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Integrating Activity Scheduling and Travel Choices in a Dynamic Network Equilibrium Framework: Concepts, Algorithms and Application

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Hooram Halat

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ABSTRACT

Integrating Activity Scheduling and Travel Choices in a Dynamic Network Equilibrium Framework: Concepts, Algorithms and Application

Hooram Halat

The purpose of the dissertation is to develop a framework for equilibration of activity-trip chain demand in an integrated system of activity scheduling and travel choices within a dynamic network equilibrium framework. Activity-based modeling systems generate detailed activity chain schedules for individuals, which have to be assigned to transportation networks. Most of the implementations of activity-based models have been separate from the dynamic traffic assignment models. However, representation of detailed individual activity schedules throughout the assignment procedure, and capturing the resulting network performance measures in generating activity schedules, would lead to schedules consistent with real dynamics of transportation networks.

Most of the studies on the integration of the activity-based modeling systems with dynamic traffic assignment systems have focused on the applications. The literature lacks a theoretical basis and rigorous analytical treatment for the integrated model; moreover, important mathematical properties of the model are not adequately investigated. This dissertation aims at bridging the gap by presenting a household activity schedule adjustment model coupled with dynamic traffic assignment, where individuals' choices are made within a user equilibrium framework. The proposed framework provides achievement of faster algorithmic convergence for the integration of activity-based and dynamic traffic assignment models by serving as an inner adjustment

process. The equilibrium problem is formulated as a fixed-point equilibrium problem, which provides a basis for investigation of the solution properties.

The input variables to the model are the individuals' detailed activity schedules (obtained from an activity-based model) which could be translated as the time-dependent origin-destination demands, while the output variables are the household members' schedules as well as path flows at equilibrium.

A primary assumption is that individuals associate a disutility to their travel patterns, which they try to minimize. First, a household disutility term is defined as a function of 1) total travel cost and 2) schedule inconsistencies. Then, the problem is formulated as a fixed-point equilibrium problem. Next, the user equilibrium conditions are defined, and a variational inequality (VI) formulation is presented, and it is shown that the solution to the VI problem meets the user equilibrium conditions. In order to investigate the solution properties of the problem formulation, the continuity and monotonicity of the involved functions is explored.

Next, a solution algorithm is proposed, and the convergence characteristics of the proposed algorithm are demonstrated through numerical results obtained from application of the proposed algorithm to a large-scale real world network.

Further, the author addresses the issue of trip chain equilibrium in a dynamic network equilibrium framework. For this purpose, the author proposes a reformulation of the trip-based demand gap function formulation for the VI formulation of the bi-criterion dynamic user equilibrium (BDUE) problem. Next, a solution algorithm is proposed for solving the BDUE problem with daily chain of activity-trips. Then, numerical results obtained from the applied algorithm to both small-scale and large-scale networks in a simulation setting are presented. The results suggest that recognizing the dependency of multiple trips in a chain and maintaining the departure time consistency of subsequent trips provide sharper drops in gap values; hence, convergence might be reached more quickly, as compared to when trips are considered independent of one another. In addition, the integrated model of schedule adjustment and dynamic traffic assignment is extended to incorporate the cancellation of activities. It is shown that incorporation of activity cancellation could improve the algorithmic convergence.

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1 INTRODUCTION

1.1 Problem Description and Motivation

The need to participate in activities gives rise to individuals' demand for travel and resulting travel patterns. That is, individuals make decisions on activities to undertake, and then distribute the chosen activities spatiotemporally over the transport network. Their activity choices and schedules lead to activity patterns that eventually are interpreted as their travel behavior.

As trip makers' lifestyles change over time, their activity patterns become more intricate, which adds to the complexity of travel behavior forecasting, hence making the evaluation and prediction of impacts of various policies on the transportation system complicated. Transportation policies impact the demand for travel by influencing individuals' activity participation choices and ways to fulfill them. This overall impact manifests itself in the form of individuals' travel mode choice changes, activity rescheduling, and even destination-switching. As a result, it is essential to capture the behavioral mechanisms associated with activity scheduling of individuals, while preserving the fundamental consistency and interdependency of trip chains that comprise individuals' daily activity patterns throughout traffic and route assignment procedures. In other words, establishing a linkage between both the supply and demand sides of transportation is essential.

Traditionally, travel demand was represented as a collection of independent single-destination one-way trips; however, transportation planners now have a better understanding of the nature of travel behavior, mainly through deeper insight into household interactions and lifestyle preferences, thus leading to increased recognition and modeling of travel demand as activity-induced demand.

Conventionally, trip-based approaches were dominantly used in transportation planning processes. These approaches consist of multiple steps, each of which contributes to either the demand side or the supply side of the transportation network. Trip-based approaches, however, typically fail to capture essential behavioral aspects of individual travelers. To overcome this, a new body of research that addresses the transportation demand side through an activity-based modeling framework has garnered increasing attention in practice. Activity-based modeling (ABM) systems recognize the mutually related decisions for sequences of trips of individuals, which leads to activity patterns (Ettema et al. (1993)). ABM also seeks to capture interpersonal and intrapersonal consistencies as well as household members' interactions such as vehicle and/or task sharing. As a result, the overall aim is to provide a rather realistic basis for travel demand management by recognizing the complexities in the travel behavior of individuals (Ettema et al. (1993); Lin et al. (2008)).

The transportation supply (performance) side has traditionally relied on the assignment of vehicular trips to the network under a steady-state assumption of time-invariant origin-destination trip rates and associated link travel times. The importance of realistically capturing the dynamics of traffic spurred the introduction of dynamic (time-dependent) traffic assignment (DTA) models. DTA models provide a superior framework compared to static assignment models by explicitly capturing traffic congestion propagation, time-dependent travel demand and supply interactions, and the effects of traffic controls such as intelligent transportation system technologies. ABM approaches typically incorporate the temporal dimension as a continuum, and analyze activity patterns considering continuous time. However, notwithstanding their considerable degree of spatial and temporal detail, ABM models of travel demand have typically been used in conjunction with static assignment tools in planning practice.

As mentioned earlier, it is essential to integrate the transportation demand and supply sides. Integrating ABM with DTA models, which has gained increasing attention in recent years, provides a modeling framework for addressing arising planning and operational challenges, as well as transportation policies such as road pricing. Poor representation of detailed individual activity patterns obtained from ABM within DTA models could lead to fundamentally biased results. In other words, capturing sequences of activities and their interdependence within DTA, as well as the detailed time-dependent network conditions within ABM, are essential steps in the integration of the two models, which motivates this dissertation.

Various studies have been conducted on the integration of transportation demand and supply sides, however, there is no rigorous framework for the equilibration of the activity trip chains formulated, or accepted in the literature. Most of the studies on the integration of the activity-based modeling systems and dynamic traffic assignment systems have consisted of ad hoc applications. The literature lacks a theoretical basis and analytical foundations for the integrated model, and various mathematical properties of proposed models are not investigated. Lack of a validated platform for computing equilibrium states for activity chains of trip-makers precludes realizing the advantages expected from the enhanced modeling realism offered by both ABM and DTA models. As a result, many of the applications of demand forecasting and policy impact analysis suffer from the absence of a robust and rigorous underlying framework. Hence, development of a unified framework that integrates ABM and DTA models is significant for contributing to improved transportation planning strategies and analytical methodologies.

Network level-of-service (LOS) attributes are used by the ABM to generate activity schedules, which are then assigned to the underlying transportation network through the DTA procedure, resulting in time-dependent traffic conditions. An ideally-equilibrated network could be viewed as one in which the produced LOS through demand assignment replicates the LOS values used in generating the demand (Lin et al. (2008)). Hence, the problem of achieving an equilibrium state for the activity chains could be treated as a variant of a fixed-point problem, therefore inducing solution feasibility, existence, and uniqueness assessment challenges. In this regard, development of a mathematical framework, along with associated algorithms that lead to solution convergence, requires further research. One of the difficulties associated with the model formulation is determination of theoretically and fundamentally robust convergence criteria. Modern network equilibrium methods typically rely on minimization of a gap measure intended to capture the distance of a current solution from the equilibrated state. Defining and computing such a gap measure in the activity modeling context is not trivial. Furthermore, inclusion of capabilities such as activity cancellation adds to the complexity associated with the integration problem. Further complications arise during the assignment of detailed individual activity chains to the network leading to in the form of increased dimensionality of the dynamic traffic assignment problem, due to temporal and spatial interdependency of individual trip sequences.

Given the dimensions of the problem and size of real-world networks addressed in practice, applications of such fully integrated system could be cumbersome or impractical on typical high-end computing workstations. High implementation time and large memory requirements associated with applications of ABM to large-scale networks impede efficient exploitation of the modeling advantages that lie in an integrated system of ABM and DTA. Hence, defining a surrogate gap measure consistent with the underlying rules of the ABM, yet not as comprehensive, helps overcome the computation and/or implementation difficulties.

In this study, a household activity schedule adjustment model coupled with the dynamic traffic assignment is considered as the aforementioned surrogate measure. The surrogate gap measure enables faster algorithmic convergence in an integrated ABM-DTA model by creating an inner adjustment platform, and therefore reducing the number of times ABM has to be performed. Figure 1 provides a conceptual representation of the integrated framework. The combined activity schedule adjustment and route choice model in this study initially assumes that sequence and number of activities planned for travelers are fixed, so only their trip departure times, durations and paths are subject to adjustment. A variation of the problem is further addressed, where the order of activities in an individual's preplanned activity schedule remains fixed; however, the number of activities could change since the individual is allowed to cancel an activity within his/her activity chain.

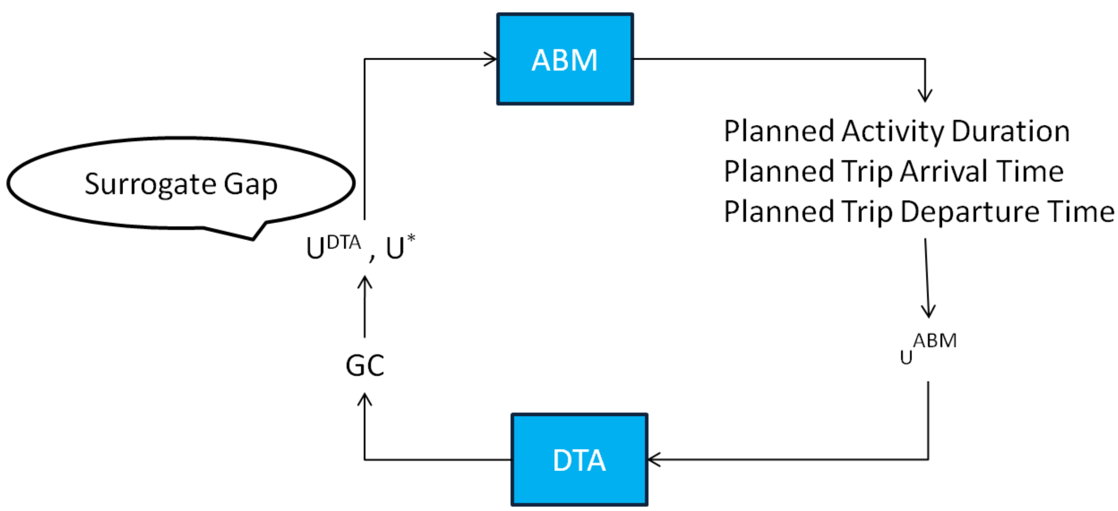


Figure 1 Integrated System of ABM and DTA

From the conceptual standpoint, it is essential that the activity schedules generated by ABM account for the temporal consistency of a sequence of activities. In this regard, each activity start time should correspond to its prior trip arrival time, and an activity end time should correspond to its next activity departure time. As a result, accounting for trip details and evaluation of trip time feasibilities leads to consistent individual schedules. Most of the ABM approaches do not explicitly control for travel time feasibilities. The individuals' activity schedules are generated by ABM according to anticipated travel times, and the generated activity patterns are assigned to the transportation network through DTA. Trip travel times obtained from the DTA could in turn be used to reschedule individuals' activities by adjusting trip departure times and activity durations.

From the implementation standpoint, maintaining consistent temporal and spatial resolutions of the model components in the integrated system is one of the main challenges. Implementing a computationally feasible time interval for modeling trip departure times and planning of daily tours within ABM, compatible with the temporal resolution of the DTA, is essential. In addition, the ability to incorporate finer-grained spatial units of analysis, such as micro-analysis zones, within the model contributes to higher-quality results. Correct representations of individuals as well as compatible spatial configurations in both models are additional challenges of the integrated system model. Achieving a desired representation requires the establishment and maintenance of consistency between individual units considered in ABM such as individual households and persons, and the individual vehicles considered in DTA. Furthermore, considering individual-level time-space constraints and realistic information availability is essential in capturing realisms of behavioral aspects and network dynamics.

The aim of this dissertation is to develop a model formulation that leads to a unified platform to bridge, in a mutually consistent manner, the detailed individual activity sequences generated by ABM, and their assignment to the transportation network by DTA. Specific study objectives are discussed in the next section.

1.2 Study Objectives and Contributions

The purpose of this dissertation is to develop a modeling framework to bridge the gap between ABM and DTA for individual travelers' activity-trip chains. The specific objectives are:

1. To develop a model formulation for an integrated system of dynamic traffic assignment and activity schedule adjustment to capture the traffic network performance under adjustment of individuals' activity schedules.
2. To address the solution properties such as existence and uniqueness of the proposed equilibrium formulation.
3. To develop a solution algorithm that entails the determination of theoretically-robust convergence criteria, leading to faster algorithmic convergence.
4. To allow for the rescheduling of individuals' activities as well as trip cancellation in a way consistent with the ABM behavioral modeling framework.
5. To develop a dynamic user equilibrium platform for the assignment of daily activity-trip chains of heterogeneous users, which incorporates a gap-based direction finding iterative procedure.
6. To circumvent the need to store memory-intensive node-to-node time-dependent shortest paths so as to provide the ability to implement the dynamic user

equilibrium algorithm to large-scale networks, while maintaining the spatial and temporal dependencies of the trip sequences.

This dissertation aims to contribute to both the theoretical concepts of an integrated equilibrium framework, as well as the implementation of such a framework to real-world large-scale networks:

1a. Unlike the existing studies in the literature, this dissertation aims at examining the theoretical aspects of the equilibration of activity trip chains within an integrated ABM-DTA system. The existing studies on this topic mostly focus on ad hoc applications of some integrated system, and lack a rigorous and robust theoretical framework. The problem is modeled as a fixed-point equilibrium problem. Theoretical difficulties associated with the integrated equilibrium framework are stated and addressed.

1b. This dissertation discusses and evaluates properties of the proposed model, such as existence and uniqueness of the solution.

1c. A solution algorithm is devised, and convergence characteristics of the process are investigated.

1d. The temporal and spatial interdependencies of activity chains are captured within a dynamic traffic assignment framework.

2a. The proposed DTA model for activity trip chains of individuals is implemented on large-scale networks, resulting in algorithmic convergence while maintaining the spatial and temporal interdependency of individual activity trips.

2b. The proposed equilibrium model of integrated DTA and activity schedule adjustment is implemented on both a small-scale and a large-scale network, resulting in algorithmic convergence.

1.3 Overview

In this chapter, the significance to both theory and practice of integrating the activity-based modeling approach and the dynamic traffic assignment approach is described. The purpose of the study and the objectives are also presented.

Chapter 2 presents a literature review of the activity-based modeling approach and dynamic traffic assignment models. Furthermore, an in-depth literature review on the integration of those two types of models is provided.

Chapter 3 presents the user equilibrium conditions for an integrated system of activity schedule adjustment and dynamic traffic assignment. Next, a variational inequality (VI) formulation of the equilibrium problem is presented, and it is shown that the solution to the VI problem formulation is equivalent to the user equilibrium conditions.

Chapter 4 defines gap functions for the VI problem formulation and presents a solution algorithm for solving the equilibrium problem. The solution procedure is implemented on both a small-scale and a large-scale network.

Chapter 5 extends the equilibrium model of Chapter 3 by incorporating activity/trip cancellation. First, the problem statement is presented, and next a solution algorithm is proposed and applied to a small-scale network.

Chapter 6 presents the dynamic network equilibrium for daily activity trip chains. A reformulation of the trip-based demand gap function formulation for the variational inequality formulation of the bi-criterion dynamic user equilibrium (BDUE) problem is presented. Next, a solution algorithm for solving the BDUE problem with daily chains of activity-trips is proposed. The solution procedure is implemented both on a small-scale and a large-scale network.

2 LITERATURE REVIEW

In this chapter, a review of the studies on the integration of ABM and DTA is provided. The review is divided into three sections. First, a review on activity-based models is provided. The second section incorporates a review of the DTA models, and the third section contains the review of studies addressing the integrated ABM and DTA problem.

2.1 Activity-based Models

Capturing individuals' activity choices, scheduling, and resulting movement patterns in transportation networks has been a topic of interest in various studies. The seminal work of Hägerstraand (1970) on time-space geography initiated a trend in analyzing human movement within the built environment. More sophisticated activity-based models provide a platform to capture individual preferences, household and spatiotemporal constraints in both activity and travel patterns of individuals. Activity-based approaches view travel as an activity-induced demand and travel behavior as a process derived from time and space constraints (Recker et al. (1986)). Studies on activity-based approaches have focused on capturing the spatial and temporal constraints in the analysis of activity demands, activity scheduling, and their connection to household members.

Root and Recker (1981) proposed a theoretical model of activity scheduling by considering a pre-travel and a travel phase. During the pre-travel phase, they construct the activity programs according to anticipated travel times and activity durations. In the travel phase, they adjust the patterns that are inconsistent with the schedules by removing, adding, or changing the sequences of activities. Their proposed method is based on a utility maximization framework. They suggested that the activity scheduling problem could be solved through multiple stages of utility maximization procedure.

Among the pioneering simulation-based approaches to the household activity scheduling problem is the Simulation of Travel/Activity Responses to Complex Household Interactive logistic Decisions (STARCHILD) model presented by Recker et al. (1986). They consider- a detailed household activity agenda in their operational model system, and modeled household members' detailed activity schedules. Their approach is also based on utility maximization theory involving individuals' decision-making procedures.

Ettema et al. (1993) presented an activity scheduling model based on Root and Recker's (1981) theoretical model. Their model treats scheduling decisions as consecutive stages of schedule construction and adaptation. They pointed out that their model differs from the STARCHILD approach in the sense that the latter requires individuals to choose among a set of feasible schedule patterns, while their approach is based on heuristic search to obtain a suboptimal solution.

Golledge et al. (1994) develop an activity generator model "SCHEDULER". In SCHEDULER, individuals are considered to initially have a mental calendar of activities from which the activities are chosen, and the sequence is determined by adopting a nearest neighbor heuristic. Then, during the mental execution, in cases where conflicts occur, the activity programs are adjusted by resequencing or replacing activities.

Ettema and Timmermans (2003) explored the effects of trip departure time choice on the entire activity pattern. They defined marginal utility for activities as a function of time of day, and used the collected data on travel diaries in Voorhout (the Netherlands) to estimate the model parameters, such that the difference between observed departure times and predicted departure times is minimized.

Ashiru et al. (2004) presented a utility-based framework of combined activity timing and duration choice. They outlined a solution algorithm for the activity scheduling and time allocation problem.

2.2 The Dynamic User Equilibrium Problem

In this section, a brief overview of the dynamic user equilibrium problem is presented, including the two main approaches to the problem, i.e. the analytical approach and the simulation-based approach.

In their seminal work, Beckmann et al. (1956) proposed a mathematical formulation for the user equilibrium (UE) traffic assignment problem. Their work was extended to address the dynamics of traffic demand, path flows and users departure time choices, which led to UE dynamic traffic assignment. Following the terminology adopted in the literature, the term dynamic user equilibrium is used in this dissertation.

Dynamic user equilibrium (DUE) models have greatly contributed to the prediction of dynamic traffic flow patterns, evaluation of advanced traffic control strategies, and evaluation of travel demand management strategies (Abdelghany and Mahmassani (2003)). These models assign the time-varying origin-destination demands to transportation networks to obtain time-varying path flows, which follow the time-dependent generalization of Wardrop's first principle (Wardrop (1952)), i.e. travelers belonging to the same origin-destination zone pair and departure time interval experience the same minimum travel cost along any used path, and none of the unused paths have a lower travel cost. The dynamic user equilibrium (DUE) problem has been studied extensively, as evident by the various analytical and simulation-based models proposed to address the problem (Peeta and Ziliaskopoulos (2001)). Peeta and Ziliaskopoulos (2001) identified four classes of approaches to DTA problems: mathematical programming, optimal control, variational inequality, and simulation-based models, of which the first three belong to analytical approaches, while the last one pertains to simulation-based approaches. They discussed that the term "simulation-based models" could be misleading, as this term generally refers to the solution methodology to the analytical formulation of the mathematical problem.

Analytical models of DUE take advantage of link/node exit functions for traffic flow propagation, as well as assumptions on link performance functions, such as convexity and continuity, to model path costs. Though analytical approaches provide a theoretically useful basis in terms of characteristics of the solution, such as existence and uniqueness as well as the satisfaction of DUE conditions, they lack proper representation of traffic flow dynamics (Peeta and Ziliaskopoulos (2001); Lu et al. (2009)). The simulation-based approach, however, is capable of representing real dynamics of network traffic flow, such as the spatial and temporal vehicle interactions, traffic flow propagation, and determination of link and path travel costs.

2.2.1 **Analytical Models**

2.2.1.1 Mathematical Programming Models

Mathematical programming DTA models provide a discrete time formulation of the problem. Various studies address the DTA problem through the mathematical programming approach for both SO and UE cases.

Merchant and Nemhauser (1978) are among the pioneers who presented a mathematical programming formulation for the DTA problem. Their model formulation is a discrete time nonlinear nonconvex mathematical programming problem, and it pertains to the fixed demand single destination SO case of the DTA problem. A piecewise linear version of the model leads to a global solution.

Carey (1987) reformulated the DTA problem as a convex nonlinear program by improving the model presented by Merchant and Nemhauser (1978). They presented extensions of their proposed model to handle multiple destinations as well as multiple commodities. The extended models, however, do not hold the convexity property of the original model due to "first-in first-out" (FIFO) requirements that need to be satisfied.

Mathematical programming formulations of DTA problems provide a mathematically robust basis for the problem; however, they suffer from loss of representation of traffic realism (Peeta and Ziliaskopoulos (2001)). In addition, the FIFO property results in non-convex constraints which, in DTA problem formulation, causes analytical tractability issues.

2.2.1.2 Optimal Control Models

The DTA formulations based on optimal control treat O-D demand rates and link flows as continuous functions of time. The constraints are similar to those of the mathematical programming model, except that they are continuous time constraints.

Friesz et al. (1989) considered continuous time link-based formulations of the DTA problem using optimal control theory for both UE and SO cases. A main drawback of their approach is the lack of realistic link performance and exit functions.

Various studies of the DTA problem using optimal control models exist in the literature; most lack traffic realism in link performance and exit functions. Ran et al. (1993) formulated the instantaneous dynamic user optimal (DUO) traffic assignment problem using the optimal control theory approach. The cost functions in their model formulation are assumed to be non-negative, differentiable, and increasing.

2.2.1.3 Variational Inequality Models

Dafermos (1980) presented a variational inequality formulation of the static traffic network equilibrium. In her study, travel demand is assumed fixed for any origin-destination pair in the network.

Friesz et al. (1993) suggested a continuous time VI formulation for the simultaneous route departure time equilibrium problem. They incorporated both the travel cost estimated by link performance function as well as the late/early arrival penalty into their definition of path costs. Existence and uniqueness properties of the solution to their proposed model could not be established.

Wie et al. (1995) proposed a discrete time VI formulation for the simultaneous route departure time equilibrium problem. In their formulation, existence of a solution under certain regularity conditions is established, though the validity of these conditions is not established.

The formulation introduced by Wie et al. (1995) is a path-based VI formulation, which requires computationally intensive complete path enumeration procedures. To overcome this issue, Ran and Boyce (1996b) formulated a link-based VI model of the problem that uses fixed departure times. Their model formulation is a discretized link-based VI formulation that incorporates exit flow capacity constraints. They discuss the computational burden associated with the capacity and oversaturation constraints in applications to real networks.

