ. This means, we can find a type below $\theta_1^{'}$ who will strictly benefit by mimicking type $\theta_1^{'}$ in the first period and not taking the specified contract from P_i in the second period. This violates upward incentive compatibility for any subset of fully separating types in the first period, which means no fully separating contract can exist in the first period.

APPENDIX B

Omitted Proofs: Chapter 2

B.1. Proof of Lemma 4

The result is a special case of Theorem 2 in Persico (2000). It suffices to show that the market research problem satisfies the assumptions of that Theorem.

First we show that for each firm i, signal s_i is *affiliated* with α. Two random variables S and A with joint density $f(s, \alpha)$ are affiliated if for any realizations $s^{'} > s$ and $\alpha^{'} > \alpha$, $f(s^{'}, \alpha^{'})f(s, \alpha) \geq f(s, \alpha^{'})f(s^{'}, \alpha)$.

Using the probability density functions of normal distributions with equal variance, for any two states $\alpha^{'} > \alpha$ and any two signal realizations $s^{'} > s$, we can see that

$$\frac{f(s^{'}, \alpha^{'})}{f(s, \alpha^{'})} = \frac{\exp\left(-\frac{(s^{'}-\alpha^{'})^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha^{'})^2}{2v}\right)} = \exp\left(\frac{(2\alpha^{'} - s^{'} - s)(s^{'} - s)}{2v}\right)$$

$$> \exp\left(\frac{(2\alpha - s^{'} - s)(s^{'} - s)}{2v}\right) = \frac{\exp\left(-\frac{(s^{'}-\alpha)^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha)^2}{2v}\right)} = \frac{f(s^{'}, \alpha)}{f(s, \alpha)}.$$

So by definition of affiliation, s_i is affiliated with α.

Given two signals S^{t_1} and S^{t_2}, we say that S^{t_1} is more *accurate* than S^{t_2} if ${F^{t_1}}^{-1}(F^{t_2}(s|\alpha)|\alpha)$ is nondecreasing in α, for every s; where $F^{t_1}(\cdot|\cdot)$ and $F^{t_2}(\cdot|\cdot)$ are cumulative distibution functions for S^{t_1} and S^{t_2}, respectively. [See Lehmann (1988).] For each firm i, the accuracy of its signal s_i is increasing in t_i. [See example 4 in Section 3.2 of Persico (1996).]

By inspection, for each firm i, $u_i(\alpha, p_i) \equiv \int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha) dF^{t_{-i}}(s_{-i}|\alpha)$ is differentiable in p_i, and the optimal action $p_i^*(s_i, t_i)$ is differentiable in s_i and t_i.

Finally, the cdf of the normal distribution with variance v_i and state α is

$$F(x|\alpha, v_i) = \int_{-\infty}^{x} \left(\frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{(z-\alpha)^2}{2v_i} \right) \right) dz,$$

which is differentiable with respect to v_i and continuous in α. Now, because $v_i = \frac{V_\alpha}{t_i} - V_\alpha$ is differentiable in t_i, it follows that $F(x|\alpha, v_i)$ is differentiable in t_i.

Thus, the conditions of Theorem 2 in Persico (2000) are satisfied by the market research problem.

B.2. Derivation of Equation 2.1

At a given signal s_i and with accuracy t_i, denote firm i's optimal price p_i^* as in Vives (1984). Define p_{-i}^* similarly.

$$u_\gamma(\alpha, p_i^*) = \int_{s_{-i}=-\infty}^{\infty} p_i^* q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF(s_{-i}|\alpha)$$

Integrating by parts:

$$\begin{aligned} u_\gamma(\alpha, p_i^*) &= p_i^* \left\{ \left[q_i(p_i^*, p_{-i}^*, \alpha, \gamma) F(s_{-i}|\alpha) \right]_{s_{-i}=-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \right\} \\ &= p_i^* \Bigg\{ \big(q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma) F(\infty|\alpha) - q_i(p_i^*, p_{-i}^*(-\infty), \alpha, \gamma) F(-\infty|\alpha) \big) \\ &\qquad - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \Bigg\} \\ &= p_i^* q_\infty - p_i^* \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \end{aligned}$$

Where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$. We take the derivative with respect to s_i. Note that when pricing functions are as in the equilibrium of Vives (1984), both $\frac{\partial q_i}{\partial p_{-i}}$ and $\frac{\partial p_{-i}^*}{\partial s_{-i}}$ are independent of s_{-i}.

$$\frac{\partial u_\gamma(\alpha, p_i^*)}{\partial s_i} = \left(q_\infty + p_i^* \frac{\partial q_\infty}{\partial p_i}\right) \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} \left(F(s_{-i}|\alpha)\right) ds_{-i} \right\}$$

We take the derivative with respect to α. Note that $\frac{\partial q_i(\infty)}{\partial p_i}$, p_i^*, and $\frac{\partial p_i^*}{\partial s_i}$ are independent of α, and that conditional on some realization α' of the state signals are normally distributed with mean α' and some variance that is independent of α. The derivative is

$$\begin{aligned}
\frac{\partial^2 u_\gamma(\alpha, p_i^*)}{\partial \alpha \partial s_i} &= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} \left(F_\alpha(s_{-i}|\alpha)\right) ds_{-i} \right\} \\
&= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} \left(-f(s_{-i}|\alpha)\right) ds_{-i} \right\} \\
&= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} + \left(\frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \right).
\end{aligned}$$