2.2.2 **Simulation-based Models**

As mentioned earlier, the term “simulation-based” refers to a solution methodology rather than model formulation (Peeta and Ziliaskopoulos (2001)). In simulation-based models, a simulator is used to model dynamics of traffic flow according to traffic theoretic relationships; hence these models can capture traffic flow propagation more realistically than analytical models.

Several solution algorithms are developed to solve both analytical and simulation-based approaches. The heuristic solution methods are of interest to this dissertation since they have been effective in addressing the computational challenges associated with large-scale dynamic traffic assignment (DTA) problems.

One of the proposed heuristic methods is the method of successive averages (MSA), which has been adopted successfully by some researchers (Tong and Wong (2000)). Tong and Wong (2000) stated that their satisfactory computational results on a small-scale network do not guarantee the same trend for larger real networks.

Another category of proposed heuristic methods applied in earlier studies (Smith and Wisten (1995); Huang and Lam (2002)), implements a path-swapping method that intuitively swaps proportions of flows from higher-cost paths to the shortest paths. The swap proportion is a function of the flow on the current path, the path cost difference between current path and least cost path, and a step size.

Lu et al. (2009) mentioned that the literature does not introduce a methodical approach for determining the step size. They emphasized the importance of choosing an appropriate step size for swapping the flows in achieving algorithmic convergence (Lu et al. (2009)).

Lu et al. (2009) also reformulated the variational inequality formulation of the DUE problem, via a gap function, as a nonlinear minimization problem (NMP), in addition to proposing a column generation based optimization procedure to solve the NMP. Their solution algorithm incorporates a simulation-based dynamic network loading procedure, which results in time-dependent path travel costs, and a path swapping descent direction method to solve the restricted NMP over a subset of feasible paths.

2.3 Integrated ABM and DTA

In this section a review of the attempts made at integrating the ABM and DTA is provided.

Abdelghany et al. (2001) developed spatial micro-assignment models of travel demand in forms of activity-trip chains. Their model formulation addresses path choices of travelers so as to maintain the intermediate destinations of planned trip sequences, as well as the final destination, activity duration at the intermediate destinations, and departure time at the origin.

Lam and Yin (2001) proposed a conceptual model of activity-based dynamic traffic assignment. Their combined model of activity and route choice encompasses a temporal elastic travel demand in a time dependent route choice framework. They proposed an iterative heuristic approach, and illustrated that the incorporation of MSA and Diagonalization methods in their solution algorithm tends to provide a solution to the dynamic user equilibrium problem.

Lam and Huang (2003) presented a mathematical programming model of combined activity-destination and user equilibrium route choices. They assumed a single homogenous user class and activity durations are considered exogenous in their modeling approach. They further presented a variational inequality formulation of activity-travel choices in networks with queues. They mentioned how their activity-destination choice model could lead to a tool for forecasting the time-dependent origin-destination demand.

Zhang et al. (2005) pointed to a gap in the literature of linking activity and travel scheduling to traffic congestion. They proposed an integrated model of daily activity scheduling with bottleneck congestion. They derived a daily utility function of schedules by capturing queueing models; given the obtained utilities, they allowed adjustment of schedules in terms of departure time and activity duration. Their model considers individuals with home to work and work to home trips (single activity participation), and they do not explicitly capture route choices in their modeling approach.

The activity chaining model presented by Kim et al. (2006) incorporates a utility model considering all individual activities, budget and time constraints. They adopted a sequential solution approach to the activity chaining problem, and then assigned the activities to the network through a DTA model to feed the updated time dependent path travel times back to the activity chaining phase.

Ramadurai and Ukkusuri (2010) presented a joint model of activity location, participation time, duration and route choices in a dynamic framework. They adopted the cell transmission model for capturing traffic flow dynamics, and approached the dynamic user equilibrium problem utilizing a modification of Wardrop's equilibrium framework. They proposed a solution algorithm, which they applied to a hypothetical network, to discuss the convergence characteristics of their modeling approach. Their model formulation does not incorporate the temporal dimension associated with dynamic user equilibrium models, however, and the defined activity utilities are assumed to be dependent on activity durations yet independent of participation time. As discussed by Chow and Recker (2012) and Chow (2014), their model considers congestion as the driving factor in activity scheduling, and therefore fails in capturing the diverse criteria associated with the process of activity scheduling.

Chow and Recker (2012) presented an inverse optimization approach to address the issue of parameter estimation for the household activity pattern problem (HAPP). They adopted the formulation proposed by Recker (1995), which views the problem of household activity pattern scheduling as a multi-objective variant of pickup and delivery problems with time windows. The adopted objective function consists of three components: the travel time between origin and destination nodes, the return home delay yielding from multiple sojourn tours, and a measure for the length of travel day.

Dynamic Network Equilibrium Problem for Daily Activity Trip Chain

Considering that travelers usually make multiple trips within a day, each of which corresponds to an activity belonging to their daily activity schedule, changing the unit of demand from a one-way trip to a chain of trips in traffic assignment models could play an important role in obtaining more accurate results (Abdelghany et al. (2001); Abdelghany and Mahmassani (2003)). Efficient and detail-compatible assignment of trip chains to transportation networks contributes substantially to improving activity-based modeling approaches and the evaluation of related functional and economic policies (Abdelghany et al. (2001)).

Existing applications of DUE models to large-scale real networks mostly consider the unit of traffic demand either as a one-way trip or as multiple independent trips. However, individuals’ travel patterns typically follow a sequence of trips linked together. As mentioned earlier, the spatial micro-assignment model developed by Abdelghany et al. (2001) considers travel demand in forms of activity-trip chains. Path choices of travelers is modeled in a way that maintains the planned sequence of trips' intermediate destinations as well as the final destination, activity duration at the intermediate destinations, and departure time at the origin. In addition, Abdelghany and Mahmassani (2003) presented a temporal-spatial micro-assignment of the travel demand with activity-trip chains. They have expanded their spatial micro-assignment model to capture other travelers' choices such as sequence of activities and departure time choice at the origin.

The MSA method, combined with the generalized cost gap-based approach considering users' heterogeneity through a bi-criterion dynamic user equilibrium model, is used in Chapter 6 as the base model to develop a solution algorithm to consider daily chains of activity-trips in the dynamic user equilibrium problem.

3 PROBLEM STATEMENT

In this chapter, a mathematical formulation for an integrated system of activity schedule adjustment and dynamic traffic assignment models is presented. First, a household disutility term is defined as a function of total travel cost and schedule inconsistency. Next, assuming that each user makes travel choices that minimize his/her experienced disutility, a user equilibrium framework is adopted, followed by definition of the user equilibrium conditions. In the next section, a variational inequality (VI) formulation is presented, and it is shown that the solution to the VI problem formulation satisfies the user equilibrium conditions.

The problem formulation presented in this chapter is designed for a scenario in which sequence and number of planned activities are assumed fixed, meaning that only the trip departure times, durations and paths are subject to change. Variations of the problem will be presented in Chapter 5 of this dissertation. A time-space diagram of individual activity sequences and travel choices is depicted in Figure 2.

3.1 User Heterogeneity

In this dissertation, trip-makers are treated differently in terms of perception of time, and the measure by which they evaluate its worth (value-of-time perceptions). Given that each traveler comes from a specific socio-demographic background, and each trip serves a particular purpose (e.g. work vs. non-work trips), capturing travelers' heterogeneity in the evaluation of travel decision costs, particularly during the assignment procedure, is essential (Jiang et al. (2011a); Jiang and Mahmassani (2013)).

Road pricing is considered to be an effective demand management strategy to control and reduce traffic congestion. In forecasting toll facility usage, the willingness of a trip-maker to pay a certain price to save on a specific unit of time is of great interest. The conventional traffic assignment models assume all trip-makers' preferences of time savings over cost to be the same (Jiang and Mahmassani (2013)). However, others have suggested that VOT varies notably across the population of trip-makers, and thus road user heterogeneity is reflected in individual reactions to toll charges (Small and Yan (2001); Brownstone and Small (2005)). A typical approach in the estimation of travel path costs is to define a generalized cost function that consists of both out-of-pocket cost (path toll) as well as path travel time converted into cost by multiplying it by the corresponding value of time (VOT). In this study, the path generalized cost (as discussed above) is utilized, which allows for capturing user heterogeneity. Capturing user's heterogeneity within DTA models is discussed in detail in Chapter 6.

3.2 Household Disutility Associated with Travel Choice

A dynamic network $G = (N, A)$ is considered with N as a finite set of nodes and A as a finite set of directed links $(u, v) \in A$, where $u, v \in N$. τ_0 denotes the earliest possible departure time from all origin nodes, σ as a small time interval during which no noticeable change in traffic conditions or travel cost happen, and K as a large value in a way that $\tau_0 + K\sigma$ covers the entire time period (planning horizon). The planning horizon is discretized into a set of small intervals $\Gamma = \{\tau_0, \tau_0 + \sigma, \tau_0 + 2\sigma, \tau_0 + 3\sigma, \dots, \tau_0 + K\sigma\}$. A total of HH households is assumed on the network, each having $M(hh)$ members, thus leading to a total of IT individual travelers. The set j is defined as the set containing the household members that make joint trips and the associated joint trips. Each individual is considered to belong to a user class m, and the set of user classes is denoted by M. The set $Ę = \{1,2, \dots, \mathrm{IT}\}$ contains individual travelers in the network.

Assuming that every household member has a set of preferred activity arrival, departure, and duration times, a measure of schedule inconsistency is developed to capture late or early arrivals (LA, EA), departures (LD, ED), and/or duration deviations for all activities. Associated with each type of schedule inconsistency are three penalty factors: a fixed penalty, FP^i, a variable penalty, P^i, and a penalty for moving beyond a certain threshold P^{b^i}.

${P_{LD}}^{i,m,tr}$: $({FP_{LD}}^{i,m,tr}, {P^b}_{LD}^{\;\;i,m,tr}, {P_{LD}}^{i,m,tr})$ Penalty associated with the late departure of trip tr of traveler i with user class m

${P_{ED}}^{i,m,tr}$: $({FP_{ED}}^{i,m,tr}, {P^b}_{ED}^{\;\;i,m,tr}, {P_{ED}}^{i,m,tr})$ Penalty associated with the early departure of trip tr of traveler i with user class m

${P_{LA}}^{i,m,tr}$: $({FP_{LA}}^{i,m,tr}, {P^b}_{LA}^{\;\;i,m,tr}, {P_{LA}}^{i,m,tr})$ Penalty associated with the late arrival of trip tr of traveler i with user class m

${P_{EA}}^{i,m,tr}$: $({FP_{EA}}^{i,m,tr}, {P^b}_{EA}^{\;\;i,m,tr}, {P_{EA}}^{i,m,tr})$ Penalty associated with the early arrival of trip tr of traveler i with user class m

${P_{LT}}^{i,m,tr}$: $({FP_{LT}}^{i,m,tr}, {P^b}_{LT}^{\;\;i,m,tr}, {P_{LT}}^{i,m,tr})$ Penalty associated with the activity duration lengthening of trip tr of traveler i with user class m

${P_{ET}}^{i,m,tr}$: $({FP_{ET}}^{i,m,tr}, {P^b}_{ET}^{\;\;i,m,tr}, {P_{ET}}^{i,m,tr})$ Penalty associated with the activity duration shortening of trip tr of traveler i with user class m

$TS_{In}^{i,m,tr}$ Threshold associated with the schedule inconsistency of type In for trip tr of traveler i with user class m

$\delta S_{In}^{i,tr}$: $(\delta D_{L,E}^{i,tr}, \delta A_{L,E}^{i,tr}, \delta T_{L,E}^{i,tr})$ Schedule inconsistency of type In for trip tr of traveler i with user class m

$\Delta S_{In}^{i,m,tr}$: $(\Delta D_{L,E}^{i,tr}, \Delta A_{L,E}^{i,tr}, \Delta T_{L,E}^{i,tr})$ Inconsistent schedule penalty of type In for trip tr of traveler i with user class m

$\Delta S^{i,m,tr}$ Inconsistent schedule penalty for trip tr of traveler i with user class m

$$\Delta S_{In}^{i,m,tr} = \begin{cases} FP_{In}^{i,m,tr} + P_{In}^{i,m,tr} * \delta S_{In}^{i,tr} & \delta S_{In}^{i,tr} < TS_{In}^{i,tr} \\ FP_{In}^{i,m,tr} + P_{In}^{i,m,tr} * \delta S_{In}^{i,tr} + P^{b}{}_{In}^{i,tr} * (\delta S_{In}^{i,tr} - TS_{In}^{i,tr}) & \delta S_{In}^{i,tr} > TS_{In}^{i,tr} \end{cases} \tag{3-1}$$

$$\Delta S^{i,m,tr} = \sum_{tr \in \varphi'^{i}} (\Delta S_{D}{}^{i,m,tr} + \Delta S_{A}{}^{i,m,tr} + \Delta S_{T}{}^{i,m,tr}) \tag{3-2}$$

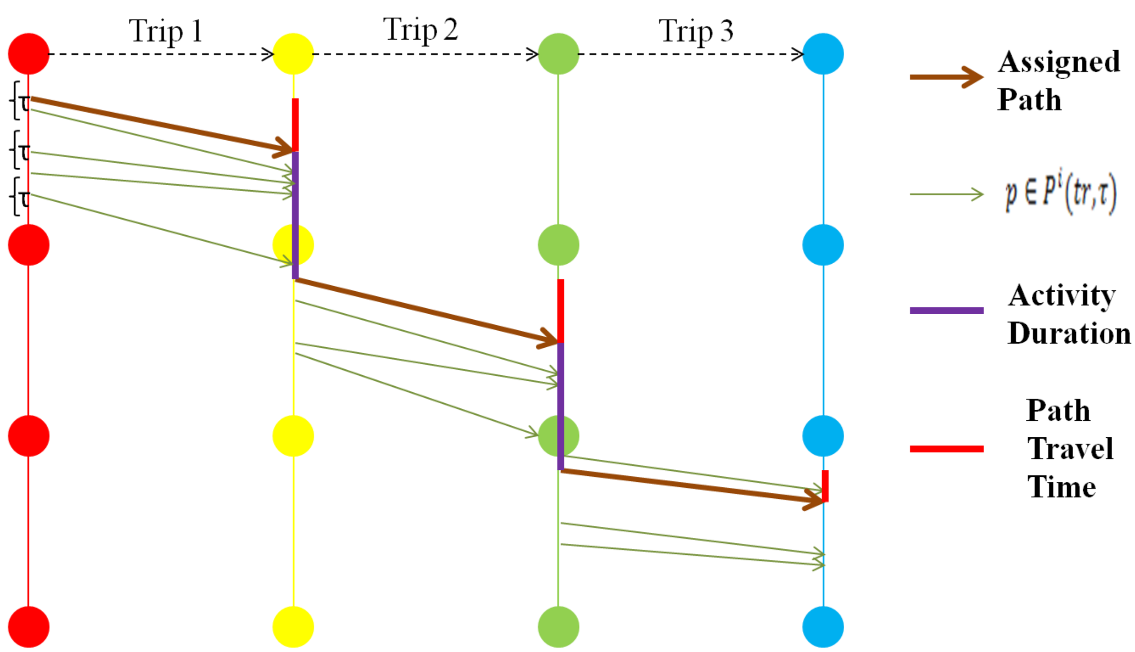


Figure 2 Time-space Network Representation

Other notations are as follows:

o Subscript belonging to an origin node

d Subscript belonging to a destination node

τ Subscript for a departure time interval

$P(o,d)$ The set of all feasible paths associated with o,d pair

$OD(i,tr)$ Function yielding the origin-destination pair associated with trip tr of individual i

p Subscript for a path $p \in p(o,d)$

$Y_{i,tr}^{\tau,p}$ Binary decision variable; equal 1 if traveler i chooses path p at departure time interval τ for its trip tr, and 0 otherwise

${\varphi'}^{i}$ The set containing trip numbers of corresponding origin destination pairs of planned trips of individual i: ${\varphi'}^{i} = \{1,2,\dots,TR(i)\}$, where $TR(i)$ is the number of trips planned for individual i

tr Subscript corresponding to trip number of traveler

$r_{OD(i,tr)}^{\tau,p}$ Number of trips from o to d departing o at time interval τ and assigned to path p

r Time-dependent vector of path flows

r' Time-dependent link flow vector

B Time dependent link-path incidence matrix

$f_{a,t}({r'}_{a,t})$ Travel time on link a at time t as a function of flow on link a at time t

$GC_{od,m}^{\tau,p}$ Path generalized cost for individuals of user class m departing path p at departure time τ

$TT_{od}^{\tau,p}$ Experienced path travel time for trips from o to d at departure time interval τ assigned to path p

$TC_{od}^{\tau,p}$ Experienced path travel cost for trips from o to d at departure time interval τ assigned to path p

The disutility associated with schedule/route choice of an individual traveler i is considered as combined total travel cost and schedule inconsistency, as follows:

$$\sum_{tr\in\varphi'^{i}}\sum_{\tau\in\Gamma}\sum_{p\in P(OD(i,tr))} Y_{i,tr}^{\tau,p} * GC_{OD(i,tr),m}^{\tau,p} + \Delta S^{i,m,tr} \qquad \forall i \in E \tag{3-3}$$

However, individuals' activity scheduling decisions are dependent on other household members. For instance, they may adjust their departure time in a manner consistent with other members who are participants of a joint trip. Hence, it is essential that the disutility is attributed to the whole household, leading to the following definition of household disutility for every household hh:

$$\sum_{i=1}^{M(hh)}\sum_{tr\in\varphi'^{i}}\sum_{\tau\in\Gamma}\sum_{p\in P(OD(i,tr))} Y_{i,tr}^{\tau,p} * GC_{OD(i,tr),m}^{\tau,p} + \Delta S^{i,m,tr} \tag{3-4}$$

Conditions and Constraints:

The decision variable $Y_{i,tr}^{\tau,p}$ should satisfy the following conditions:

(3-5)

$$\sum_{\tau\in\Gamma}\sum_{p\in P(OD(i,tr))} Y_{i,tr}^{\tau,p} = 1 \quad \forall i \in E, tr \in \varphi'^{i}$$

$$Y_{i,tr}^{\tau,p} = Y_{i',tr'}^{\tau,p} \quad \forall j \in J,\ \forall(i,tr) \in j,\ \forall(i',tr') \in j \tag{3-6}$$

$$Y_{i,tr}^{\tau,p} \in \{0,1\} \quad \forall i \in E, tr \in \varphi'^{i}, \tau \in \Gamma \tag{3-7}$$

Equation (3-5) ensures that all trips are assigned exactly one path and a single departure time. Equation (3-6) maintains the consistency of travel choices of travelers who have joint trips, and constraint (3-7) restricts the decision variable to be a binary variable taking values 0 or 1.

The travel/route decisions of all individuals in the network result in path flows, which in turn yield the path travel times and generalized costs indicated below:

$$\sum_{i=1}^{IT}\sum_{tr\in\varphi'^{i}} Y_{i,tr}^{\tau,p} = r_{od}^{\tau,p} \quad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d) \tag{3-8}$$

$$r' = B.r \tag{3-9}$$

$$TT_{od}^{\tau,p} = \sum_{(a,t)\in(p,\tau)} f_{a,t}\left(r'_{a,t}\right) \quad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d) \tag{3-10}$$

$$GC_{od,m}^{\tau,p} = \alpha(m) * TT_{od}^{\tau,p} + TC_{od}^{\tau,p} \quad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d), m \in M \tag{3-11}$$

Equation (3-8) yields the flow on route p and departure time τ, which is dependent on decision variables $Y_{i,tr}^{\tau,p}$ of all travelers in the network. Equation (3-9) converts the dynamic path flows to dynamic link flows. Equation (3-10) relates the time dependent path travel times to the dynamic link flows. In equation (3-10), (a, t) represents links (a) that belong to the path (p), and associated link departure times (t), which starting from the first link's departure time (τ) are dynamically updated for each subsequent link through the addition of previous links dynamic travel time to (τ). Equation (3-11) contains the definition of time dependent path generalized costs, which captures the heterogeneity of users. The following notations are also defined:

Dep_i^{tr} Adjusted departure time of trip tr of individual i

A_i^{tr} Desired arrival time of trip tr of traveler i

D_i^{tr} Desired departure time of trip tr of traveler i

T_i^{tr} Desired duration of activity following trip tr of traveler i

$T_i^{tr^{Min}}$ Minimum duration of activity following trip tr of traveler i

In addition, the following scheduling constraints are defined for $\forall i \in Ȩ, tr \in \varphi'^{i}$:

$$Dep_i^{tr} \geq \tau * \sum_{p \in P(OD(i,tr))} Y_{i,tr}^{\tau,p} \quad \forall i \in Ȩ, tr \in \varphi'^{i}, \tau \in \Gamma \tag{3-12}$$

$$Dep_i^{tr} \geq Dep_i^{tr-1} + \sum_{\tau \in \Gamma} \sum_{p \in P(OD(i,tr))} Y_{i,tr-1}^{\tau,p} * TT_{i,tr-1}^{\tau,p} + T_i^{tr^{Min}} \quad \forall i \in Ȩ, tr > 1 \in \varphi'^{i} \tag{3-13}$$

$$Dep_i^{tr} \geq 0 \quad \forall i \in Ȩ, tr \in \varphi'^{i} \tag{3-14}$$

Inequality (3-12) ensures that the value of Dep_i^{tr} is as large as the selected departure time τ. Inequality (3-13) ensures that the subsequent trips of a traveler (i) have temporal consistencies, i. e. no trip departs earlier than the previous trip. Constraint (3-14) defines the feasible set for the departure time variable.

In order to simplify the equations, a new term is defined to represent the travel time that a traveler experiences on path p: $ETT_i^{tr} = \sum_{\tau \in \Gamma} \sum_{p \in P(OD(i,tr))} Y_{i,tr}^{\tau,p} * TT_{i,tr}^{\tau,p}$

$$\delta S_{LA}^{i,tr} = \begin{cases} Dep_i^{tr} + ETT_i^{tr} - A_i^{tr} & Dep_i^{tr} + ETT^{i,tr} \geq A_i^{tr} \\ 0 & Dep_i^{tr} + ETT^{i,tr} < A_i^{tr} \end{cases} \tag{3-15}$$

$\forall i \in E, tr \in \varphi'^{i}$

$$\delta S_{EA}^{i,tr} = \begin{cases} A_i^{tr} - Dep_i^{tr} + ETT_i^{tr} & A_i^{tr} \geq Dep_i^{tr} + ETT^{i,tr} \\ 0 & A_i^{tr} < Dep_i^{tr} + ETT^{i,tr} \end{cases} \tag{3-16}$$

$\forall i \in E, tr \in \varphi'^{i}$

$$\delta S_{LD}^{i,tr} = \begin{cases} Dep_i^{tr} - D_i^{tr} & Dep_i^{tr} \geq D_i^{tr} \\ 0 & Dep_i^{tr} < D_i^{tr} \end{cases} \tag{3-17}$$

$\forall i \in E, tr \in \varphi'^{i}$

$$\delta S_{ED}^{i,tr} = \begin{cases} D_i^{tr} - Dep_i^{tr} & D_i^{tr} \geq Dep_i^{tr} \\ 0 & D_i^{tr} < Dep_i^{tr} \end{cases} \tag{3-18}$$

$\forall i \in E, tr \in \varphi'^{i}$

$$\delta S_{LT}^{i,tr} \tag{3-19}$$

$$= \begin{cases} Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} - T_i^{tr} & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} \geq T_i^{tr} \\ 0 & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} < T_i^{tr} \end{cases}$$

$\in E, tr \in \varphi'^{i}$

$$\delta S_{ET}^{i,m,tr} \tag{3-20}$$

$$= \begin{cases} T_i^{tr} - Dep_i^{tr+1} + ETT_i^{tr} + Dep_i^{tr} & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} \geq T_i^{tr} \\ 0 & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} < T_i^{tr} \end{cases}$$

$\in E, tr \in \varphi'^{i}$

Equations (3-15) to (3-20) define the schedule inconsistency terms as functions of trip departure time.

In this section, a disutility function associated with travel patterns of individual travelers is defined. Assuming that each traveler makes travel choices that minimize the experienced disutility, a user equilibrium framework is adopted in the next section based on the following conceptual framework: In the real world, individuals plan their activity schedules, and make travel choices to accomplish their activity sequences, but the plan may not be executed as intended due to congestion, accidents, transit delays, etc. As a result, individuals try to make adjustments to their activity/travel choices to minimize the associated disutility. The adjustment process continues until no further improvement can be achieved, meaning that the system has reached an equilibrium state. Therefore, at the equilibrium state, no traveler can decrease his or her disutility by switching paths or departure times (schedule) associated with his or her trips. In the next section, the user equilibrium problem is formulated based on this defined disutility for travelers.

3.3 Model Formulation

3.3.1 **The Fixed-point Equilibrium Problem**

Various transportation network problems have been examined using variations of the fixed-point problem formulation. Cantarella (1997) formulated the equilibrium assignment problem as a fixed-point problem. Cascetta and Postorino (2001) proposed a fixed-point modeling approach to the origin-destination demand estimation problem. Fixed-point formulations are particularly useful when there is no analytical form of the function available, evaluations of the function are computationally expensive, or the problem size is large. Furthermore, they provide a basis for solution algorithms as well as analysis of problem properties such as solution existence and uniqueness (Dennis Jr and Schnabel (1996); Kelley (2003); Bierlaire and Crittin (2006)). Dennis Jr and Schnabel (1996) provided a comprehensive discussion of fixed-point formulations. They stated that problems with more than 50 variables are considered large size problems, and finding a solution for them without having a proper starting point, or mild nonlinearity, is not guaranteed. Moreover, Dennis Jr and Schnabel (1996) mentioned that fixed-point problems are complex phenomena often captured by computer simulation models rather than closed form analytical functions, leading to the unavailability of derivatives. In addition, there are problem instances that have high computational costs associated with their function evaluations, requiring efficient algorithms.

Several approaches have been proposed for solving systems of nonlinear equations, or equivalently, fixed-point problems. Iterative approaches have been widely used in solving fixed-point problems. These approaches start with an initial point s_0 and update the point at each iteration by moving towards a direction aimed at approaching convergence. *Newton*'s method is among the most widely used solution methods for this class of problems. However, *Newton*'s method requires the evaluation of first order derivatives, which is not feasible in problems with large number of unknowns or when computational evaluations of the derivatives are expensive or unavailable (Dennis Jr and Schnabel (1996); Kelley (2003); Bierlaire and Crittin (2006)). To overcome this problem, variations of *Newton*'s method, such as *quasi-Newton* and *Inexact Newton* methods provide alternative approaches without the need to compute derivatives. Quasi-Newton methods consider the linear approximation of the function. One of the shortcomings of quasi-Newton methods is that their convergence ability is limited by proximity of the initial starting point to the optimal solution. Inexact Newton methods find the direction at each iteration rather than approximating the derivatives. Existing approaches to solve Inexact Newton formulations mostly use Krylov iterative linear methods. Krylov-Newton methods eliminate the need for matrix-matrix products by building each iteration through matrix-vector products as discussed by Kelley (2003). Bierlaire and Crittin (2006) stated that although Krylov-Newton methods provide a robust theoretical basis, they do not provide a good platform for problems with noisy functions.

3.3.2 **Fixed-point Equilibrium Formulation**

In this dissertation, equilibrium is defined as the state at which no household can improve their utility by adjusting their trip departure time, activity duration and/or route choice. The disutility as defined in the previous section is to be minimized for each traveler. Hence, the problem can be conceptually formulated as a fixed-point equilibrium problem in the closed, bounded, and convex path flow space Ω:

$$r^* = R(SA\left(SP\left(C(r^*)\right)\right)) \qquad (3\text{-}21)$$

Where,

r^* Vector of optimal path flows

C Vector of path costs

SP Vector of schedule penalties

SA Vector of adjusted schedules

R Route assignment operator

r^* is the equilibrium dynamic path flow vector obtained from dynamic traffic assignment. The path cost function C is a function of vector of dynamic path flows r, and produces time dependent path generalized costs (or travel times). As discussed earlier, individuals are assumed to associate a schedule disutility based on their travel costs, and as a result, adjust their activity schedules in terms of departure times and activity durations to reduce the experienced disutility. SP is the vector of schedule penalties (schedule disutility) that is considered to be a function of dynamic path costs. SA is the schedule adjustment function, which yields as output the vector of desired trip departure times, arrival times and activity durations for all activities of travelers. The route assignment operator $R(s)$ produces as output the flow on every path at each departure time interval, based on adjusted users' schedules. Given the size and dynamics associated with transportation networks, simulation tools take the lead in appropriately capturing the behavioral realisms that lie within transportation phenomena. Therefore, a state-of-the-art dynamic traffic simulation and assignment tool can be used to capture the time dependent path flows and path costs.

The user equilibrium conditions are defined next. A set ψ^i is considered for each individual i, which is the set of all the feasible schedule and path combinations for the entire trip chain of the traveler. In this section, each schedule and path combination is denoted by (s, p), where s denotes a feasible schedule, including departure times for each trip and activity durations for each activity, and p represents a feasible path. Note that here (s, p) does not represent a schedule and path for a single trip of the individual; rather, it captures the entire chain of trips of the traveler. ${U_i}^{(s,p)}(X)$ is defined as the disutility for individual i associated with travel pattern (s, p) under a feasible travel pattern assignment of X to all travelers in the network. ${U_i}^*(X)$ is the minimum disutility calculated for traveler i, where travelers are assigned by travel pattern X into the network. This disutility is associated with the optimal travel pattern $(s, p)^*$ of individual i.

$$U_i(X) = (C_i(X) + SP_i(X)) \qquad (3\text{-}22)$$

User Equilibrium Conditions:

$$X_i^{(s,p)} \times \left({U_i}^*(X) - {U_i}^{(s,p)}(X)\right) = 0 \qquad \forall i \in \mathrm{E}_{\iota}, (s, p) \in \psi^i \qquad (3\text{-}23)$$

$${U_i}^*(X) \leq {U_i}^{(s,p)}(X) \qquad \forall i \in \mathrm{E}_{\iota}, (s, p) \in \psi^i \qquad (3\text{-}24)$$

$$\sum_{(s,p)\in\psi^i} X_i^{(s,p)} = 1 \qquad \forall i \in \mathrm{E}_{\iota} \qquad (3\text{-}25)$$

$$X_i^{(s,p)} = \{0,1\} \qquad \forall i \in \mathrm{E}_{\iota}, (s, p) \in \psi^i \qquad (3\text{-}26)$$

Equation (3-22) defines experienced disutility of traveler i under travel pattern assignment X for all travelers in the network as the sum of travel cost and schedule inconsistency penalties. Equation (3-23) states that if a certain travel pattern is assigned to a traveler, it is the optimal pattern. Similarly, if the disutility associated with the travel pattern is larger than the optimal disutility, then the travel pattern should not be selected for the traveler at the equilibrium condition. Inequality (3-24) constrains the optimal disutility of a traveler (i) to be less than or equal to all possible disutilities associated with feasible travel patterns of the individual i. Equation (3-25) enforces that exactly one single travel pattern should be assigned to an individual traveler (i), and condition (3-26) restricts the variable ($X_i^{(s,p)}$) to be a binary variable. The feasible set constructed by the equations (3-25) and (3-26) is denoted by Ξ.

The author now defines a variational inequality (VI) formulation satisfying the user equilibrium state conditions. It is further shown that finding a solution to the equilibrium problem is equivalent to finding the solution to the VI problem.

Variational Inequality Formulation: find a feasible travel pattern assignment vector X^* such that the inequality defined by inequality (3-27) holds for all X in the feasible set (Ξ) defined by equations (3-25) and (3-26).

$$(SP + C)_{(s,p)^*} . (X - X^*) \geq 0 \qquad (3\text{-}27)$$

$$U(X^*) \circ (X - X^*) \geq 0 \qquad (3\text{-}28)$$

Proof of Equivalency

Rewriting equation (3-23) by multiplying X by each disutility term in the parentheses and moving the second term on the left side to the right side:

$$X_i^{(s,p)^*} \times U_i^{\,*}(X^*) = X_i^{(s,p)^*} \times U_i^{\,(s,p)}(X^*) \qquad \forall i \in E,(s,p) \in \psi^i \tag{3-29}$$

In inequality (3-24) one could multiply both sides by the nonnegative term X:

$$X_i^{(s,p)} \times U_i^{\,*}(X^*) \leq X_i^{(s,p)} \times U_i^{\,(s,p)}(X^*) \qquad \forall i \in E,(s,p) \in \psi^i \tag{3-30}$$

Subtracting $U_i^{\,*}(X^*) \times X_i^{(s,p)^*}$ from both sides of the equation (3-30), and substituting it on the right side using equation (3-29):

$$U_i^{\,*}(X^*) \times X_i^{(s,p)} - U_i^{\,*}(X^*) \times X_i^{(s,p)^*} \leq U_i^{\,(s,p)}(X^*) \times X_i^{(s,p)} - U_i^{\,(s,p)}(X^*) \times X_i^{(s,p)^*} \quad \forall i \in E,(s,p) \in \psi^i \tag{3-31}$$

Adding both sides over all individuals, their path choices, and possible schedules:

$$\sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{\,*}(X^*) \times X_i^{(s,p)} - U_i^{\,*}(X) \times X_i^{(s,p)^*} \leq \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{\,(s,p)}(X^*) \times (X_i^{(s,p)} - X_i^{(s,p)^*}) \tag{3-32}$$

$$[\sum_{i=1}^{IT} U_i^{\,*}(X^*) \times \sum_{(s,p)\in\psi^i} (X_i^{(s,p)} - X_i^{(s,p)^*})] \leq \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{\,(s,p)}(X^*) \times (X_i^{(s,p)} - X_i^{(s,p)^*}) \tag{3-33}$$

According to equation (3-25) the left side of inequality (3-33) is zero, therefore the defined user equilibrium conditions yield the VI formulation. Now we need to prove that the solution to the VI formulation satisfies the user equilibrium conditions defined by ((3-23)-(3-26)). Since X^* is restricted to be from the feasible set Ξ, it already meets the conditions in equations (3-25) and (3-26). Therefore, we need to prove that the solution to the VI problem meets the conditions in equation (3-23) and inequality (3-24).

The VI formulation can be stated as follows by expanding inequality (3-28):

$$\sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i{}^{(s,p)}(X^*)\times X_i^{(s,p)} - \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i{}^{(s,p)}(X^*)\times X_i^{(s,p)^*} \geq 0 \tag{3-34}$$

$U_i{}^*(X)$ is defined as the minimum disutility for each traveler *i* over all possible travel patterns *(s,p)*:

$$U_i{}^*(X) = Min_{(s,p)}\, U_i{}^{(s,p)}(X^*) \leq U_i{}^{(s,p)}(X^*) \qquad \forall i \in \mathcal{E}, (s,p) \in \psi^i \tag{3-35}$$

$(s^\wedge, p^\wedge)$ is defined as the following:

$$(s^\wedge, p^\wedge) = Argmin\ _{(s,p)} U_i{}^{(s,p)}(X^*) \quad \forall i \in \mathcal{E} \tag{3-36}$$

Based on $(s^\wedge, p^\wedge)$ definition the travel pattern assignment X' is defined that satisfies constraints (3-25) and (3-26) with the following specification:

$$X_i^{(s,p)'} = \begin{cases} 0 & (s,p) \neq (s^\wedge, p^\wedge) \\ 1 & (s,p) = (s^\wedge, p^\wedge) \end{cases} \ \forall i \in \mathcal{E}, (s,p) \in \psi^i \tag{3-37}$$

Therefore

$$X_i^{(s,p)'} \times \left(U_i^{(s,p)}(X^*) - U_i^{*}(X)\right) = 0 \qquad \forall i \in E, (s,p) \in \psi^i \tag{3-38}$$

Multiplying $X_i^{(s,p)'}$ by each of the two terms in the parenthesis, we get:

$$X_i^{(s,p)'} \times U_i^{(s,p)}(X^*) = X_i^{(s,p)'} \times U_i^{*}(X) \qquad \forall i \in E, (s,p) \in \psi^i \tag{3-39}$$

Now substituting the right side of equation (3-39) in the expanded VI formulation of (3-34), which is valid for each feasible X (such as X'):

$$\sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{*}(X) \times X_i^{(s,p)'} - \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{(s,p)}(X^*) \times X_i^{(s,p)^*} \geq 0 \tag{3-40}$$

Rearranging the summation in the first term, we get:

$$\sum_{i=1}^{IT} U_i^{*}(X) \times \sum_{(s,p)\in\psi^i} X_i^{(s,p)'} - \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} U_i^{(s,p)}(X^*) \times X_i^{(s,p)^*} \geq 0 \tag{3-41}$$

As X' is a feasible travel pattern assignment based on (3-25), $\sum_{(s,p)\in\psi^i} X_i^{(s,p)'}$ can be assumed as 1. Similarly, as X^* is a feasible travel pattern assignment, this value based on (3-25) can be substituted by $\sum_{(s,p)\in\psi^i} X_i^{(s,p)^*}$ for each traveler i.

$$\sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} (U_i{}^{(s,p)}(X^*) - U_i{}^*(X)) \times X_i^{(s,p)^*} \leq 0 \tag{3-42}$$

All the terms in the summation in inequality (3-42) are nonnegative based on our X^* definition and Equation (3-35). Therefore the inequality (3-42) can be held if and only if $\left(U_i{}^{(s,p)}(X^*) - U_i{}^*(X)\right) \times X_i^{(s,p)^*} = 0$.

Based on Lu (2007) and Patriksson (2013) a function G needs to satisfy two conditions to be defined as a gap function for any VI formulation. The two conditions are as follows:

$G(X) \geq 0 \quad \forall\, x \in \Xi$

$G(X^*) = 0$

In the next section, the author defines such gap function for the presented VI formulation in this study.

3.4 Equivalent Gap Function:

$$G(X) = \sum_{i=1}^{IT} \sum_{(s,p)\in\psi^i} X_i^{(s,p)} * (U_i{}^{(s,p)}(X) - U_i{}^*(X)) \tag{3-43}$$

In the gap function presented above, $X_i^{(s,p)}$ is either zero or one since it belongs to the feasible set Ξ. Also, $U_i{}^*(X)$ is the minimum disutility at any feasible X, thus it is always smaller than or equal to $U_i{}^{(s,p)}(X)$. As a result, the defined gap function is greater than or equal to zero. Assuming X^* as the solution to the VI problem, and since it satisfies condition (3-23), $G(X^*)$ is zero. Moreover, X belongs to Ξ, therefore the defined gap function meets the user equilibrium state conditions defined by (3-23) to (3-26). The defined gap function can be interpreted as a measure to evaluate distance between the optimal state and any feasible state, minimization of which would result in the equilibrium state.

3.5 Discussion of The Solution Properties

KAKUTANI'S Fixed-point Existence Theorem.

According to Cantarella (1997), an upper semi-continuous point to set map $\psi(x)$ defined over a compact convex and non-empty set X with $\{y = \psi(x)\} \subseteq X, \forall x \in X, and\ \{y = \psi(x)\}$ non-empty, closed and convex, has at least one fixed-point in set $X: \exists x^* \in X: x^* = \psi(x)$.

BROUWER'S Fixed-point Existence Theorem.

According to Cantarella (1997), a continuous function $\psi(x)$ defined over a compact convex and non-empty set X with $\psi(x) \subseteq X, \forall x \in X$ has at least one fixed-point in set $X: \exists x^* \in X: x^* = \psi(x)$

According to the fixed-point existence theorems, in order for the above fixed-point formulation to have solution(s), all the defined functions or maps have to be continuous (or upper semi-continuous).

Continuity in Vector Space

A function $y = \psi(x) \quad R^n \to R^n$ is continuous at $x_0 = (x_1, x_2, \dots, x_N) \epsilon R^n$ if and only if:

$$\lim_{x \to x_0} \psi(x) = y_0$$

Where, $y_0 = (y_1, y_2, \dots, y_N) \epsilon R^n$

In other words, if the limits exist when approaching x_0 from all feasible directions in Cartesian space, and if their values are equal to the function value, the function is continuous.

Lemma 1. The vector of path costs is in general a discontinuous function of path flow vector.

Discontinuity of the path cost function for general traffic conditions is discussed in detail by Alibabai (2011). Although the path cost function is a discontinuous function of the path flow vector in general (Alibabai (2011)), Cantarella (1997) states that the path cost function is continuous if the costs do not tend to infinity, even when flows are close to or higher than capacities.

Lemma 2. The vector of schedule penalties is a discontinuous function of the vector of path costs.

The vector of schedule penalties is a discontinuous function of path costs if at a $C = C^*$ it follows the following form:

$$SP(C) = \begin{cases} SP & C = C^* \\ SP + \Delta & C = C^* \pm \delta \end{cases}$$

Example Consider an individual traveler making trips 1 and 2 (Figure 3). Assume a disutility measure for the traveler as a simplified function of late arrival. The penalties associated with the disutility function are assumed to be ${FP_{LA}}^i = 0.5, {P^*_{LA}}^i = 0.02, {P_{LA}}^i = 0.01$. We assume path travel time value of the first trip at departure time 8:00 ($D_1^1 = 8{:}\,00$) is 30 min, and for the second trip at 10:00 ($D_1^2 = 10{:}\,00$) is 20 min. If the traveler has a preferred arrival time of 8:30 to activity 2, a maximum late arrival of 5 min, and a preferred arrival time of 10:20 to activity 3, the optimal schedule would be obtained at ($D_1^1 = 8{:}\,00, D_1^2 = 10{:}\,00$). Now approaching the path cost of path 1 at departure time D_1^1 from the right, the result is the following schedule penalties:

$$SP(C) = \begin{cases} 0 & C^1 = 30 \\ 0.5 + 0.01 * \delta & C^1 = 30 + \delta \end{cases}$$

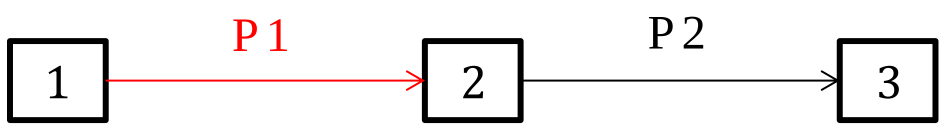


Figure 3 Small Network Example P1: Path 1 P2: Path 2

As it can be seen in the above example, the vector of individuals schedule penalty is a discontinuous function of the vector of path costs.

According to Lemmas 1 and 2, the existence of a solution for the proposed fixed-point equilibrium formulation cannot be guaranteed. If solution(s) exist(s), the uniqueness of it is dependent on the monotonicity of the involved functions. The following is a brief evaluation of the monotonicity of the above functions.

Monotonocity

Definition: A vector function is considered monotone if increasing the value of any of the elements in the function input does not lead to decrease in the value of any of the output elements.

Lemma 3. The vector of path costs is not a pointwise monotone function of the path flow vector.

One might define a continuous form for the vector of path costs as a function of the path flow vector. However, as discussed by Alibabai (2011), it can be shown that the vector of path travel times (cost) is not generally a monotonous function of the path flow vector.

Lemma 4. The vector of schedule penalties is a nonmonotonic function of the vector of path costs.

Consider an individual traveler making trips 1 and 2 (Figure 3). We assume a disutility measure for traveler as a simplified function of late and early arrival. The penalties associated with the disutility function are assumed to be ${FP_{LA}}^{i} = 0.5,\ {P^{*}_{LA}}^{i} = 0.02,\ {P_{LA}}^{i} = 0.01,\ {FP_{EA}}^{i} = 0.3,$ ${P^{*}_{EA}}^{i} = 0.01,\ {P_{EA}}^{i} = 0.01$. We assume path travel time value of first trip of traveler at departure time 8:00 is 25 min, and for the second trip at 10:00 is 20 min. If the traveler has a preferred arrival time of 8:30 to activity 2, a maximum late arrival of 5 min, and a preferred arrival time of 10:20 to activity 3, the optimal schedule will be obtained at $(D_1^1 = 8{:}\,00, D_1^2 = 10{:}\,00)$. Now approaching the path cost of path 1 at departure time D_1^1 from the right, results in the following schedule penalties:

$$SP(C) = \begin{cases} 0.31 & C^1(D_1^1) = 29 \\ 0.00 & C^1(D_1^1) = 30 \\ 0.51 & C^1(D_1^1) = 31 \end{cases}$$

According to Lemmas 3 and 4, one cannot guarantee the uniqueness of the solution for the equilibrium problem presented in this chapter.

3.6 Summary

In this chapter, the author aimed to provide a robust conceptual and mathematical framework for the integration of ABM and DTA. For this purpose, a framework was proposed that provides an inner adjustment process for an ABM-DTA integrated model (surrogate gap). The proposed surrogate measure captures the individuals' activity scheduling and travel choices in a dynamic network equilibrium framework. A user equilibrium framework for the path and schedule choices of individual travelers was defined for this purpose. A variational inequality formulation was presented as the variant formulation of the equilibrium problem, and it was shown that the solution to the VI formulation meets the user equilibrium conditions. Further, the equivalent gap function for the VI formulation was defined to provide a gap-based solution approach. In the end, the solution properties of the proposed fixed-point problem formulation are explored through an analysis of continuity and monotonocity of some of the involved functions. It is shown that there is no guarantee that a solution exists for the problem due to discontinuity of the functions. Moreover, the nonmonotonocity of the functions are shown through examples, which leads to the fact that there might be multiple solutions for the problem, provided that a solution exists. However, the solution to the problem, as well as its convergence characteristics, can be shown numerically, and are presented in the next chapter..

4 METHODOLOGY AND IMPLEMENTATION

In this chapter, a solution algorithm is proposed for the problem stated in Chapter 3. First, two gap functions are defined, and then a solution approach is presented, the objective of which is to minimize the defined gaps.

4.1 Introduction

The problem formulation presented in Chapter 3 is a combinatorial problem, where every individual faces a choice set consisting of routes and departure times. In a network with multiple feasible paths between each origin-destination pair as well as departure time intervals, the size of such choice set ψ^i is very large. In particular, the size grows to infinity in a continuous setting. Additionally, the constraints imposed by the interdependencies of household members and the spatial and temporal interdependencies of trip chains add to the complexity of the problem. Hence, we decouple the problem into two sub-problems of dynamic traffic assignment and household activity schedule adjustment, and we approach the problem from a network equilibrium perspective.

The utility resulting from the assignment of an activity plan to the traffic simulator would differ from the estimated utility of the activity plan in a non-equilibrium state. Therefore, the difference between individual experienced utilities and corresponding minimum possible values can be used as a gap measure to assess the convergence pattern. Two gap measures, one for the supply side and the other for the demand side, are defined as follows:

On the supply side:

$$\sum_{o \in O} \sum_{d \in D} \sum_{\tau=1}^{T} \sum_{m=1}^{M} \sum_{p \in p(o,d)} r_{od,m}^{\tau,p} \left[GC\,_{od,m}^{\tau,p}(\alpha, r) - GC^{*\tau}_{od,m} \right] \tag{4-1}$$

On the demand side:

$$\sum_{i=1}^{IT} \Delta S^{i} \tag{4-2}$$

As mentioned earlier, most of the existing solution methods to fixed-point formulations require computations of derivatives, or involve computationally intensive procedures of finding search directions. In problems of large sizes, such as the problem in this study, either with significant noise or when there is no closed form defined, such methods should be avoided. In the problem presented here, both the vector of feasible schedules and the set of feasible paths include a large number of variables. Therefore, the author adopts a descent direction method to solve the problem. The descent direction method starts from an initial feasible solution, and iteratively updates the solution by moving towards a direction identified to reduce the objective function value. An appropriate step size is also incorporated in the updating procedure to overcome the fluctuations. In order to evaluate the convergence characteristics of the descent direction method, one has to prove that the adopted direction strictly decreases the objective function at every iteration. However, such proof requires evaluation of the derivatives, which is computationally burdensome in the problem presented in this study. Moreover, the existence of high levels of temporal and spatial correlations within the problem of activity scheduling in a dynamic network equilibrium framework makes the computation of derivatives very difficult. Therefore, one might not be able to provide a mathematical proof of convergence of the algorithm for the problem. However, the convergence characteristics could be illustrated through analytical results.

A bi-level solution approach to the problem is adopted, where the lower-level schedules are treated as fixed, and only path assignments get updated, while the upper-level schedule adjustments take place with the paths, thus travel times remain unchanged.