B.3. Proof of Lemma 5

By Lemma 4, it suffices to show that $\exists t'$ s.t. $\frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0 \ \forall t < t'$.

$$\begin{aligned}
CMP(t,\gamma) + CRD(t,\gamma) &= \frac{1-\gamma}{2-\gamma t} t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2} t^2 \\
\frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] &= \frac{t\left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right)}{(2 - t\gamma)^3} \\
\therefore \frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0 &\Leftrightarrow \left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right) < 0.
\end{aligned}$$

Suppose $t \leq \frac{1}{2}$. Then $t^2\gamma^3 + t\,(4 + 2\gamma - 6\gamma^2) - 4$ is maximized on the domain $0 \leq \gamma < 1$ at $\gamma = 0$. At $\gamma = 0$

$$t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4 = 4t - 4 < 0.$$

The result follows.

B.4. Proof of Lemma 6

$$CMP(t,\gamma) + CRD(t,\gamma) = \frac{1-\gamma}{2-\gamma t}t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2$$

At $\gamma = 0$, $CMP(t,0) + CRD(t,0) = \frac{t}{2}$. Since $\lim_{\gamma\to 1}\left[CMP(t,\gamma) + CRD(t,0)\right] = 0$ and both $CMP(t,\gamma)$ and $CRD(t,\gamma)$ are continuous, it must be that if there exists some γ s.t. $CMP(t,\gamma) + CRD(t,\gamma) > \frac{t}{2}$, then there exist two values of γ such that $CMP(t,\gamma) + CRD(t,\gamma) = \frac{t}{2}$. There are two solutions γ^* for $CMP(t,\gamma^*) + CRD(t,\gamma^*) = \frac{t}{2}$:

$$\gamma^* = \frac{1}{2} - \frac{t}{4} \pm \frac{\sqrt{t^3 - 4t^2 + 36t - 32}}{4\sqrt{t}}$$

If $t < 1$, the solutions are real-valued and interior exactly when

$$t^3 - 4t^2 + 36t - 32 \geq 0.$$

When $t = 1$, the smaller of the two solutions is not interior, but the higher solution is interior. The left hand side of this expression is increasing in t, strictly negative at $t = 0$ and strictly positive at $t = 1$. The results follow.

APPENDIX C

Derivations in the Numerical Example in Chapter 3

Given $D_i(p_i, p_{-i}) = 10 - 2p_i + p_{-i}$, we can invert it to get the inverse demand function $P_i(q_i, q_{-i}) = 10 - \frac{2}{3}q_i - \frac{1}{3}q_{-i}$. Plugging in $q_{-i} = 0$ gives us the monopoly inverse demand function which is $P_i(q_i, 0) = 10 - \frac{2}{3}q_i$, inverting this again gives us the monopoly demand function $D_i^M(p_i) = 15 - \frac{3}{2}p_i$.

To calculate monopoly profits $\pi_i^M(c_i)$, we look at the maximization problem

$$\max_{p_i} (p_i - c_i)\left(15 - \frac{3}{2}p_i\right)$$

The first-order condition for this problem is $15-3p_i+\frac{3}{2}c_i = 0$, which yields $p_i^M(c_i) = 5+\frac{1}{2}c_i$ and $\pi_i^M(c_i) = \left(5 - \frac{1}{2}c_i\right)\left(\frac{15}{2} - \frac{3}{4}c_i\right)$. Therefore $\pi_i^M(c_1 = 8) = \frac{3}{2}$; and $\pi_i^M(c_0 = 0) = \frac{75}{2}$

To calculate duopoly prices $p_i^B(c_i, c_{-i})$ and profits $\pi_i^B(c_i, c_{-i})$, we look at the maximization problem

$$\max_{p_i} (p_i - c_i)(10 - 2p_i + p_{-i})$$

The first-order condition for this problem is $10 - 4p_i + p_{-i} + 2c_i = 0$, which yields the best-response functions $p_i^{BR}(p_{-i}, c_i) = \frac{5}{2} + \frac{1}{4}p_{-i} + \frac{1}{2}c_i$, and by simultaneously solving the two best-response functions we get $p_i^B(c_i, c_{-i}) = \frac{10}{3} + \frac{8}{15}c_i + \frac{2}{15}c_{-i}$ and $\pi_i^B(c_i, c_{-i}) = \left(\frac{10}{3} - \frac{7}{15}c_i + \frac{2}{15}c_{-i}\right)\left(\frac{20}{3} - \frac{14}{15}c_i + \frac{4}{15}c_{-i}\right)$. Hence, $p_i^B(c_2, c_1) = \frac{142}{15} < \frac{19}{2} = c_2$, and $p_i^B(c_1, c_0) = \frac{38}{5} < 8 = c_1$. Also, $\pi_i^B(c_2, c_2) = \frac{1}{18}$ and $\pi_i^B(c_1, c_1) = \frac{8}{9}$.