4.2 Convergence Criteria

The objective in this chapter is to obtain consistent schedules with realistic travel times, which are affected by the decisions of all system users. To obtain this consistency, the planned schedule is compared with the experienced (simulated) one, so as to reduce the gap between them. At the beginning of each day, individuals decide their planned daily trip chain schedule based on anticipated travel time. The daily trip chain schedule consists of a set of travel decisions made by different agents. These decisions include activities, destinations, departure time, transportation mode and route decisions. However, due to congestion and pricing, the experienced travel time may be different from the expected travel time, which would lead to an experienced schedule which is different from the planned one. To obtain a better travel experience, individuals could modify their schedule based on the updated travel time by changing departure time, activity duration and maybe routes. Then this modified schedule is fed into the DTA models to get an updated travel time, which leads to new schedule.

Two different measures to represent the inconsistency between planned and experienced utilities are considered in this chapter to monitor the speed of convergence. The first gap measure is the *inconsistent schedule penalty,* as defined in the objective function (3-2). Another important measure of inconsistency between planned and experienced schedules is the *number of households* *with unrealistic schedules*. Individuals plan their daily schedules based on anticipated travel times in the network. However, in the presence of congestion, experienced travel times might be longer than they anticipate. In some cases, travelers can accommodate these fluctuations in travel time by shortening activity durations. However, in some cases, travel times might be so long that the travelers cannot reach the destination of their current trip before their next planned trip starts, unless they could have unrealistic negative activity duration. In those cases, the schedule with negative activity duration is flagged as unrealistic. The number of households with unrealistic schedules is selected as a measure of inconsistency, as it reveals the impact of the experienced travel times on the planned schedules. As the number of households and individuals with unrealistic schedules decreases, the gap between planned travel times and experienced travel times is also likely decreasing.

In addition to schedule inconsistency measures, the DTA relative gap is defined to track the performance of the mesoscopic simulation model, and the integration convergence in terms of travel time. The DTA relative gap is the relative difference between the generalized cost of the actual path and the optimal path from the last iteration, where the generalized cost includes monetized travel time (using value of time), and travel cost in terms of tolls.

$$DTA\ relattive\ gap = \frac{Experienced\ generalized\ cost - Optimal\ generalized\ cost}{Experienced\ generalized\ cost} \tag{4-3}$$

In the next section, the proposed solution algorithm is presented in Figure 4, followed by description of adopted strategies and steps of the algorithm.

4.3 Algorithm

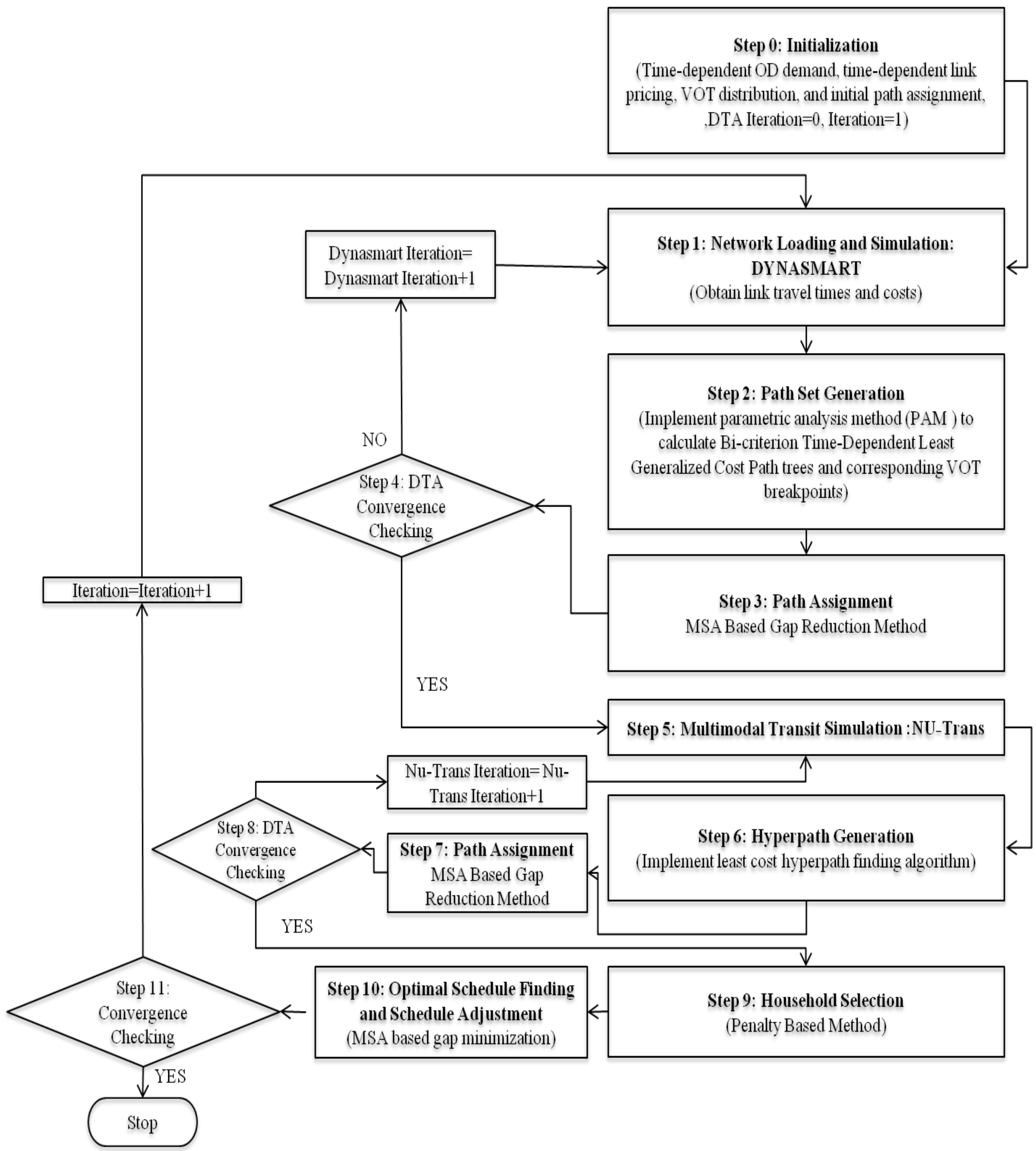


Figure 4 Overall Steps of the Solution Algorithm

4.3.1 Schedule Adjustment Strategies

To select households for schedule adjustment, different selection strategies are proposed. Note that if we keep changing the schedule of all households in each iteration, in addition to computational difficulties, there might be continual fluctuations in some users' schedules, possibly resulting in diverging solutions between iterations.

1. Unrealistic-only method: This selection method adjusts the schedules just for households with unrealistic schedules as defined in the research framework section.
2. Random selection method: This selection method, in addition to all households with unrealistic schedules, adjusts other households randomly with a certain probability. Therefore, at each iteration, the household with a realistic schedule might be selected for the schedule adjustment with a probability of $\left[\frac{Beta}{1+level\ 2\ iteration\ number}\right]$.
3. Penalty-based method: This selection method selects households based on the ratio of each household's total inconsistent schedule penalty to the maximum inconsistent schedule penalty of the whole population. Households with higher total inconsistent schedule penalties have a higher chance of being selected for the schedule adjustment in this method. The probability of being adjusted is equal to:

$\left[\frac{Household's\ inconsistent\ schedule\ penalty}{(1+level\ 2\ iteration\ number)\times Max\ household\ inconsistent\ schedule\ penalty\times beta2}\right]$. Note that Beta2 is a user specified input parameter, which is used to control the number of households selected.

4.3.2 Path Swap Strategies

In the first iteration of the integration, the roadway simulator finds the least generalized cost paths for all vehicles; these paths are then stored throughout the integration. In higher iterations of equilibrium, the roadway simulator only swaps the stored simulated paths to the optimal path, for the probabilistically selected individuals based on the generalized cost gap. Two route swap strategies are proposed and tested, and the details of these strategies are described as follows.

1. All travelers with global MSA strategy. This strategy incorporates the iteration number into the MSA factor. It swaps the simulated path to the optimal path for all individuals with a probability of $\left[\frac{DTA\ relative\ gap}{DTA\ iteration\ number+Integration\ iteration\ number}\right]$
2. All travelers without global MSA strategy. This strategy does not consider the integration iteration number when calculating the MSA factor. Therefore, it swaps the simulated path to the optimal path for all individuals with a probability of $\left[\frac{DTA\ relative\ gap}{DTA\ iteration\ number+1}\right]$

   In the remainder of this section, we denote the DTA iteration number by $n_{DTA}$and the integration iteration number by $n_{Integration}$.

4.3.3 Steps of the Algorithm

Step 0: Travelers' trip itineraries obtained from the ABM model according to anticipated time dependent travel times, set $n_{DTA} = 1$, and $n_{Integration} = 1$

Step 1: Load the activity-trip chain demand to the road network using a dynamic traffic assignment and simulation tool (DYNASMART) (Jayakrishnan et al. (1994); Mahmassani (2001); Halat et al. (2016)) to obtain an initial vector of path flows $r^{n_{DTA}}$

Step 2: Find the search direction (Auxiliary paths) by performing the time dependent least generalized cost path finding

Step 3: Calculate the route choice probability, update the traveler's path by switching travelers' path to newly found least cost paths, and update the path flows $r^{n_{DTA}+1}$ (MSA-based gap reduction method)

$$Route\ Swap\ Probablity\text{:}\ \frac{1}{1+\ n_{DTA}} * \frac{(\text{experienced generalized path cost}-\text{least generalized path cost})}{\text{experienced generalized path cost}}$$

Or

$$Route\ Swap\ Probablity\text{:}\ \frac{1}{1+\ n_{DTA}+\ n_{Integration}} * \frac{(\text{experienced generalized path cost}-\text{least generalized path cost})}{\text{experienced generalized path cost}}$$

Step 4: If the average relative gap < threshold, or $\text{Iteration}_{\text{Dynasmart}}$ =Max $\text{Iteration}_{\text{Dynasmart}}$, go to step 5; otherwise go to step 1

$$Relative\ Gap = \frac{(\text{experienced generalized path cost} - \text{least generalized path cost})}{\text{experienced generalized path cost}}$$

Step 5: Load the demand to the network using a multimodal dynamic traffic assignment and simulation tool (NU-Trans) (Verbas et al. (2015); Verbas et al. (2016))

Step 6: Perform the time dependent least cost hyper path finding algorithm

Step 7: Update path assignments (MSA-Based Gap Reduction Method)

Step 8: If the average relative gap < threshold, or $\text{Iteration}_{\text{NU-Trans}}$ =Max $\text{Iteration}_{\text{NU-Trans}}$, go to step 8; otherwise go to step 5

Step 9: Select the households for schedule adjustment adopting a selection strategy described earlier coupled with the MSA factor $\frac{1}{1+n_{Integration}}$

Step 10: Find the optimal schedules, and adjust the schedules of the selected households (MSA Based Gap Reduction Method)

Step 11: If the average inconsistent schedule penalty<threshold, stop; otherwise go to step 1

First, starting from a feasible desired schedule obtained from an ABM model, a dynamic network loading and simulation is performed. Then, a least generalized cost path finding procedure is performed to find the set of least cost paths, and update the path flows given a particular step size. The resulting time-dependent network conditions are obtained from the simulation of updated path flows. An optimal schedule-finding procedure is performed in the next level to find the set of feasible optimal schedules given the dynamic network conditions. An appropriate step size is used to modify the schedules of the selected subset of the households with certain deviations from their preferred schedule.

This approach allows us to capture:

- *Individual schedule consistency*: Activity start time should correspond to the preceding trip arrival time and activity end time should correspond to the following trip departure time. In the existing ABMs, certain steps have been made to ensure a partial consistency between departure and arrival times, as well as duration at the entire-tour level. The proposed approach in this study, however, allows one to include trip details, and control for feasibility of travel times within the tour framework. Certain attempts to incorporate trip departure time choice in a framework of trip chains have been made within DTA models, DYNASMART in particular. However, these attempts were limited to the tour level only, and also required a simplified representation of activity duration profiles. This constraint was specifically addressed in the course of the current study by developing a schedule adjustment algorithm and corresponding software module.
- *Physical flow process properties*: These “hard” constraints apply to the network loading and flow propagation aspects in DTA procedures. Physical principles, such as

conservation of vehicles at nodes, are adhered to strictly (e.g. no vehicles should simply be lost or otherwise disappear from the system). Thus, travel times that are used to equilibrate the schedule are fully consistent with the DTA network state.

- *Equilibrium travel times*: Travel times between activities in the schedule generated by the demand model should correspond to realistic network travel times for the corresponding origin, destination, departure time, and route generated by the traffic simulation model with the given demand. While most of the ABMs include a certain level of demand-supply equilibration, they are limited to achieving stability in terms of average travel times. There is no control for consistency within the individual daily schedule. The challenge is to couple this constraint with the previous one, i.e. ensure individual schedule continuity with equilibrium travel times. This is addressed in the current study by monitoring schedule inconsistency in the equilibration.
- *Realistic activity timing and duration*: Activities in the daily schedule have to be placed according to behaviorally-realistic temporal profiles. Each activity has a preferred start time, end time, and duration formalized as a utility function with multiple components. In the presence of congestion and pricing, travelers may deviate from the preferred temporal profiles (including even cancel or change order of activity episodes). However, this rescheduling process should obey utility-maximization rules over the entire schedule and cannot be effectively modeled by simplified procedures that adjust departure time for each trip separately. None of the existing operational ABMs explicitly control for activity durations, although some of them control for entire-tour durations, or the duration of the activity at the primary destination. DTA models that incorporate departure time choice

have been bound to a simplified representation of temporal utilities and limited to trip chains in order to operate within a feasible dimensionality of the associated choices when combined with the dynamic route choice. This constraint expresses consistency between activity start and end times as controlled by the schedule adjustment module.

4.4 Integration Components

The major components used to implement the integration of DTA and schedule adjustment modules are a roadway simulator (DYNASMART), a transit simulator (NU-TRANS), a schedule adjustment module, and the Tour Processor.

DYNASMART performs the roadway simulation and dynamic traffic assignment. It models the evolution of traffic flows in a traffic network resulting from the decisions of individual travelers seeking to fulfill a chain of activities at different locations in a network, over a given planning horizon. Each DTA run consists of an outer loop and an inner loop. In the outer loop, the time dependent shortest paths (TDSP) are calculated, and an initial traffic assignment and simulation is performed. Once the outer loop is completed, the inner loop swaps the simulated paths of selected travelers to optimal paths found among the existing paths of the last outer iteration. This method reaches the user equilibrium faster than generating new paths at each iteration, as the shortest path calculation is time consuming. The overall framework of DYNASMART on activity trip chains is discussed in depth in Chapter 6 of this study.

NU-TRANS is a multi-modal transit network analysis and evaluation tool. With its three main components, it is capable of finding a multi-modal, time-dependent least cost hyperpath (i.e. optimal strategy) for a transit passenger with a given origin, destination, departure time and user-specific attributes. Additionally, it assigns passengers probabilistically to an elementary path derived from the optimal strategy, and simulates pedestrians, bicyclists, passengers and transit vehicles second-by second on a large-scale transit network. More details on the transit simulator used in this study (NU-TRANS) can be found in (Verbas et al. (2015); Verbas et al. (2016)).

The Tour Processor rearranges the ABM output into vehicle trip chains subsequently loaded onto the highway and transit networks by DYNASMART and NU-TRANS, respectively. The Tour Processor takes the information from ABM as the input file, and reorganizes them into the format required by the multi-modal microsimulation. It also takes care of data transfers inside the multi-modal microsimulation between the NU-TRANS and DYNASMART (such as stop specific dwell times, park-and-ride and kiss-and-ride intermediate transit destinations, experienced travel time of buses, etc.). The schedule adjustment module is called by the Tour Processor to generate new travelers' activity schedules based on the simulated travel time.

The schedule adjustment module adjusts the individual activity-travel schedules (activity start and end times, or equivalently trip arrival and departure times) generated by the ABM based on the individual simulated travel times produced by DYNASMART and NU-TRANS, to ensure the schedule consistency for each person and household. For each household, the total schedule inconsistent penalty is calculated as described in Section 3.2. This penalty is used as the objective function of the Individual Schedule Adjustment Model.

The DTA Gap in DYNASMART and NU-TRANS is defined as the difference between the generalized cost of the actual path and the optimal path, calculated based on the latest traffic simulation.

A methodology and solution algorithm was introduced in Sections 4.1 to 4.4. As was discussed in Chapter 3, the existence or uniqueness of a solution to the equilibrium problem defined in this dissertation are not guaranteed. However, the algorithmic convergence of the proposed solution method could be demonstrated through numerical results. In the next section, the above methodology, including different schedule adjustment strategies, are applied for the integration on a test network as the case study. Note that the above-mentioned convergence criteria are incorporated to assess these different strategies. Further, the author aims to represent the convergence characteristics and applicability of the proposed solution algorithm to large-scale real-world networks through numerical results.

4.5 Numerical Results

4.5.1 Sub-area Test Network

The network considered in this chapter is the Chicago sub-area network (Figure 5), which is extracted from the Chicago full regional network. There are 13 freeways, 334 arterials, 137 nodes, and 57 traffic analysis zones (TAZ) in the network. The simulation runs start at 3:00 AM and lasts for 24 hours. A total of 84,954 travelers from 34,170 households are considered for the simulation. Each simulated traveler has at least one activity in its schedule, and the maximum number of activities considered here is 21. The results presented here for the sub-area test network are based on 10 iterations, with the MSA factor of $^{1}/_{1 + Iteration}$ for the schedule adjustment and $^{1}/_{1 + Iteration_{DTA} + Iteration}$ for the DTA module. For more detailed discussion of the numerical convergence under various MSA strategies, we refer the reader to Xiang (Alex) Xu (2017).

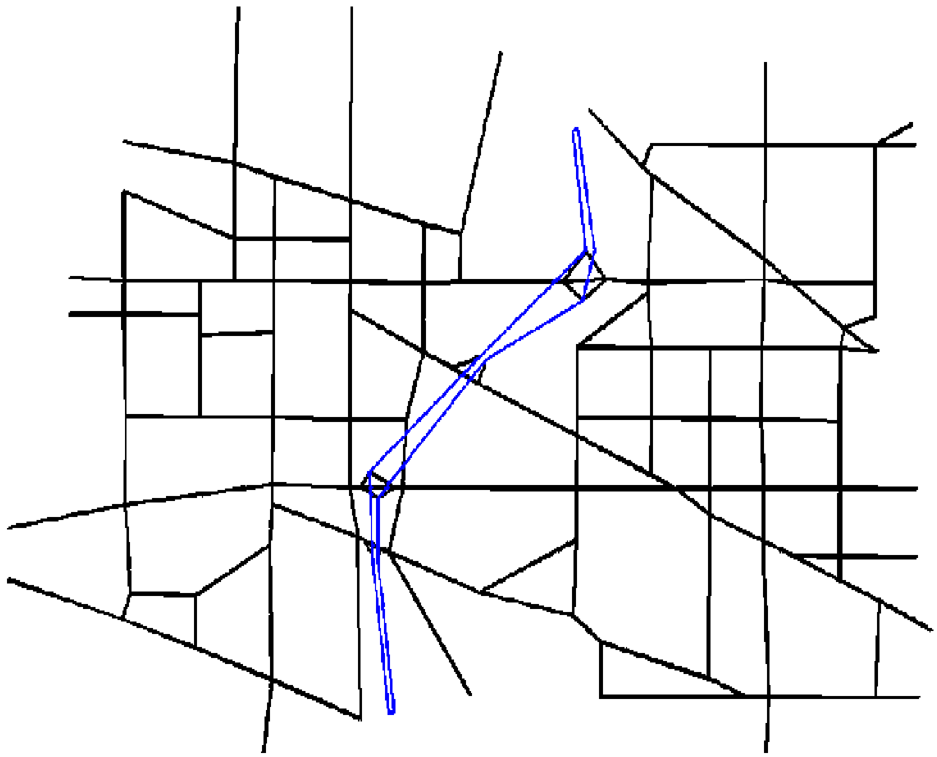
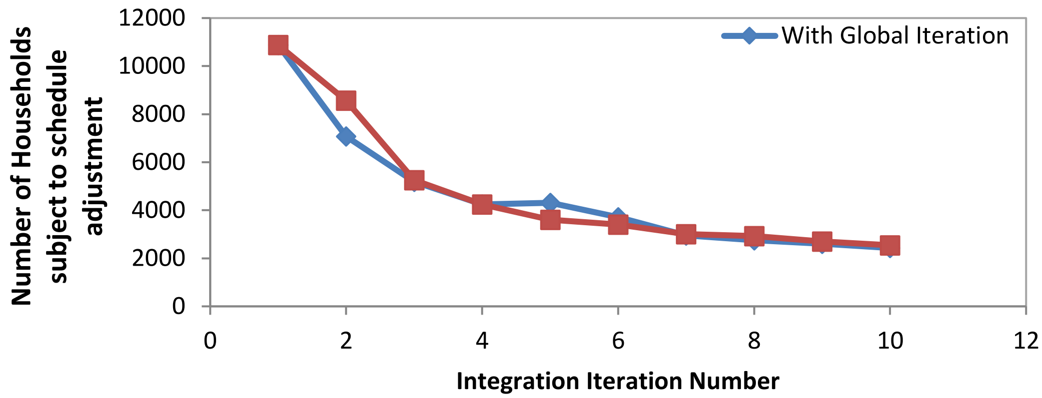
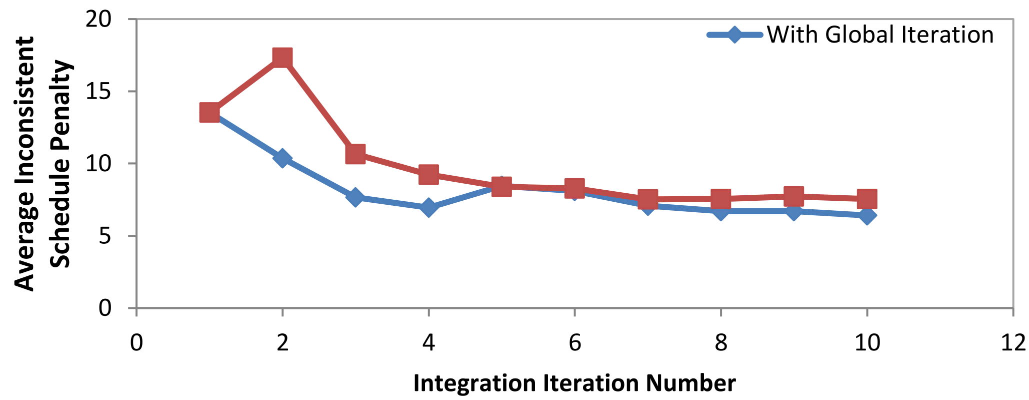


Figure 5 Sub-area Network Configuration

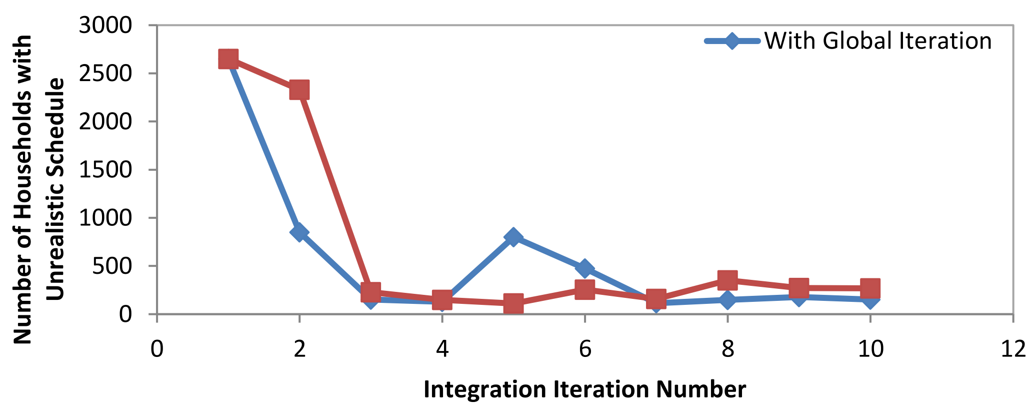
To show the effects of different MSA factors, two runs are set up for 10 iterations of the integrated framework. Both runs calculate the schedule adjustment probability based on the random selection method: (1) households with unrealistic schedules are selected with 100% chance (2) other households are selected randomly with a probability of 25% in the first iteration of level 2 integration. This probability decreases with the MSA factor for higher iterations. The first scenario considers both DTA iteration number and integration (global) iteration number to calculate the MSA factor for the roadway simulator, while the second scenario only considers the DTA iteration number. The number of households selected for the schedule adjustment is similar in these two scenarios.



(a) Number of households with modified schedule



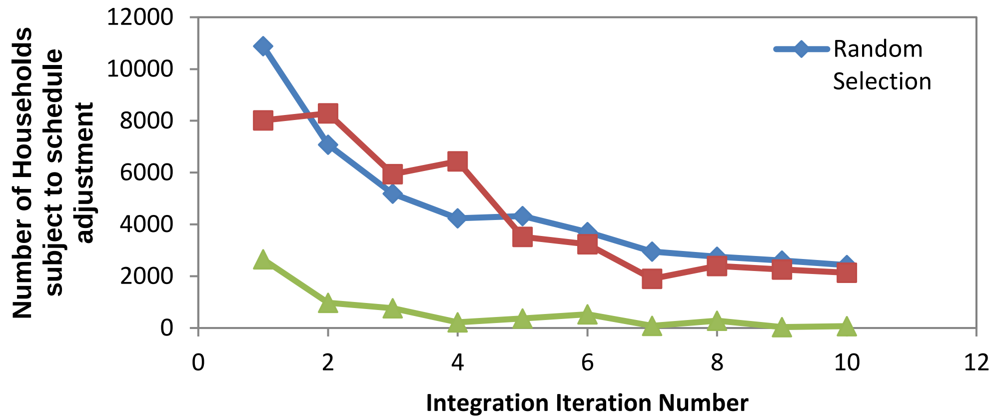
(b) Average inconsistent schedule penalty



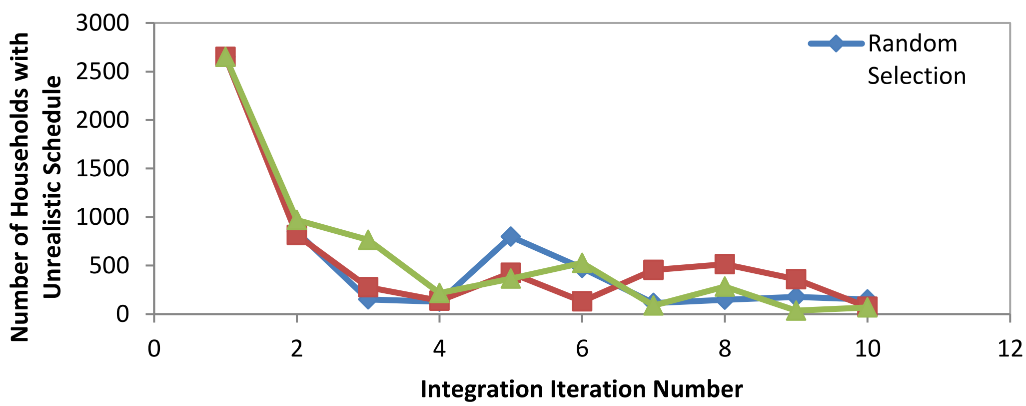
(c) Number of households with unrealistic schedules

Figure 6 Comparison of scenarios w/-w/o Integration iteration number in DTA MSA

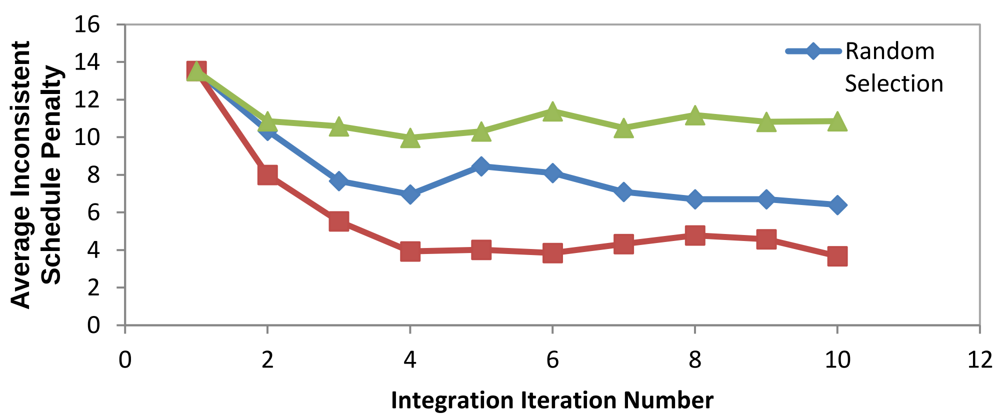
Figure 6(a) and 6(b) compare the performance of the two scenarios in terms of the number of households, subject to schedule adjustment and the average inconsistent schedule penalty, respectively. Figure 6(c) compares the other gap measure, the number of households with unrealistic schedules. All measures decrease faster and have a smaller final value in the first scenario wherein the MSA factor includes the integration iteration number. Based on the findings in Figure 6, the MSA factor that includes the iteration number of integration is selected for the subsequent numerical results.



(a) Number of households with modified schedule



(b) Average inconsistent penalty



(c) Number of households with unrealistic schedules

Figure 7 Comparison of different selection strategies for the schedule adjustment

Figure 7 compares the performance of three schedule adjustment selection strategies. The unrealistic-only method adjusts the schedule for households with unrealistic schedules. The random selection method, in addition to households with unrealistic schedules, selects other households randomly with a probability of 25% in the first iteration. The penalty-based method selects households with unrealistic schedules with 100% probability and other households with a probability calculated based on the inconsistent schedule penalty. Figure 7(a) shows that the unrealistic-only method selects the least number of households at each iteration, while the random method selects the most. Although the three methods select a different number of households, the number of households with unrealisic schedules are reduced in similar patterns (see Figure 7(b)). The final values in all three scenarios are close to each other. However, the penalty-based method generates greater and faster reduction in the average inconsistent schedule penalty, as shown in Figure 7(c). This shows that defining two convergence measures helps to distinguish these different strategies. Based on this figure, we select the penalty-based approach for the numerical results on the large-scale network presented in the next section.

4.5.2 **Chicago Full Regional Network**

In this section, the applicability of the proposed solution algorithm to a large-scale network, namely the Chicago full regional network, is presented. The network is displayed in Figure 8; it covers part of Illinois, Indiana, and Wisconsin, and is bound by Lake Michigan to the east. There are 1400 freeway corridors including I-90, I-94, I-55, I-80, etc., as well as 36,722 arterials. The network has 13,093 nodes, 40,443 links and 1,961 traffic analysis zones (TAZ). A total of 2,262,300 travelers are considered, each having at least one trip in their itineraries. The simulations start at 3:00 AM, and last for 24 hours. Note that our focus here is to show the applicability of the algorithm to large-scale networks, meaning we do not aim to analyze different MSA factors or household selection strategies.

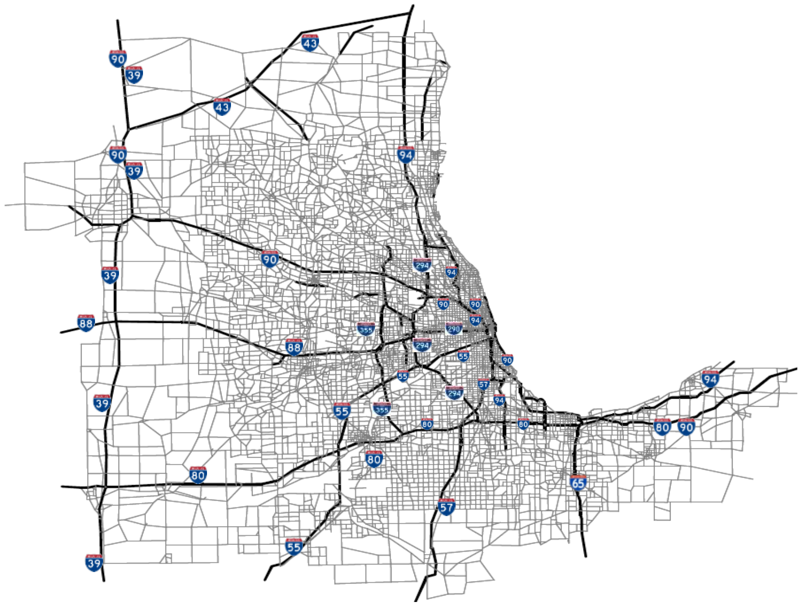


Figure 8 Chicago Full Regional Network Configuration

Figure 9 represents variations of a measure of gap with the iteration number. The represented measure is the number of households/travelers with unrealistic schedules (negative activities), where one or more of their trip travel times are so long that they cannot accommodate it by shortening their activity duration. As can be seen, both the number of travelers and number of households with unrealistic schedules decrease as the iteration number increases.

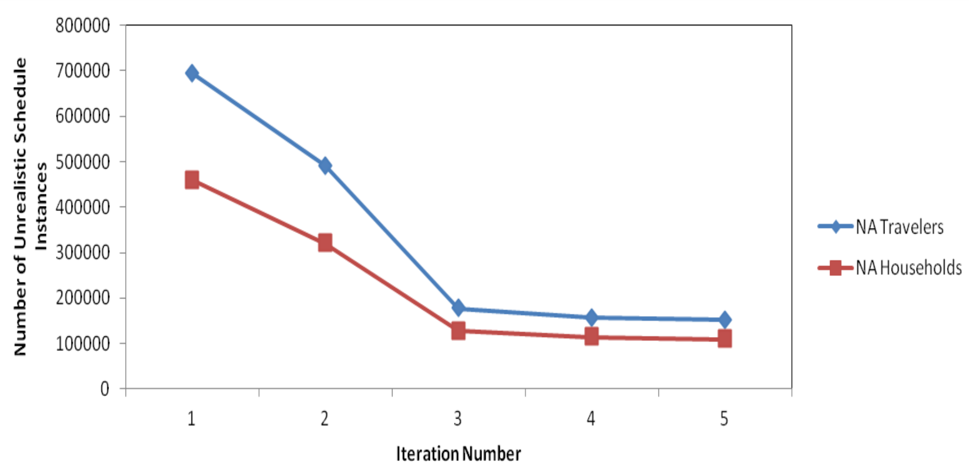


Figure 9 Number of Households and Travelers with Negative Activity (NA)

4.6 Summary

In this chapter a solution algorithm was proposed for the equilibrium problem defined in Chapter 3. The proposed algorithm was performed on two networks of different sizes, and it was shown that the adopted methodology results in reduction in the gap measures (algorithmic convergence). The next chapter provides a variation of the equilibrium problem addressed in Chapters 3 and 4, by allowing individuals to cancel (remove) and activity from their preplanned activity chains.

5 INCORPORATION OF ACTIVITY CANCELLATION

In this chapter, the disutility and equilibrium models presented in Chapter 3 are extended through the incorporation of activity/trip cancellation. Unlike the problem presented in Chapter 3, the number of activities/trips of a given individual is dynamic; however, the order of the activities remains fixed. In other words, individuals have a set of preplanned activities, which they do not wish to reorder, but can cancel (remove) one activity from their preplanned activity chain. The presentation in this chapter follows the same general structure as the previous chapter. Many details and steps are similar; however, they are repeated here for completeness.

5.1 Problem Statement

A dynamic network is considered, where τ_0 is assumed as the earliest possible departure time from all origin nodes, σ as a small time interval during which no noticeable change in traffic conditions happens, and K as a large value in a way that $\tau_0 + K\sigma$ covers the entire planning horizon. The planning horizon is considered to be discretized into a set of smaller intervals $\Gamma = \{\tau_0, \tau_0 + \sigma, \tau_0 + 2\sigma, \tau_0 + 3\sigma, \ldots, \tau_0 + K\sigma\}$. A total of HH households is assumed to comprise the network, each having $M(hh)$ members, resulting in total of IT individual travelers. The set j is defined as the set containing the household members that make joint trips and the associated joint trips, and the set Ȩ $= \{1,2, \ldots, \mathrm{IT}\}$ comprises individual travelers in the network. In the formulation presented in this section, our assumption is that no loops exist in the traveler's agenda.

It is assumed that each household member i has a set of preferred activity arrival A_i^{od}, departure D_i^{od}, and duration time $T_i^{od}$for each trip from o to d within his/her agenda. The author associates measures of schedule inconsistency, capturing the early (E) or late (L) arrivals/departures to/from and/or duration deviations, for all activities. Associated with each type of schedule inconsistency, three penalty factors are considered: a fixed penalty (FP^i), a variable penalty (P^i), and penalty for above a certain threshold (${P^b}^i$).

${P_{LD}}^{i,m,od}$: (${FP_{LD}}^{i,m,od}, {P^b}_{LD}{}^{i,m,od}, {P_{LD}}^{i,m,od}$) Penalty associated with the late departure of trip form o to d of traveler i with user class m

${P_{ED}}^{i,m,od}$: (${FP_{ED}}^{i,m,od}, {P^b}_{ED}{}^{i,m,od}, {P_{ED}}^{i,m,od}$) Penalty associated with the early departure of trip form o to d of traveler i with user class m

${P_{LA}}^{i,m,od}$: (${FP_{LA}}^{i,m,od}, {P^b}_{LA}{}^{i,m,od}, {P_{LA}}^{i,m,od}$) Penalty associated with the late arrival of trip form o to d of traveler i with user class m

${P_{EA}}^{i,m,od}$: (${FP_{EA}}^{i,m,od}, {P^b}_{EA}{}^{i,m,od}, {P_{EA}}^{i,m,od}$) Penalty associated with the early arrival of trip form o to d of traveler i with user class m

${P_{LT}}^{i,m,od}$: (${FP_{LT}}^{i,m,od}, {P^b}_{LT}{}^{i,m,od}, {P_{LT}}^{i,m,od}$) Penalty associated with the activity duration lengthening of trip form o to d of traveler i with user class m

${P_{ET}}^{i,m,od}$: (${FP_{ET}}^{i,m,od}, {P^b}_{ET}{}^{i,m,od}, {P_{ET}}^{i,m,od}$) Penalty associated with the activity duration shortening of trip form o to d of traveler i with user class m

$TS_{In}^{i,m,od}$ Threshold associated with the schedule inconsistency of type In for trip form o to d of traveler i with user class m

$\delta S_{In}^{i,od}$: $(\delta D_{L,E}^{i,od}, \delta A_{L,E}^{i,od}, \delta T_{L,E}^{i,od})$ Schedule inconsistency of type In for trip from o to d of traveler i with user class m

$\Delta S_{In}^{i,m,od}$: $(\Delta D_{L,E}^{i,od}, \Delta A_{L,E}^{i,od}, \Delta T_{L,E}^{i,od})$ Inconsistent schedule penalty of type In for trip from o to d of traveler i with user class m

$\Delta S^{i,m,od}$ Inconsistent schedule penalty for trip from o to d of traveler i with user class m

$$\Delta S_{In}^{i,m,od} = \begin{cases} FP_{In}^{i,m,od} + P_{In}^{i,m,od} \times \delta S_{In}^{i,od} & \delta S_{In}^{i,od} < TS_{In}^{i,od} \\ FP_{In}^{i,m,od} + P_{In}^{i,m,od} \times \delta S_{In}^{i,od} + P{^b}_{In}^{i,od} \times (\delta S_{In}^{i,od} - TS_{In}^{i,od}) & \delta S_{In}^{i,od} > TS_{In}^{i,od} \end{cases} \quad (5\text{-}1)$$

$$\Delta S^{i,m,od} = \Delta S_D{}^{od} + \Delta S_A{}^{od} + \Delta S_T{}^{od} \quad (5\text{-}2)$$

A time space diagram of individual activity sequences and travel choices is depicted in Figure 10.

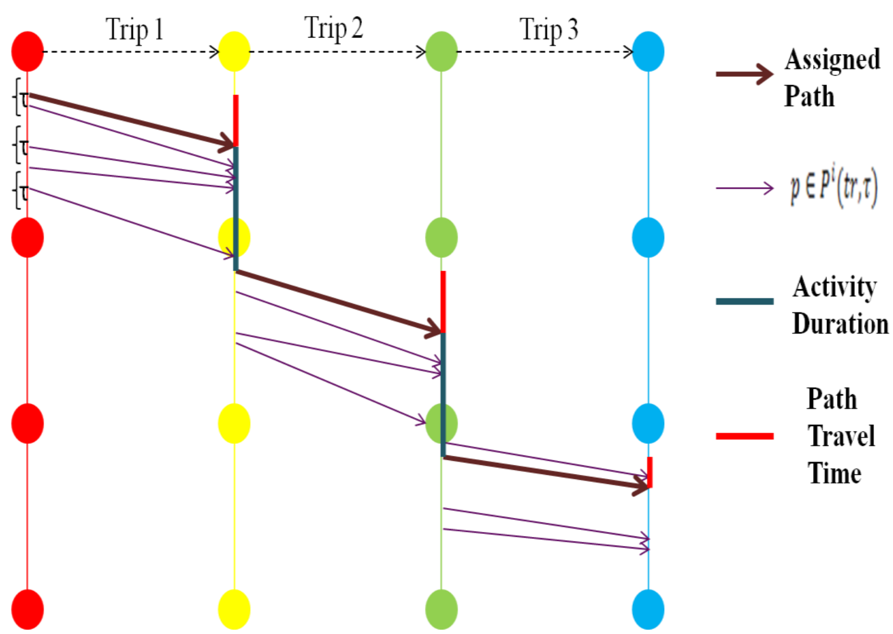


Figure 10 Time Space Network Representation

Other notations are as follows:

o Subscript belonging to an origin node

d Subscript belonging to a destination node

τ Subscript for a departure time interval

o_0 Subscript denoting origin of the first trip

d_0 Subscript denoting destination of the last trip

$P(o,d)$ The set of all feasible paths associated with o, d pair

p Subscript for a path $p \in p(o,d,)$

tr Subscript corresponding to trip tr of traveler

$TR(i)$ Number of trips of traveler i

φ^i The set containing feasible origin destination pairs (in a predefined order, i.e. activities can only be skipped for cancellation purposes) associated with planned trips of individual i

$X_{i,od}^{\tau p}$ Binary decision variable equal 1 if traveler i chooses to have a trip from o to d on path p at departure time interval τ, and 0 otherwise

$r_{od}^{\tau,p}$ Number of trips from o to d departing o at time interval τ and assigned to path p

r Vector of path flows

r' Link flow vector

B Link-path incidence matrix

$f_{a,t}(r'_{a,t})$ Travel time on link a at time t as a function of flow on link a at time t

$GC_{od,m}^{\tau,p}$ Path generalized cost for individuals of user class m departing path p at departure time τ

$TT_{od}^{\tau,p}$ Experienced path travel time for trips from o to d at departure time interval τ assigned to path p

$TC_{od}^{\tau,p}$ Experienced path travel cost for trips from o to d at departure time interval τ assigned to path p

ΔU_i Disutility function of traveler i associated with deviations from preferred schedule

We define the following scheduling equations for $\forall i \in$ Ę, $od \in \varphi^i$:

The disutility associated with the schedule/route choice of an individual traveler i can be considered as the summation of travel cost and schedule inconsistency as follows:

$$\sum_{od \in \varphi^i} \sum_{\tau \in \Gamma} \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \times GC_{od,m}^{\tau,p} + X_{i,od}^{\tau,p} \times \Delta S^{i,m,od} \qquad \forall i \in Ę \tag{5-3}$$

However, individuals' activity scheduling decisions are dependent on other household members. For instance, they may adjust their departure time in a manner consistent with other members who are participants of a joint trip. Hence, it is essential that the disutility be attributed to the whole household, leading to the following definition of household disutility for every household hh:

$$\sum_{i=1}^{M(hh)} \sum_{od \in \varphi^i} \sum_{\tau \in \Gamma} \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \times GC_{od,m}^{\tau,p} + X_{i,od}^{\tau,p} \times \Delta S^{i,m,od} \tag{5-4}$$

The decision variable $X_{i,tr}^{\tau,p}$ should satisfy the following conditions:

$$0 \leq \sum_{\tau \in \Gamma} \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \leq 1 \qquad \forall i \in Ę, od \in \varphi^i \tag{5-5}$$

(5-6)

$$TR(i) - 1 \leq \sum_{od\in\varphi^i} \sum_{\tau\in\Gamma} \sum_{p\in P(o,d)} X_{i,od}^{\tau,p} \leq TR(i) \qquad \forall i \in \text{Ę}$$

$$\sum_{o_0 d\in\varphi^i} \sum_{\tau\in\Gamma} \sum_{p\in P(o_0,d)} X_{i,o_0 d}^{\tau,p} = 1 \qquad \forall i \in \text{Ę} \tag{5-7}$$

$$\sum_{od\in\varphi^i} \sum_{\tau\in\Gamma} \sum_{p\in P(o,d)} X_{i,od}^{\tau,p} - \sum_{dw\in\varphi^i} \sum_{\tau\in\Gamma} \sum_{p\in P(d,w)} X_{i,dw}^{\tau,p} = 0 \qquad \forall d \in \Xi^i \tag{5-8}$$

$$\sum_{od_0\in\varphi^i} \sum_{\tau\in\Gamma} \sum_{p\in P(o,d_0)} X_{i,0d_0}^{\tau,p} = 1 \qquad \forall i \in \text{Ę} \tag{5-9}$$

$$X_{i,od}^{\tau,p} = X_{i\prime,od}^{\tau,p} \quad \forall j \in J,\ \forall (i,od) \in j,\ \forall (i',od) \in j \tag{5-10}$$

$$X_{i,od}^{\tau,p} \in \{0,1\} \quad \forall i \in \text{Ę}, od \in \varphi^i, \tau \in \Gamma \tag{5-11}$$

Inequality (5-5) ensures that all trips by the individual are assigned at most one path. Due to the author's assumption on allowing at most one of the traveler's trips to be cancelled, there should be exactly one path assigned to trips that are not cancelled, and a maximum of $TR(i) - 1$ paths assigned to traveler i. Equation (5-6) ensures that a traveler has at most one cancelled trip, and all the other trips of the traveler are assigned exactly one path. Equation (5-7) enforces that a traveler has exactly one trip out of the origin of the first trip in the traveler's trip chain, with exactly one path assigned to it. Equation (5-8) ensures the connectivity of the consecutive trips of the traveler, where Ξ^i is the set including all destination nodes in the travel agenda of traveler i.

Equation (5-9) enforces that a traveler has exactly one trip to the final destination, with exactly one path assigned to it. Equation (5-10) maintains the consistency of travel choices of travelers who have joint trips. Equation (5-11) restricts the decision variable to be a binary variable, which only takes values 0 or 1.

The travel/route decisions of all individuals in the network result in path flows, which in turn yield the path travel times and generalized costs:

$$\sum_{i=1}^{IT} \sum_{od \in \varphi^i} X_{i,od}^{\tau,p} = r_{od}^{\tau,p} \qquad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d) \tag{5-12}$$

$$r' = B.r \tag{5-13}$$

$$TT_{od}^{\tau,p} = \sum_{(a,t) \in (p,\tau)} f_{a,t}\left(r'_{a,t}\right) \qquad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d) \tag{5-14}$$

$$GC_{od,m}^{\tau,p} = \alpha(m) * TT_{od}^{\tau,p} + TC_{od}^{\tau,p} \qquad \forall\, o \in O, d \in D, \tau \in \Gamma, p \in P(o,d), m \in M \tag{5-15}$$

Equation (5-12) relates the flow on route p and departure time τ to decision variables $X_{i,tr}^{\tau,p}$ of all travelers in the network. Equation (5-13) yields the dynamic link flows, and equation (5-14) defines the dynamic path travel time as dependent on the dynamic link flows. In equation (5-14), (a,t) consists of the links (a) that belong to the path (p) as well as the associated link departure times (t), which for each link is defined by the addition of previous links' dynamic travel times to the first link's departure time (τ).

Dep_i^{tr} Adjusted departure time of trip tr of individual i

A_i^{od} Desired arrival time of the trip from o to d of traveler i

D_i^{od} Desired departure time of the trip from o to d of traveler i

T_i^{od} Desired duration of activity at d following the trip from o to d of traveler i

$T_i^{tr^{Min}}$ Minimum duration of activity following trip tr of traveler i

Additionally, the following scheduling constraints are defined for $\forall i \in Ȩ, od \in \varphi^i$:

$$Dep_i^{od} \geq \tau \times \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \quad \forall i \in Ȩ, od \in \varphi^i, \tau \in \Gamma \tag{5-16}$$

$$\sum_{od \in \varphi^i} \sum_{\tau \in \Gamma} \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \times (Dep_i^{od} + TT_{i,od}^{\tau,p} + T_i^{od^{Min}}) \leq \sum_{dw \in \varphi^i} \sum_{\tau \in \Gamma} \sum_{p \in P(o,d)} X_{i,od}^{\tau,p} \times Dep_i^{dw} \quad \forall i \in Ȩ, \forall d \in \Xi^i \tag{5-17}$$

$$Dep_i^{od} \geq 0 \qquad \forall i \in Ȩ, od \in \varphi^i \tag{5-18}$$

For the sake of simplicity, a new term is introduced to represent the travel time that a traveler experiences on path p: $ETT_i^{od} = \sum_{\tau \in \Gamma} \sum_{p \in P(od)} X_{i,od}^{\tau,p} * TT_{i,od}^{\tau,p}$

Inequality (5-16) enforces that Dep_i^{tr} be as large as the selected departure time τ. Constraint (5-17) ensures the temporal consistency of the subsequent trips of a traveler (i), i. e. no trip can depart earlier than its previous trip. Constraint (5-18) restricts the departure time variable to be nonnegative.

$$\delta S_{LA}^{i,od} = \begin{cases} Dep_i^{od} + ETT_i^{od} - A_i^{od} & Dep_i^{od} + ETT^{i,od} \geq A_i^{od} \\ 0 & Dep_i^{od} + ETT^{i,od} < A_i^{od} \end{cases} \tag{5-19}$$

$$\forall i \in E, od \in \varphi^i$$

$$\delta S_{EA}^{i,od} = \begin{cases} A_i^{od} - Dep_i^{od} + ETT_i^{od} & A_i^{od} \geq Dep_i^{od} + ETT^{i,od} \\ 0 & A_i^{od} < Dep_i^{od} + ETT^{i,od} \end{cases} \tag{5-20}$$

$$\forall i \in E, od \in \varphi^i$$

$$\delta S_{LD}^{i,od} = \begin{cases} Dep_i^{od} - D_i^{od} & Dep_i^{od} \geq D_i^{od} \\ 0 & Dep_i^{od} < D_i^{od} \end{cases} \tag{5-21}$$

$$\forall i \in E, od \in \varphi^i$$

$$\delta S_{ED}^{i,od} = \begin{cases} D_i^{od} - Dep_i^{od} & D_i^{od} \geq Dep_i^{od} \\ 0 & D_i^{od} < Dep_i^{od} \end{cases} \tag{5-22}$$

$$\forall i \in E, od \in \varphi^i$$

$$\delta S_{LT}^{i,od} \tag{5-23}$$

$$= \begin{cases} Dep_i^{dw} - ETT_i^{od} - Dep_i^{od} - T_i^{od} & Dep_i^{dw} - ETT_i^{od} - Dep_i^{od} \geq T_i^{od} \\ 0 & Dep_i^{dw} - ETT_i^{od} - Dep_i^{od} < T_i^{od} \end{cases}$$

$$\forall i \in E, od \in \varphi^i$$

$$\delta S_{ET}^{i,od} = \tag{5-24}$$

$$= \begin{cases} T_i^{od} - Dep_i^{dw} + ETT_i^{od} + Dep_i^{od} & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} \geq T_i^{tr} \\ 0 & Dep_i^{tr+1} - ETT_i^{tr} - Dep_i^{tr} < T_i^{tr} \end{cases}$$

$$\forall i \in E, od \in \varphi^i$$

Equations ((5-19)-(5-24)) represent the schedule inconsistency terms, which are defined as functions of departure times.

In this section, a disutility function associated with an individual's travel pattern is defined. Assuming each individual traveler as a user who makes travel choices to minimize experienced disutility, a user equilibrium framework is adopted in the next section based on the following conceptual framework stated in Chapter 3, which is reiterated here. In the real world, individuals make travel choices to participate in a sequence of activities. However, when executed, their travel experience may not be consistent with the planned schedules due to congestion, accidents, transit delays, etc. As a result, they try to minimize the associated disutility by making adjustments to their activity/travel choices. They continue the adjustment process until they cannot achieve any further improvement, meaning that the system reaches an equilibrium state. Therefore, an ideally-equilibrated network can be viewed as the one in which individuals' planned schedules will be replicated when executed. In other words, at the equilibrium state, none of the travelers are able to decrease their disutility by changing departure times (schedule) or switching paths associated with their trips. In the next section, a user equilibrium problem formulation is presented based on the defined disutility.

5.2 Model Formulation

The equilibrium problem can be conceptually formulated as a fixed-point equilibrium problem in the closed, bounded, and convex path flow space Ω:

$$r^* = R(AC(SA\left(SP\left(C(r^*)\right)\right))) \qquad (5\text{-}25)$$

r^* Vector of optimal path flows

C Vector of path costs

SP Vector of schedule penalties

SA Schedule adjustment operator

AC Activity cancellation function

R Path assignment operator

r^* represents the equilibrium dynamic path flow vector obtained from the dynamic traffic assignment model. The path cost function C produces time-dependent path generalized costs (or travel times) and is a function of the vector of dynamic path flows r. SP is the vector of schedule penalties (schedule disutility) and is considered to be a function of dynamic path costs. SA is the schedule adjustment function, which produces the vector of desired trip departure times, arrival times, and activity durations for all activities of travelers. AC is the activity cancellation operator, which applies a particular selection strategy in choosing the travelers as well as the trips to be cancelled, resulting in updated activity sequences for the selected travelers. Path assignment operator R yields the flow on all paths at each departure time interval based on adjusted travelers' schedules.

Problem Challenges

Since the existence of a solution for a fixed-point formulation depends on continuity of the functions, there is no guarantee that a solution exists for this problem formulation. That is, the problem formulation presented here comprises an activity cancellation function, which leads to discontinuities; thus, the problem might not have any solution.

As mentioned earlier, an ideally-equilibrated network can be viewed as one in which individuals' planned schedules will be replicated when executed. Hence, one might approach the equilibrium problem by attempting to minimize the gap between the experienced travel patterns and optimal/desired paths and schedules. In this problem formulation, every user faces a choice set of departure times and routes. The existence of multiple feasible departure times and paths connecting each origin-destination pair makes the size of the choice set considerably large. Moreover, the size of the choice set grows to infinity in a continuous setting. In addition, complexities arise due to constraints imposed by the interdependencies of household members as well as the spatial and temporal interdependencies of trip chains, which have to be taken into account in the dynamic user equilibrium assignment of trip chains to the transportation network. Hence, we break the problem into two sub-problems and we define two gap measures, one for the dynamic traffic assignment and one for the household activity schedule adjustment problem.

$$\sum_{o \in O} \sum_{d \in D} \sum_{\tau=1}^{T} \sum_{m=1}^{M} \sum_{p \in p(o,d)} r_{od,m}^{\tau,p} \left[GC_{\ od,m}^{\ \tau,p}(\alpha, r) - GC^{*\tau}_{\ od,m}\right] \tag{5-26}$$

$$\sum_{i=1}^{IT} \Delta S^i \tag{5-27}$$

A network equilibrium framework is adopted in approaching the problem, which is discussed in the following section.

5.3 Methodology

As mentioned earlier, the solution approaches to fixed-point formulations often involve derivative computations or require computationally burdensome direction-finding search procedures. In large size problems, ones with noisy functions, or when there is no closed form defined, such as in this study, these solution approaches should be avoided. Therefore, a descent direction method is adopted to solve the problem. As discussed in Chapter 4, the descent direction method is based on the following framework: starting from an initial feasible solution, the solution gets iteratively updated by moving towards a direction that results in reductions in the objective function. In order to overcome the fluctuations, an appropriate step size is also incorporated in the updating procedure. Evaluations of the convergence characteristics of the descent direction method involve proving that the objective function strictly decreases at every iteration given the adopted direction. However, derivative evaluations are required in performing such a proof, which as discussed earlier is computationally intensive for the current problem. Moreover, the existence of high levels of temporal and spatial correlations within the problem of activity scheduling in a dynamic network equilibrium framework makes the evaluation of derivatives very difficult. Therefore, providing a mathematical proof of the convergence characteristics of the algorithm might not be possible. However, the analytical results provide a sufficient platform for presenting convergence characteristics.

In the proposed approach, first the activity agendas are treated (sequence, number and duration of activities) as fixed, so only the path assignments are updated. Next, schedule adjustments and trip cancellations are performed, with the paths remaining unchanged. In other words, having obtained a feasible desired schedule from an ABM model, a dynamic network loading and simulation is performed. Then, a least generalized cost path finding procedure is executed to find the set of least-cost paths. The path assignments are then updated using a step size, and trip chains are simulated according to newly assigned paths to obtain the resulting time-dependent network conditions. Given the dynamic network conditions, an optimal schedule-finding procedure is performed to find the set of feasible optimal schedules (trip departure times). A subset of the households is then selected according to their deviations from preferred schedules and, using an appropriate step size, the schedules are updated. The selected subset of households whose schedules could not be improved through departure time adjustments is analyzed for possible adjustments through trip cancellation. Note that not all activities/trips of an individual (household member) are considered cancellable. For instance, trips to/from home are not cancellable. In addition, each selected individual is restricted to have at most one cancelled activity.

5.3.1 **Selection Strategies**

At the beginning of the day, individual travelers make travel plans according to anticipated network conditions (travel times). In the presence of congestion, accidents, or transit delays, they might experience travel patterns inconsistent with their planned schedules, which results in making schedule adjustments in order to minimize their disutility. In some instances of schedule inconsistency, users can adjust their schedules by shortening the activity durations. However, there might be instances of significantly long travel times, which do not allow the completion of the current trip before the subsequent trip’s planned start time. In other words, these travelers should be able to have unrealistic negative activity durations to be able to accommodate the activity in their schedule (as discussed in Chapter 4). In this chapter, the terms *travelers with unrealistic schedules* or *negative activity (NA) duration* are used to represent the latter instance of inconsistency. In the household selection approach for schedule (trip departure time) adjustments, all the households with unrealistic schedules are selected. In addition, a penalty-based method (as discussed in Chapter 4) is implemented to probabilistically select from the rest of the households. The number of households and travelers with unrealistic schedules also provides a measure of gap for the evaluation of convergence characteristics of the approach.

Once schedule adjustment is performed on all the households in the selected subset for schedule adjustment, the next step is to find the households whose inconsistent schedules could not be adjusted by departure time modifications. This group of households is referred to as *non-adjustable households* in this chapter. The non-adjustable households' cancellable activities are then evaluated to find the activities to be cancelled. Two strategies are considered for trip selection. First, the latest arrival approach (LAA) involves choosing the cancellable activity/trip with the latest arrival time to be cancelled. Second, the total penalty approach (TPA) involves choosing the cancellable activity/trip that, if cancelled, the traveler experiences the least total schedule penalty as compared to the other cancellation scenarios.

In the next section, the proposed algorithm is illustrated in Figure 11.

5.3.2 Algorithm

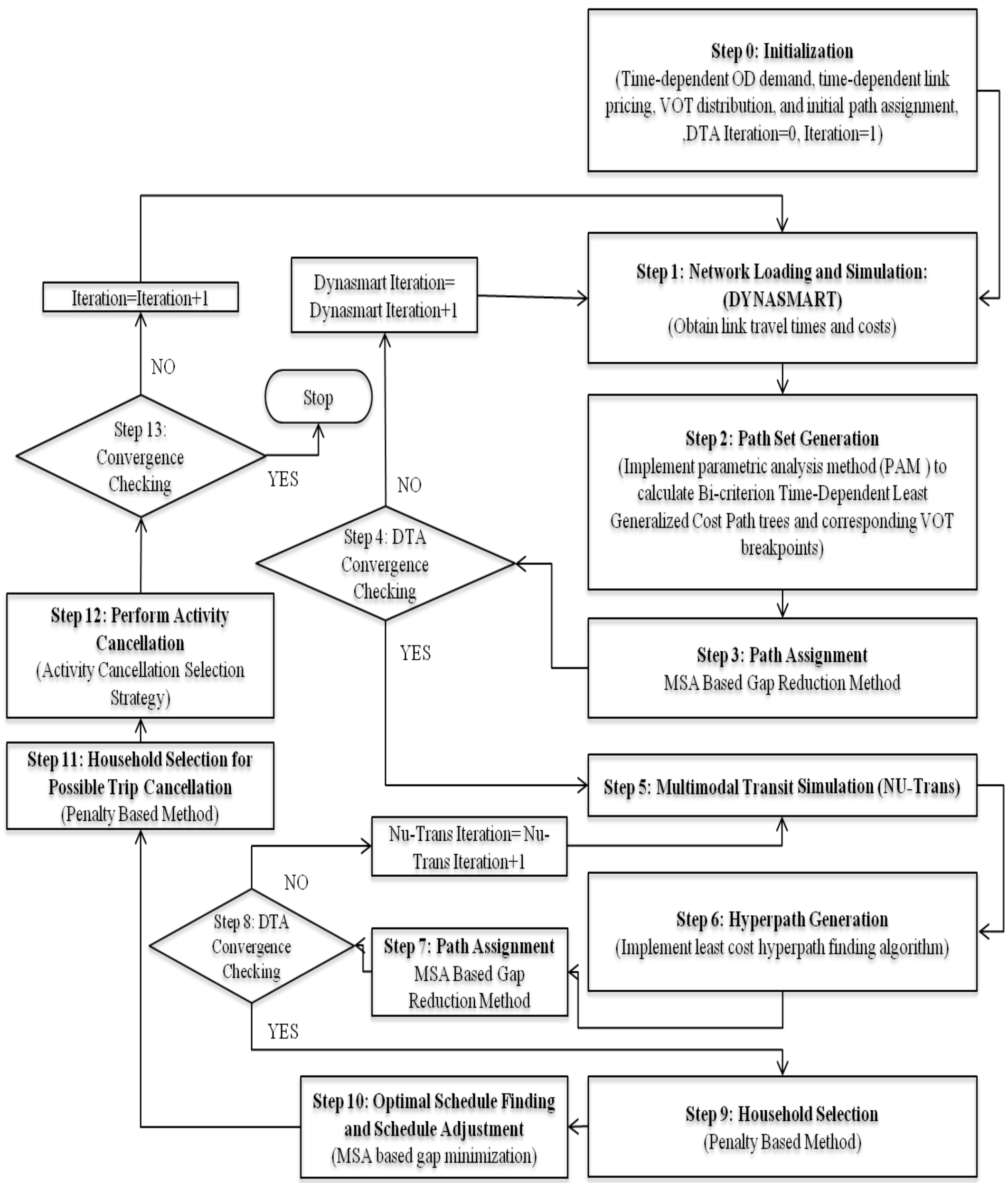


Figure 11 Overall Steps of the Solution Algorithm

Step 0: Travelers' trip itineraries obtained from the ABM model according to anticipated time dependent travel times, set $n_{DTA} = 1$, and $n = 1$

Step 1: Load the activity-trip chain demand to the road network using a dynamic traffic assignment and simulation tool (DYNASMART) (Jayakrishnan et al. (1994); Abdelghany et al. (2001); Halat et al. (2016)) to obtain an initial vector of path flows $r^{n_{DTA}}$

Step 2: Find the search direction (auxiliary paths) by performing the time-dependent least generalized cost path finding

Step 3: Calculate the route choice probability, and update the traveler's path by switching travelers' path to the newly found least cost paths, update the path flows $r^{n_{DTA}+1}$ (MSA-based gap reduction method)

$$Route\ Swap\ Probablity\colon \frac{1}{1+n_{DTA}} * \frac{(\text{experienced generalized path cost} - \text{least generalized path cost})}{\text{experienced generalized path cost}}$$

Step 4: If the average relative gap < threshold, or $\text{Iteration}_{\text{Dynasmart}}$ =Max $\text{Iteration}_{\text{Dynasmart}}$, go to step 5; otherwise go to step 1

$$Relative\ Gap = \frac{(\text{experienced generalized path cost} - \text{least generalized path cost})}{\text{experienced generalized path cost}}$$

Step 5: Load the demand to the network using a multimodal dynamic traffic assignment and simulation tool (NU-Trans) (Verbas et al. (2015); Verbas et al. (2016))

Step 6: Perform the time-dependent least-cost hyperpath-finding algorithm

Step 7: Update path assignments (MSA-Based Gap Reduction Method)

Step 8: If the average relative gap < threshold, or $\text{Iteration}_{\text{NU-Trans}}$ =Max Iteration $_{\text{NU-Trans}}$, go to step 8; otherwise go to step 5

Step 9: Select the households with unrealistic schedules (negative activity duration), and calculate the schedule adjustment probability for other households using the ratio of inconsistent penalty to maximum inconsistent penalty, coupled with the MSA factor $\frac{1}{1+iteration}$

Step 10: Find the optimal schedules, and adjust the schedules of the selected households (MSA Based Gap Reduction Method)

Step 11: Among the selected households for schedule adjustment, find the households with inconsistent schedules, meaning those that could not be modified through departure time and duration adjustments (non-adjustable households)

Step 12: Apply a trip selection strategy on the selected households for activity cancellation to find the trip to be cancelled, and cancel the trip

Step 13: If the average inconsistent schedule penalty<threshold, stop; otherwise go to step 1

5.4 Numerical Results

The network considered in this chapter is the Chicago sub-area network (Figure 12), which is extracted from the Chicago full regional network. There are 13 freeways, 334 arterials, 137 nodes, and 57 traffic analysis zones (TAZ) in the network. The simulation runs start at 3:00 AM, and last for 24 hours. A total of 76,499 travelers from 34,144 households are considered for the simulation. Each simulated traveler has at least one activity in its schedule, and the maximum number of activities considered here is 21. The results presented here for the sub-area test network are based on 10 iterations, and the MSA factor of $^1/_{1+iteration}$ for the schedule adjustment, and $^1/_{1+Iteration_{DTA}}$ for the DTA module.

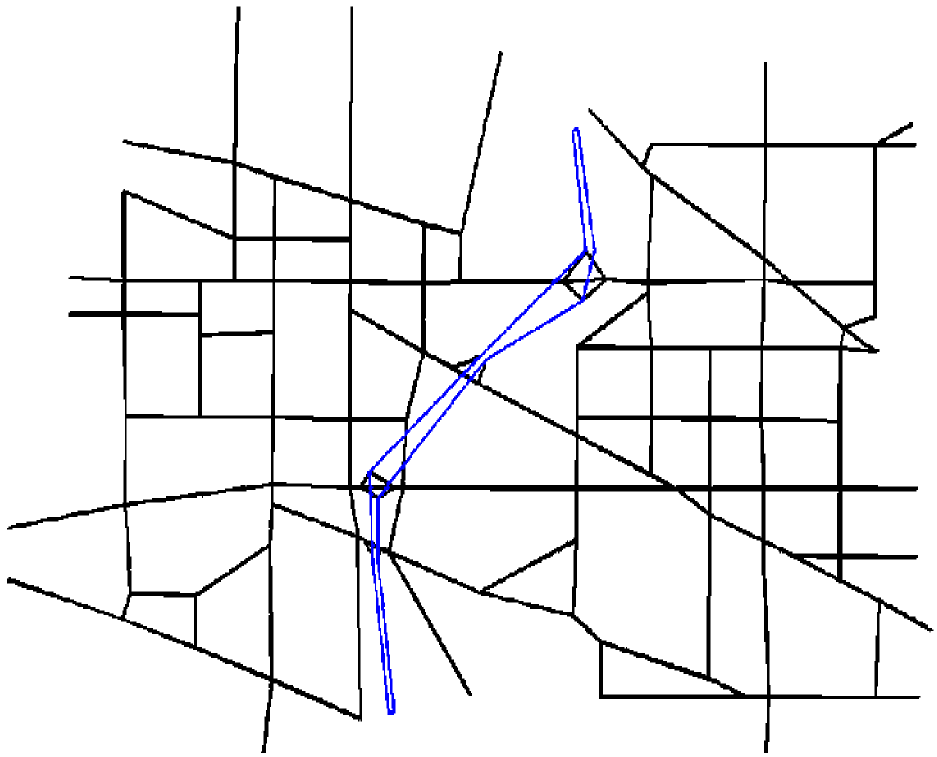


Figure 12 Chicago Sub-area Network

At the beginning of the simulation, the road network simulator (DYNASMART) runs for 4 DTA iterations to obtain equilibrated path assignments. The details on trip chain equilibrium in a dynamic traffic assignment framework are discussed in Chapter 6 of this study. At the higher iterations, however, the road network simulator is performed for only one DTA iteration. In order to achieve better convergence, the MSA factor is applied in calculating the schedule adjustment and path swapping probabilities. The schedule adjustment probability is calculated based on a penalty-based method as follows: (1) households who have unrealistic schedule are selected with 100% chance while (2) other households are selected according to a probability calculated based on the ratio of the household's total inconsistent schedule penalty to the maximum inconsistent schedule penalty of the entire population. This probability decreases with the MSA factor for higher iterations accordingly. Two activity selection scenarios for cancellation (latest arrival approach (LAA), and total penalty approach (TPA)) are considered (as discussed earlier in this chapter), and the results are compared to a scenario that no activity cancellation (demarcated as the None scenario) occurs. Figure 13 and Figure 14 represent variations of the two gap measures with the increase in the iteration number. The number of households with unrealistic schedules is depicted in Figure 13. It is evident that this gap measure decreases as the iteration number increases for both scenarios that include activity cancellation. A decreasing trend for the scenario with no cancellation can be observed as well; however, there are some fluctuations in this decreasing pattern. Figure 14 represents the variations of the average inconsistent schedule penalty with an increase in the iteration number. This gap measure also has a decreasing trend for LAA and TPA scenarios, and the decreasing trend for the None scenario is associated with fluctuations. In addition, the TPA strategy tends to have faster convergence compared to the LAA scenario.

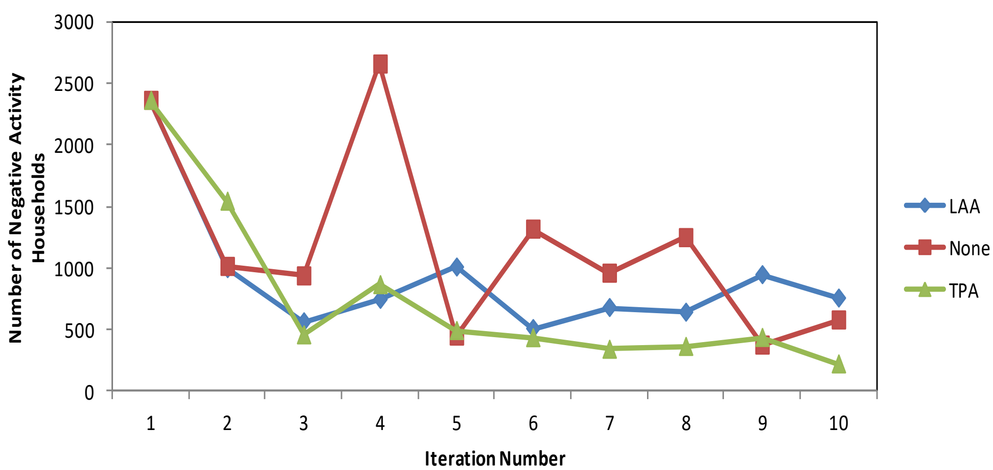


Figure 13 Number of Unrealistic Schedule Households

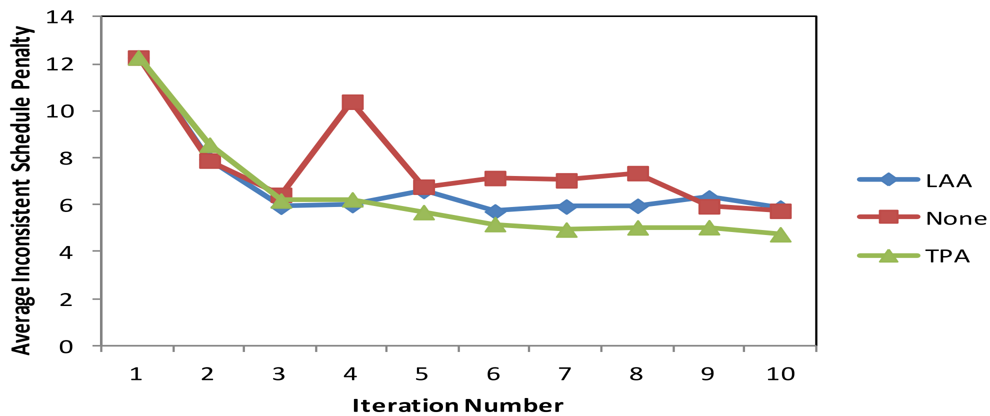


Figure 14 Average Inconsistent Schedule Penalty

5.5 Summary

In this chapter, a disutility associated with schedule inconsistency and total travel time of each individual traveler was defined. Then, assuming that each individual makes travel choices to minimize his/her disutility, a user equilibrium framework was adopted. Next, a fixed-point equilibrium model formulation was presented for the equilibrium problem, followed by the definition of the gap function. A heuristic solution algorithm incorporating an MSA-based gap minimization approach was then adopted. The integrated model presented in this chapter starts with a preplanned activity agenda (obtained from an ABM model), which consists of activity locations, sequences, desired departure times, and durations. The agendas are executed using dynamic traffic assignment and simulation tools. Given the dynamic network conditions, an optimal schedule-finding procedure was adopted to find the set of feasible optimal schedules (trip departure times). A subset of the households was then selected according to deviations from their preferred schedules, and using an appropriate step size, their schedules were modified. The selected subset of households whose schedules could not be improved through departure time adjustments was analyzed for possible adjustments through trip cancellation. Strategies for selecting household members to have cancelled activities and selecting activities for cancellation were discussed. The two tested activity-selection strategies for cancellation resulted in better numerical convergence compared to the scenario where none of the activities were cancelled. Also, the TPA represented faster convergence compared to the LAA. The author's objective here was to show the improvement in algorithmic convergence due to incorporation of activity cancellation. The author did not aim at optimizing the convergence; therefore, there could be other strategies that would result in better convergence.

6 DYNAMIC NETWORK EQUILIBRIUM FOR DAILY TRIP CHAINS

As mentioned earlier, the efficient and detail-compatible assignment of trip chains to transportation networks contributes substantially to the application of activity-based modeling approaches, as well as the evaluation of various functional and economic policies (Abdelghany et al. (2001)). Since the unit of traffic demand is considered to be either a one-way trip or multiple independent trips in most of the existing applications of dynamic network equilibrium models, capturing the daily activity-trip chains within dynamic traffic assignment models requires further attention.

This chapter includes the development of a simulation-based dynamic network traffic equilibrium model and algorithm for the assignment of activity-trip chains demand. The trip chain of each individual trip-maker is defined by the departure time at the origin, sequence of activities in a pre-specified order, activity locations, including intermediate destinations and the final destination, and the activity duration at each of the intermediate destinations. The equilibrium problem in this chapter could also be viewed as a variation of the fixed-point equilibrium problem; however, the choice dimensions are different than the ones in the previous chapters. Compared to the problems addressed in the previous chapters, neither rescheduling nor cancellation of activities are allowed.

6.1 Introduction

Travel demand analysis and forecasting in practice takes advantage of activity-based approaches to generate travel patterns of individuals. An individual activity schedule consists of a sequence and timing of activities, as well as details such as the activity purpose, location, duration, and the transportation mode to the activity. The behavioral aspects that influence planning of activity sequences by trip-makers are studied extensively in the demand forecasting arena; however, the assignment of the constructed activity schedules to transportation networks requires further scrutiny.

Applications of dynamic network equilibrium models have mostly considered the unit of traffic demand either as a one-way trip or as multiple independent trips. However, individuals' travel patterns typically follow a sequence of trips chained together with intervening activities at intermediate destinations. The intricate nature of trip sequences adds to the complexity of the path assignment procedure in real network applications, in terms of memory and time requirements. Spatial and temporal dependencies between subsequent trips necessitate time- and memory-consuming calculations and storage of node-to-node time-dependent least generalized cost path trees, which is not feasible given the size of actual networks and today's technology. The proposed algorithm in this chapter circumvents the need to store memory-intensive node-to-node time-dependent shortest paths by implementing a destination-based time-dependent least generalized cost path finding algorithm, while maintaining the spatial and temporal dependencies of the subsequent trips.

6.1.1 **Bi-criterion Dynamic User Equilibrium**

As discussed in previous chapters, the adopted approach in the estimation of travel path costs is defining a generalized cost function that consists of both 1) an out of pocket cost (path toll) and 2) a path travel time converted to cost by multiplying it by the corresponding value-of-time (VOT). As a result, the process of finding the least generalized cost path may result in different paths for travelers with different values of time. In the literature of network equilibrium, heterogeneity is considered either by treating users as discrete classes, each of which is associated with a VOT range (Yang et al. (2002); Han and Yang (2008)), or by treating VOT as a random variable with a probabilistic distribution across users (Leurent (1993)). Lu et al. (2008) developed a bi-criterion dynamic user equilibrium (BDUE) model that takes into account users' heterogeneity by assuming VOT as a continuously distributed random variable across the users. In order to generate the extreme efficient path set in their solution algorithm, a bi-criterion time dependent least generalized cost path set algorithm is applied, and the set of breakpoints corresponding to different user classes is determined. In this chapter, the bi-criterion dynamic user equilibrium (BDUE) model presented by (Lu et al. (2008)) is applied. Again, to account for users' heterogeneity, path cost is treated as a generalized path cost function defined by both path toll and weighted path travel time by traveler's VOT.

6.1.2 **Overall Framework**

The author has proposed a reformulation of the trip-based demand gap function formulation for the variational inequality formulation of the bi-criterion dynamic user equilibrium BDUE problem. Next, a solution algorithm is proposed for solving the BDUE problem for daily chain of activity-trips. Implementation of the proposed algorithm for very large networks circumvents the need to store memory-intensive node-to-node time-dependent shortest path trees by implementing a destination-based time-dependent least generalized cost path-finding algorithm, while maintaining the spatial and temporal dependencies of subsequent trips. Then, numerical results obtained from the applied algorithm to both small-scale and large-scale networks in a simulation setting are presented. The results suggest that recognizing the dependency of multiple trips of a chain and maintaining the departure time consistency of subsequent trips provide sharper drops in gap values; hence, the convergence could be achieved faster (compared to when trips are considered independent of each other).

In addition, a planning horizon segmentation procedure is implemented to facilitate calculations of the time-dependent least generalized cost path trees. The planning horizon (typically one day for activity-based model) can be divided into segments of varying lengths, and different least generalized time interval lengths can be considered for different segments. The least generalized time interval length is a period of simulation time in which the variations in travel costs can be neglected. The segmentation procedure is particularly of value in large-scale DTA models with long simulation horizons, in which the time-dependent least generalized cost path (TDLGCP) tree calculation times could be extremely high. To overcome the substantial TDLGCP calculation times of large-scale networks, one could choose shorter least generalized time interval lengths to find the time-dependent least cost path trees of the peak hours compared to non-peak hours such as midnight, and hence reduces calculation times substantially.

The overall procedure performed here is an iterative dynamic traffic simulation, via a dynamic traffic simulator (DYNASMART) (Jayakrishnan et al. (1994); Mahmassani (2001)), and a dynamic traffic assignment algorithm (discussed later in this chapter). First, the individuals' daily chains of activities are loaded onto the network by a dynamic network loading procedure, then the trip sequences are simulated using the prevailing travel times. Next, the simulated trips are evaluated for route swaps using a user equilibrium dynamic traffic assignment model, for which the proposed solution algorithm is presented in the course of this dissertation. The aforementioned steps continue till the convergence criterion is satisfied, or a pre-specified maximum number of iterations is reached.

6.2 Model Formulation

Consider a time-dependent network $G = (N, A)$ with N as a finite set of nodes and A as a finite set of directed links $(i, j) \in A$, where $i, j \in N$. Assuming $t0$ as the earliest possible departure time from all origin nodes, σ as a small time interval during which no noticeable change in traffic conditions or travel cost happens, and M as a large value in a way that $t0 + M\sigma$ covers the entire time period (planning horizon). The planning horizon is discretized into a set of small intervals $\Gamma = \{\tau_0, \tau_0 + \sigma, \tau_0 + 2\sigma, \tau_0 + 3\sigma, \dots, \tau_0 + M\sigma\}$. Other notations are as follows:

o Subscript belonging to an origin node

d Subscript belonging to a destination node

τ Subscript for a departure time interval

α Continuously distributed value of time $\alpha = [\alpha min, \alpha max]$

m Subscript for a vehicle class

$p^m(o, d, t)$ The set of all feasible paths for vehicles of class m belonging to o, d, τ triplet

p Subscript for a path $p \in p^m(o, d, \tau)$

r_{odp}^{τ} Number of trips from o to d departing o at time interval τ and assigned to path p

r Path flow vector

Ω The feasible set of path flow vector

$GC_{odp}^{\tau,m}$ Experienced path travel cost from o to d at departure time interval τ for a specific VOT (class m) assigned to path p

$TT_{odp}^{\tau,m}$ Experienced path travel time for trips from o to d at departure time interval τ assigned to path p

$TC_{odp}^{\tau,m}$ Experienced path travel cost for trips from o to d at departure time interval τ assigned to path p

$\Pi_{od}^{\tau,m}$ Least travel cost from o to d at departure time interval τ for a specific VOT

v Subscript corresponding to a vehicle

V Number of vehicles in the network

tr Subscript corresponding to trip tr of vehicle

$TR(v)$ Number of trips for vehicle v

$bin(v,tr)$ Function yielding the associated o,d,τ,m,p to trip tr of vehicle v with class m

$x(v,tr)$ Binary variable equal to 1 if $bin(v,tr) = (o,d,\tau,m,p)$, and 0 otherwise

$DGC_{bin(v,tr)}$ Experienced path travel cost for trip tr of vehicle v

$DGC_{bin(v,tr)}^{*}$ Least path travel cost for trip tr of vehicle v

As discussed earlier, the experienced generalized cost of travelers who depart from origin o to destination d along path p and have VOT α follows the following formulation:

$$GC_{odp}^{\tau}(\alpha) = TC_{odp}^{\tau} + \alpha \times TT_{odp}^{\tau} \tag{6-1}$$

Lu et al. (2009) reformulated the DUE problem via a gap function as a non-linear minimization problem. In this chapter, a gap function for the VI formulation of the BDUE problem with chains of activity-trips is presented. Multiple trips of a vehicle are characterized by the sequence of activities, activity durations, locations of intermediate destinations, and the location of the final destination. In the equilibrium framework presented here, the order of locations and durations of activities are known a priori, and the routes and trip departure times are adjusted accordingly. As presented in Jiang and Mahmassani (2013), the flow pattern r^* as the solution to the BDUE problem is equivalent to finding the solution to the following variational inequality formulation:

$$\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in\Gamma}\sum_{m=1}^{M}\sum_{p\in p(o,d,t)^m} GC_{odp}^{\tau,m}(\alpha,r^*)\times[r_{odp}^{\tau,m}(\alpha)-r_{odp}^{\tau,m}(\alpha)^*]\geq 0 \qquad (6\text{-}2)$$

$$\forall r^m(\alpha)\in\Omega^m(\alpha), and\ \forall\alpha\in[\alpha^{min},\alpha^{max}]$$

Adopting the formulation suggested in Lu et al. (2009) the equivalent gap function for the above formulation will be formulated as:

$$\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in\Gamma}\sum_{m=1}^{M}\sum_{p\in p(o,d,t)^m} r_{odp}^{\tau,m}\,[GC_{odp}^{\tau,m}(\alpha,r)-\Pi_{od}^{\tau,m}] \qquad (6\text{-}3)$$

To consider a daily chain of activity-trips, the gap function proposed in this chapter is as follows:

$$\sum_{v=1}^{V}\sum_{tr=1}^{TR(v)} DGC_{bin(v,tr)} - DGC^{*}_{bin(v,tr)} \tag{6-4}$$

Now it is demonstrated that the above proposed gap function for the VI formulation of BDUE problem is equivalent to the gap function presented in Lu et al. (2009) under the following assumption.

Assumption:

The trip departure time interval of every trip by a traveler maintains the same time interval as the trip departure time obtained from the traffic simulation. In other words, each trip is assumed to depart its origin at the assigned (simulated) time interval, meaning that assignment of paths with less travel times to the previous trips of the chain does not change the departure time interval of the current trip, and this shift in departure time interval is neglected.

$$\sum_{v=1}^{V}\sum_{tr=1}^{TR(v)} DGC_{bin(v,tr)} - DGC^{*}_{bin(v,tr)} =$$

$$\sum_{v=1}^{V}\{(GC_{bin(v,1)}(\alpha,r) + GC_{bin(v,2)}(\alpha,r) + \cdots + GC_{bin(v,TR(v))}(\alpha,r)) - \tag{6-5}$$

$$(GC^{*}_{bin(v,1)}(\alpha,r) + GC^{*}_{bin(v,2)}(\alpha,r) + \cdots + GC^{*}_{bin(v,TR(v))}(\alpha,r))\}$$

$$= (GC_{bin(1,1)}(\alpha,r) + GC_{bin(1,2)}(\alpha,r) + \cdots + GC_{bin(1,TR(1))}(\alpha,r)$$

$$+ GC_{bin(2,1)}(\alpha,r) + GC_{bin(2,2)}(\alpha,r) + \ldots + GC_{bin(2,TR(2))}(\alpha,r)$$

$$+ \ldots + GC_{bin(V,1)}(\alpha,r) + GC_{bin(V,2)}(\alpha,r)$$

$$+ \ldots + GC_{bin(V,TR(V))}(\alpha,r)) - (GC^{*}_{bin(1,1)}(\alpha,r)$$

$$+ GC^{*}_{bin(1,2)}(\alpha,r) + \cdots + GC^{*}_{bin(1,TR(1))}(\alpha,r) + GC^{*}_{bin(2,1)}(\alpha,r)$$

$$+ GC^{*}_{bin(2,2)}(\alpha,r) + \cdots + GC^{*}_{bin(2,TR(2))}(\alpha,r) \; + \cdots$$

$$+ GC^{*}_{bin(V,1)}(\alpha,r) + GC^{*}_{bin(V,2)}(\alpha,r) + \cdots + GC^{*}_{bin(V,TR(V))}(\alpha,r))$$

For any given (o,d,τ,m,p):

$$GC_{bin(v,tr)}(\alpha,r) = GC^{\tau,m}_{odp} \quad \forall v,tr \in \{v,tr | \big(bin(v,tr) = (o,d,\tau,m,p)\big)\}$$

$$x(v,tr) = \begin{cases} 0 & \forall v,tr \notin \{v,tr | \big(bin(v,tr) = (o,d,\tau,m,p)\big)\} \\ 1 & \forall v,tr \in \{v,tr | \big(bin(v,tr) = (o,d,\tau,m,p)\big)\} \end{cases} \tag{6-6}$$

$$r_{bin(v,tr)} = \sum_{v=1}^{V} \sum_{tr=1}^{TR(v)} x(v,tr)$$

$$\rightarrow \; r^{\tau,m}_{odp} = \; r_{bin(v,tr)} \qquad \forall v,tr \in \{v,tr | \big(bin(v,tr) = (o,d,\tau,m,p)\big)\}$$

$$\rightarrow Total\ GC = r_{bin(v,tr)} \times GC_{bin(v,tr)}(\alpha, r)\ \ \forall v, tr$$
$$\in \{v, tr | (bin(v,tr) = (o,d,\tau,m,p))\}$$

$$Total\ GC = \sum_{\forall v,tr \in \{v,tr | (bin(v,tr)=(o,d,\tau,m,p))\}} GC_{bin(v,tr)}(\alpha, r) \tag{6-7}$$
$$= r_{odp}^{\tau,m} \times GC_{odp}^{\tau,m}\ \ \forall o, d, \tau, m, p$$

$$GC_{bin(v,tr)}^{*}(\alpha, r) = \Pi_{od}^{\tau,m} \qquad \forall v, tr \in \{v, tr | (bin(v,tr) = (o,d,\tau,m,p))\} \tag{6-8}$$

$$\sum_{\forall v,tr \in \{v, tr | (bin(v,tr) = (o,d,\tau,m,p))\}} GC_{bin(v,tr)}^{*}(\alpha, r) \tag{6-9}$$
$$= r_{odp}^{\tau,m} \times \Pi_{od}^{\tau,m} \qquad \forall o, d, \tau, m, p$$

Therefore, it is shown that under the above assumption, the gap measures for independent trips and daily chains of activity-trips are identical. However, if the shift in departure time interval due to the path swap of previous trips is taken into account, the (o, d, τ, m, p) of a given (v, tr) might face a change in τ due to the assignment of paths with different travel times to any of vehicle v trips $\in \{1 \dots tr - 1\}$. Therefore, here it is shown that in this case, the gap measure for the daily trip chain is different from the trip-based gap measure, which calls for a modified approach. Consider the following notations:

$T_{v,tr}$ Assigned trip time for trip tr of vehicle v

$t_{v,tr}$ Planned departure time interval of trip (tr) of vehicle (v)

$t'_{v,tr}$ Updated departure time interval of trip (tr) of vehicle (v)

$bin'(v,tr)$ Associated (departure time updated) o, d, τ', m, p to trip tr of vehicle v with class m

$$t'_{v,tr} = (\sum_{tr=1}^{tr-1} T_{v,tr})/(time\ interval\ length) \quad \forall v = 1,..,V, \forall tr = 2,..,TR(v) \tag{6-10}$$

$$GC^*_{bin(v,tr)} = GC^*_{bin\prime(v,tr)} = \Pi^{\tau\prime,m}_{od} \neq \Pi^{\tau,m}_{od} \ \ \forall v, tr \in \{v, tr | t'_{v,tr} \neq t_{v,tr}\} \tag{6-11}$$

$$\sum_{v=1}^{V} \sum_{tr=1}^{TR(v)} DGC_{bin(v,tr)} - DGC^*_{bin(v,tr)} = \sum_{v=1}^{V} \sum_{tr=1}^{TR(v)} DGC_{bin(v,tr)} - DGC^*_{bin\prime(v,tr)} = \tag{6-12}$$

$$\sum_{v=1}^{V} \{(GC_{bin(v,1)}(\alpha, r) + GC_{bin(v,2)}(\alpha, r) + \cdots + GC_{bin(v,TR(v))}(\alpha, r)) - (GC^*_{bin(v,1)}(\alpha, r)$$
$$+ GC^*_{bin(v,2)}(\alpha, r) + \cdots + GC^*_{bin(v,TR(v))}(\alpha, r))\} =$$

$$(GC_{bin(1,1)}(\alpha,r) + GC_{bin(1,2)}(\alpha,r) + \cdots + GC_{bin\left(1,TR(1)\right)}(\alpha,r) + GC_{bin(2,1)}(\alpha,r)$$
$$+ GC_{bin(2,2)}(\alpha,r) + \ldots + GC_{bin\left(2,TR(2)\right)}(\alpha,r) + \ldots + GC_{bin(V,1)}(\alpha,r)$$
$$+ GC_{bin(V,2)}(\alpha,r) + \ldots + GC_{bin\left(V,TR(V)\right)}(\alpha,r)) - (GC^{*}_{bin(1,1)}(\alpha,r)$$
$$+ GC^{*}_{bin'(1,2)}(\alpha,r) + \cdots + GC^{*}_{bin'(1,TR(1))}(\alpha,r) + GC^{*}_{bin(2,1)}(\alpha,r) + GC^{*}_{bin'(2,2)}(\alpha,r)$$
$$+ \cdots + GC^{*}_{bin'(2,TR(2))}(\alpha,r) \ + \cdots + GC^{*}_{bin(V,1)}(\alpha,r) + GC^{*}_{bin'(V,2)}{}'(\alpha,r) + \cdots$$
$$+ GC^{*}_{bin'(V,TR(V))}(\alpha,r)) =$$

$$\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in\Gamma}\sum_{m=1}^{M}\sum_{p\in p(o,d,t)^m} r^{\tau,m}_{odp}\,[GC^{\ \tau,m}_{\ odp}] -$$
$$(GC^{*}_{bin(1,1)}(\alpha,r) + GC^{*}_{bin\prime(1,2)}(\alpha,r) + \cdots + GC^{*}_{bin\prime\left(1,TR(1)\right)}(\alpha,r)$$
$$+GC^{*}_{bin(2,1)}(\alpha,r) + GC^{*}_{bin\prime(2,2)}(\alpha,r) + \cdots + GC^{*}_{bin\prime\left(2,TR(2)\right)}(\alpha,r) + \cdots$$
$$+GC^{*}_{bin(V,1)}(\alpha,r) + GC^{*}_{bin'(V,2)}(\alpha,r) + \cdots + GC^{*}_{bin'(V,TR(V))}(\alpha,r)) \neq$$

$$\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in\Gamma}\sum_{m=1}^{M}\sum_{p\in p(o,d,t)^m} r^{\tau,m}_{odp}\,[GC^{\ \tau,m}_{\ odp}\,(\alpha,r) - \Pi^{\tau,m}_{od}]$$

As it is shown here, the gap function proposed by Lu et al. (2009) ignores the shift in departure time interval due to the path swap of previous trips. Therefore, a trip-chain-based formulation of the gap function is proposed here, and in order to address the aforementioned issue, which is called temporal inconsistency in this chapter, a solution algorithm is proposed.

The adopted approach is the simulation-based approach developed in Lu et al. (2009) for solving the DUE problem , in which they reformulated the DUE problem via a gap function as a non-linear minimization problem (NMP). They solved the NMP by a column generation-based approach, which augments the subset of extreme efficient or non-dominated paths in the outer loop, and solves the restricted NMP defined by a subset of feasible paths by a path-swapping descent-direction method in the inner loop. The general framework for the simulation-based BDUE algorithm is shown in Figure 15. The proposed solution algorithm in this section corresponds to steps 2, 3, and 4 of the flowchart in Figure 15.

6.3 Algorithm

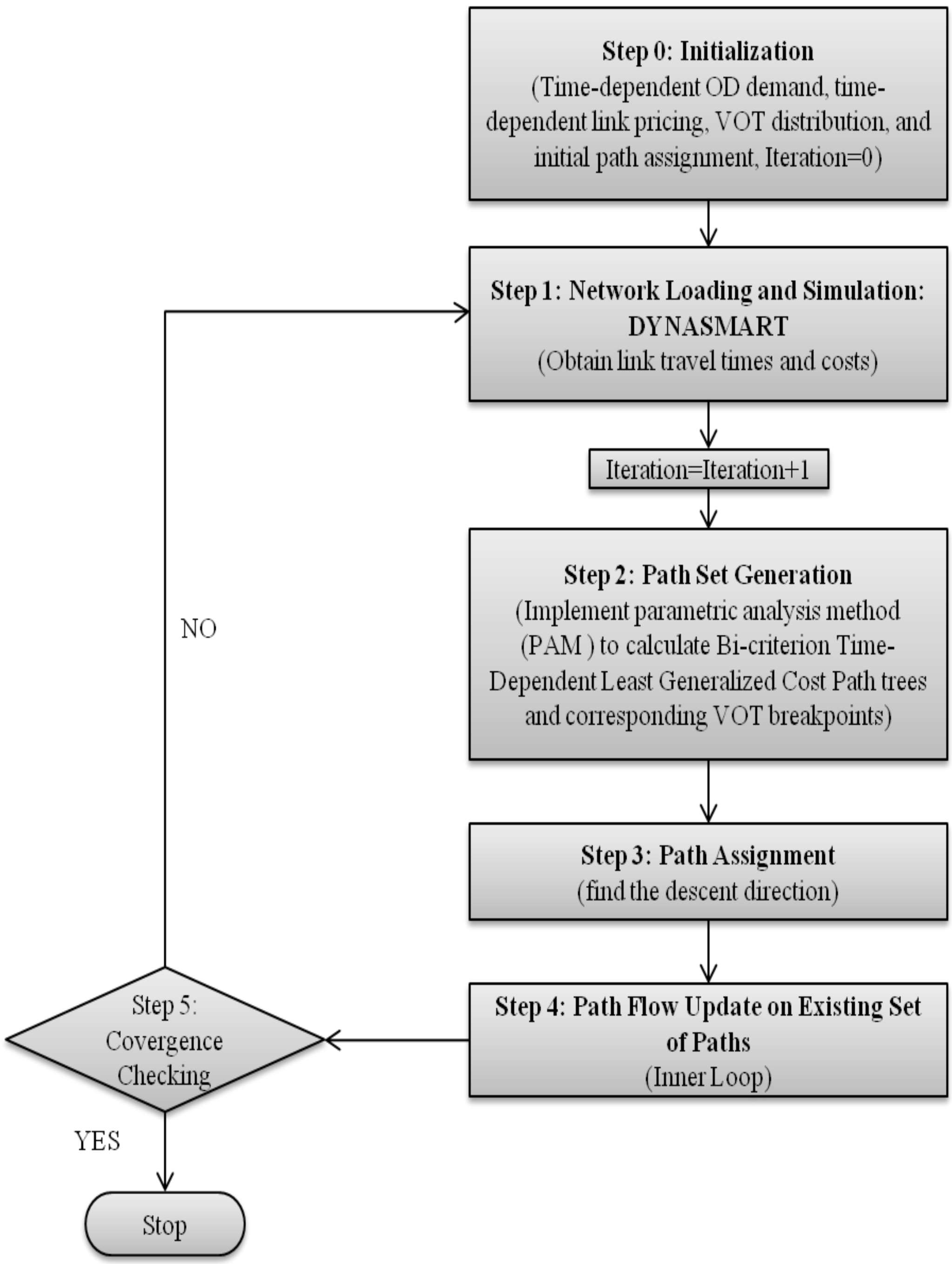


Figure 15 Simulation-based BDUE Algorithm

The outer loop applies a bi-criterion time-dependent least generalized cost path (BTDLGCP) algorithm to construct the extreme efficient path set, and obtain the breakpoints corresponding to various VOT ranges. In the proposed solution algorithm, the parametric analysis method (PAM) presented by Mahmassani et al. (2005) is implemented at each outer iteration for a single destination zone, all the origin zones, and all the departure time intervals. For each vehicle that has at least one trip to that single destination zone, the BTDLGCP associated with that trip's origin zone, departure time interval, and corresponding to the VOT subinterval that the vehicle's VOT belongs to, is stored. The procedure continues until all destination zones, origin zones, and departure time intervals are covered. Hence, all trips of a single vehicle are associated with a least generalized cost path obtained from PAM.

In the next step, beginning from the first trip (tr=1) of a given vehicle (v), the generalized cost of the stored least generalized cost path for that trip (tr) is compared with the experienced generalized cost of the path of the corresponding o, d, τ, m. If the experienced generalized cost exceeds the least generalized cost, a probabilistic choice rule is applied to decide whether trip tr of vehicle v path should be updated or not. A route swap decision is made using the proportional difference between the experienced path cost and the least path cost, namely the relative gap, i.e.

$$\frac{Generalized\ cost\ of\ the\ experiened\ path - Generalized\ cost\ of\ the\ coresponding\ least\ cost\ path}{Generalizedcost\ of\ the\ experiened\ path}.$$

In addition, a step size is chosen according to the Method of Successive Averages (MSA) to account for the effects of least-cost path flow updates of the previous iterations. The chosen MSA factor in here is $\frac{1}{(1+iteration)}$.

If the trip is to be assigned a new path, the tr trip of the vehicle v would be assigned to the least generalized cost path. The same procedure is applied to trip $tr + 1$; however, the assigned o, d, τ, m to trip $tr + 1$ is evaluated for a possible update based on the departure time shift caused by new path(s) assigned to any of trip(s) $\{1,2, .. tr\}$. In cases where trip $(tr + 1)$'s departure time interval is shifted due to new assigned paths to one or more of the previous trips (e.g. trips $1, 2$ ... $tr - 1$), an alternative vehicle (v') with a trip (tr') with departure time interval equal to trip $tr + 1$ of vehicle v's updated (shifted) departure time interval is found. The experienced generalized cost of the trip (tr) of vehicle v is compared with the least generalized cost of the alternative vehicle (v') and trip (tr') associated with o, d, τ', m. The procedure continues until all trips by the vehicle are taken care of. In cases where there is no alternative vehicle found, the vehicle's trip might maintain its experienced path, or be assigned the least generalized cost path associated with o, d, τ, m. The details of the solution algorithm are as follows:

Step 0: $d = 1$

Step 1: Implement PAM for destination (d)

Step 2: $o = 1$

Step 3: $\tau = 1$

Step 4: $m = 1$

Step 5: $\forall (v, tr) \in \{v, tr | \big(bin(v, tr) = (o, d, \tau, m, p)\big)\}$ Store the bi-criterion least generalized path associated with $\big(bin(v, tr) = (o, d, \tau, m, p)\big)$

Step 6: $if\ m$<total number of users' classes for o, d, τ, $m = m + 1$, and go to Step 5 $else$, $m = 1$, and go to Step 7

Step 7: $if\ \tau$<total number of assignment intervals, $\tau = \tau + 1$, and go to Step 5

$else, \tau = 1$, and go to Step 8

Step 8: $if\ o$<total number of origins, $o = o + 1$, and go to Step 5

$else, o = 1$, and go to Step 9

Step 9: If d<total number of destinations, $d = d + 1$, and go to Step 2

If d=total number of destinations, go to Step 10

Step 10: $v = 1, tr = 1$

Step 11: $if\ GC_{bin(v,tr)}(\alpha, r) > \Pi_{od}^{\tau,m} \Rightarrow SwapProb = \frac{1}{1+iteration} * \frac{GC_{bin(v,tr)}(\alpha,r) - \Pi_{od}^{\tau,m}}{GC_{bin(v,tr)}(\alpha,r)}$

$else\ SwapProb = 0$

Step 12: Draw a random number

Step 13: $if\ SwapProb > random\ number \Rightarrow$

Assign the time dependent least generalized cost path to trip (tr) of vehicle (v)

$Else$, keep the experienced path

Step 13: Update departure time interval for tr+1 as τ'

Step 14: $if\ tr < TR\ (v), tr = tr + 1$

$else, \quad v = v + 1, tr = 1$, and go to Step 11

Step 15: $if\ \tau' \neq \tau$, find a vehicle (v') with a trip (tr') with departure time (τ'), and go to Step 16,

$else, \ ' = v, tr' = tr \Rightarrow \tau' = \tau$, and go to Step 11

Step 16: $if\ GC_{bin(v,tr)}(\alpha, r)$>$\Pi_{od}^{\tau',m}$, assign the least generalized cost path of trip (tr') of vehicle (v') to trip (tr) of vehicle (v) , and go to Step 13

The next section includes results from applying the proposed method. First, a comparison of the convergence measure trend is presented for two cases, i.e. when the departure time inconsistency is ignored and when the departure time inconsistency is incorporated in the model. The proposed solution algorithm can be applied to large-scale networks by aggregating origin and destination nodes to zones to overcome the time- and memory-demanding least-cost path calculation procedure.

6.4 Numerical Results

The Chicago full regional network is considered for the performance evaluation of the proposed method. The static network is prepared by Chicago Metropolitan Agency for Planning (CMAP). This static network (originally in TransCAD) is transformed into the DYNASMART format. While this network will be used for the final evaluation and analysis, a sub-network is extracted from this network for testing purposes. The extracted sub-network, referred to as the sub-area test network, is a small-scale network. The general specifications of these networks are presented in the following sub-sections. The demand uses a format based on daily trip chains, and is obtained from CT-RAMP Activity-Based Model (ABM). A total of 2,000,481 vehicles are simulated by DYNASMART on the Chicago full regional network, with 4,864,686 trips, and 156,492 vehicles are considered for the sub-area test network, which has a total of 287,705 trips. Note that the demand for the full Chicago network is based on a 25 % sample.

6.4.1 Small-Scale Network

The sub-area test network is a small-scale network extracted from the Chicago full regional network. Figure 16 represents the sub-area test network. This network consists of 13 freeways, 334 arterials, 137 nodes, and 57 traffic analysis zones (TAZ). The simulation runs start at 3:00

AM, and last for 24 hours. There are 156,492 vehicles loaded to the network at different time intervals, and the total number of trips planned for all vehicles is 287,705. Note that in this network, a portion of loaded vehicles is related to external demand (by passing vehicles in the extracted network). These vehicles do not have trip chains, and their role is to provide enough congestion in such a small network as the background traffic. Each simulated vehicle has at least one activity in its schedule, and the maximum number of activities considered here is 21. The results presented here for the sub-area test network are based on 6 outer loop iterations including 2 inner loop iterations, and the MSA factor of $1/_{1 + iteration}$.

First, the author considers how the departure time consistency of subsequent trips affects convergence of the solution algorithm in the subarea test network. The first case presented in Figure 17 represents the results from a scenario in which there is no reliability measure involved in the generalized cost function. In other words, individuals; risk-taking/aversion behaviors towards unreliable travel time paths are not captured differently. Figure 18 demonstrates results from the same network settings as the previous case, except that the reliability measure is incorporated. The reliability measure primarily captures individuals' risk-aversion behaviors in terms of their willingness to pay an extra toll or experience larger average travel time to avoid the paths with unreliable travel times. In order to capture the reliability, the model presented by (Jiang et al. (2011b)) is implemented and tested on both the small-scale and large-scale networks. Note that, in this model, a linear relation between the average travel time and standard deviation of travel time per unit of distance is used to estimate travel time unreliability.

Figure 17 and Figure 18 display the gap values with respect to both outer and inner loop iterations. There are 6 outer loop iterations, demarcated with black triangles, and two inner loops following each outer loop, and these are demarcated by red squares. As can be observed in both figures, the gap values drop more rapidly when temporal consistency of trips during the equilibrium procedure is captured. In addition, according to Figure 17 (b) and Figure 18 (b), the gap trends resulting from outer loops show a descending pattern with an increase in the iteration number, whereas when temporal consistency is ignored, a chaotic pattern is observed in the gap values from outer loop iterations as is shown in Figure 17 (a) and Figure 18 (a). There are some fluctuations in the patterns presented in Figure 18 due to including reliability measures in calculations of path costs.

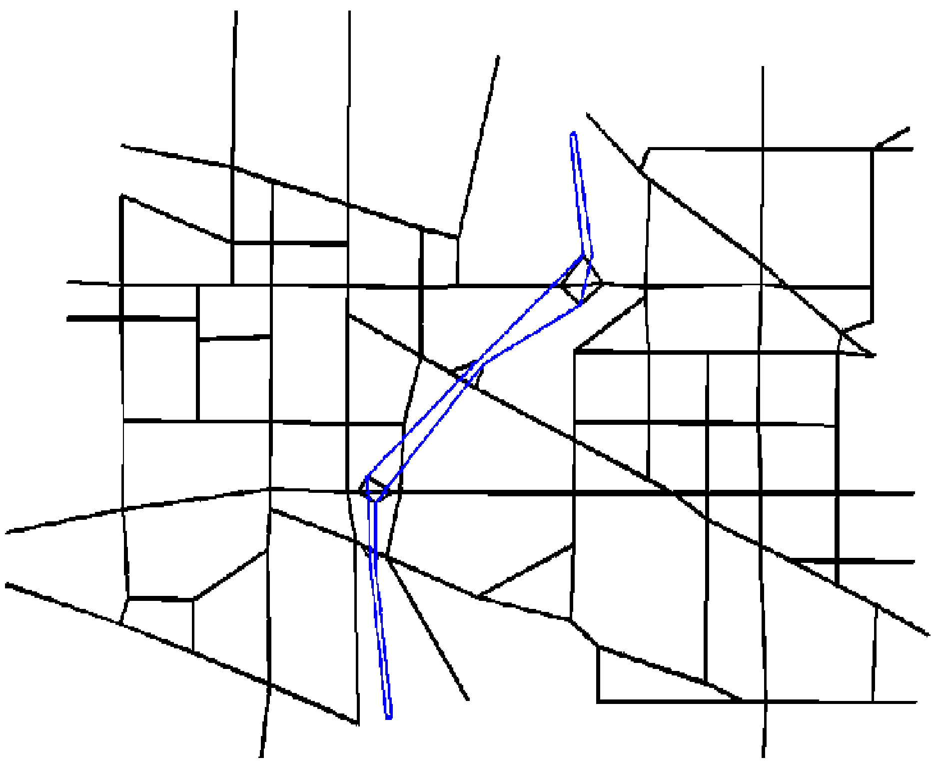


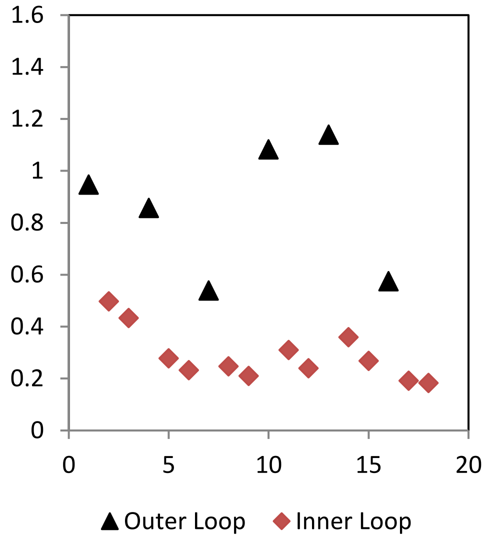
Figure 16 Small-scale Network Configuration

The main purpose of the sub-area network extraction is to perform several tests, which the general settings for our final evaluations and analysis on the full network are based on. Various combinations of scenario settings considered are: number of inner loop iterations, number of outer loop iterations, the step size (MSA factor), and reliability related scenarios. According to our test runs, the following settings are chosen for the Chicago full regional network:

Number of outer loop iterations: 4

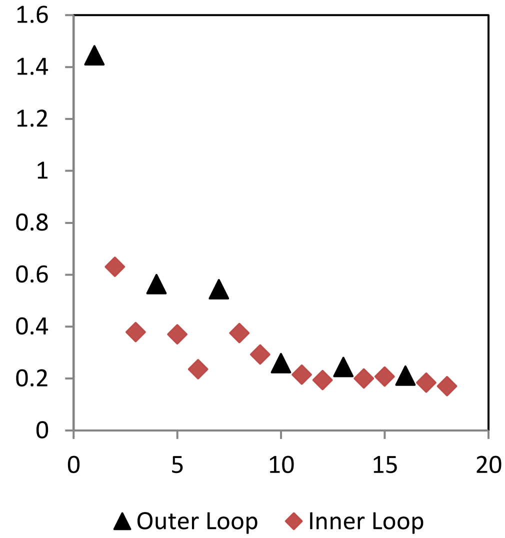
Number of inner loop iterations: 2

MSA factor: $^{1}/_{1\,+\,iteration}$



a) Gap Values for Outer and Inner Loop Iterations

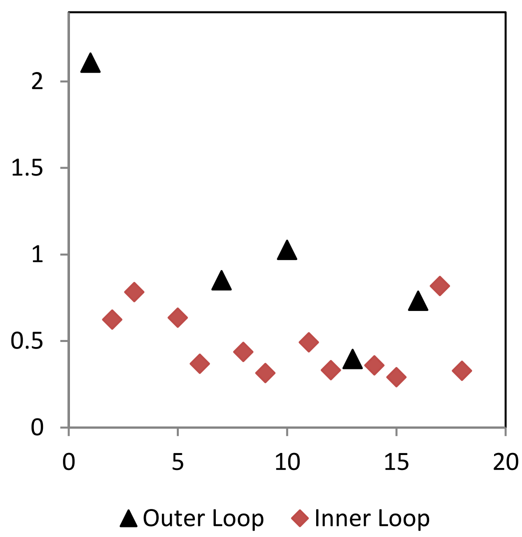
Departure Time Inconsistency

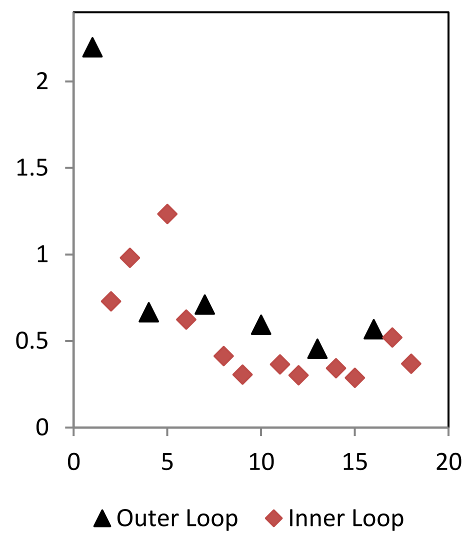


b) Gap Values for Outer and Inner Loop Iterations

Departure Time Consistency

Figure 17 Small-scale Network Gap Values When Reliability Measure Not Considered





a) Gap Values for Outer and Inner Loop Iterations

Departure Time Inconsistency

b) Gap Values for Outer and Inner Loop Iterations

Departure Time Consistency

Figure 18 Small-scale Network Gap Values When Reliability Measure Is Considered

6.4.2 Large-Scale Network

Figure 19 represents the Chicago full regional network, which is a large-scale network. The network covers a part of state of Illinois, Wisconsin, and Indiana, and is bound by Lake Michigan to the east. This network consists of 1,400 freeway corridors including I-90, I-94, I-55, I-80, etc., and 36,722 arterials. The network has 13,093 nodes, 40,443 links and 1,961 traffic analysis zones (TAZ). Traffic flow models used for the arterials and freeways of the network are based on calibrated single- or dual-regime modified Greenshields models. The simulated traffic signal setting used for the signalized intersections is a multi-phase actuated traffic signal control model with maximum green time (Gmax) of 72 seconds and minimum green time (Gmin) of 6 seconds. The current network also includes 144 tolled links with a fixed price. The location of these tolled links is shown in Figure 19.

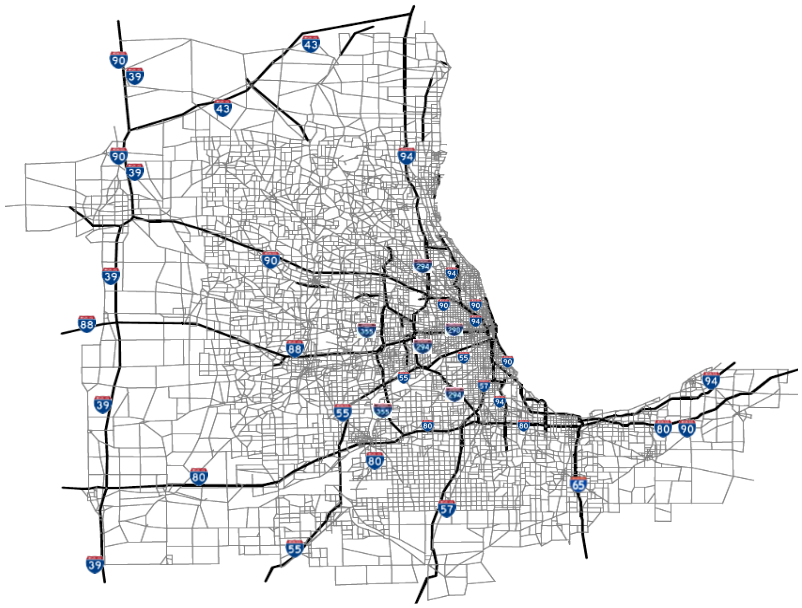
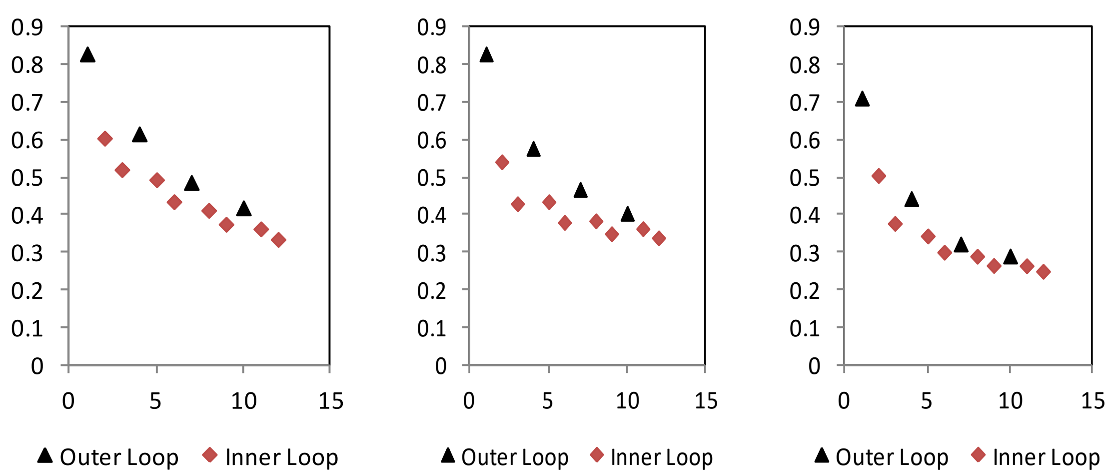


Figure 19 Large-scale Network Configuration

The simulation horizon for the Chicago full regional network starts at 3:00 AM, and lasts for 24 hours. There are 2,000,481 vehicles loaded onto the network at different time intervals throughout the simulation horizon, and the total number of trips planned for all vehicles is 4,864,686. Each simulated vehicle has at least one activity in its schedule, and the maximum number of activities considered here is 22. Three sets of runs are performed for the full regional network, for which the results are shown in Figure 20. The first two sets include reliability measures, and they differ in the MSA factor used within inner loop iterations. The first case has the MSA factor of ${}^{\mathbf{1}}/_{(\mathbf{1}+\boldsymbol{Iteration_{In}})}$, whereas the second case incorporates the MSA factor of ${}^{\mathbf{1}}/_{\mathbf{1}+\boldsymbol{Iteration}}$, where $\boldsymbol{Iteration_{In}}$ is the counter for the inner loop iterations, and $\boldsymbol{Iteration}$ is the counter for the outer loop iterations.

In addition, 6 time segments are considered, which cover the entire planning horizon (24 hours). The time segments are applied in the least generalized cost tree calculations. Each segment has a different least generalized cost time interval length. Using different least generalized cost time intervals for different time periods of the 24-hour simulation horizon significantly reduces the computation effort required to calculate the least generalized cost trees. In this approach, finer time intervals are used for AM and PM peak-hour periods, while coarser time intervals are employed during the mid-night or mid-day time periods. For the Chicago full network application, the least generalized cost path-finding calculation time is improved by a factor of seven using the modified approach instead of the standard approach. For the simulation runs presented in this paper, 6 time segments are considered with the following start times: 3:00 AM, 6:00 AM, 10:00 AM, 2:00 PM, 8:00 PM, and 10:00 PM. The least generalized cost time interval lengths are 30, 10, 20, 10, 30, and 300, respectively.



a) Gap Values for Outer and Inner Loop Iterations-Reliability-Inner Loop MSA Factor: $1/_{1+Iteration_{In}}$	b) Gap Values for Outer and Inner Loop Iterations-Reliability-Inner Loop MSA Factor: $1/_{1+Iteration}$	c) Gap Values for Outer and Inner Loop Iterations-No Reliability- Inner Loop MSA Factor: $1/_{1+Iteration}$

Figure 20 Generalized Cost Gap Values-Different Scenarios-Chicago Full Regional Network

Figure 20 includes the gap values resulting from the applied iterative algorithm to the Chicago full regional network. Decreasing trends in the gap values can be observed as the number of iterations increases. Both the outer loop iterations and inner loop iterations of all 3 scenarios demonstrate converging patterns in the gap values. Comparing Figure 20 (a) with Figure 20 (b) shows a slight sharper drop in the gap values when the chosen inner loop MSA factor is $\mathbf{1}/_{\mathbf{1}+\boldsymbol{Iteration}}$ compared to $\mathbf{1}/_{\mathbf{1}+\boldsymbol{Iteration_{In}}}$. However, the difference in their speed of convergence is not significant, and this pattern cannot be generalized to other networks, since there is no systematic way of determining the optimal step size.

6.5 Summary

In this chapter, the variational inequality formulation of the gap-based BDUE problem was formulated for a daily chain of activity-trips. It was shown that the proposed gap function formulation in this chapter is equivalent to the gap function formulation in the existing literature, if the departure time interval of each trip within a chain is assumed independent of the other trips departure time interval. However, accounting for the temporal consistency of trips during the path assignment procedure could lead to different gap function formulations. In order to capture the temporal inconsistency among interdependent trips in a chain, an appropriate solution algorithm is proposed, which is also applicable to large-scale networks.

The main contribution of the proposed approach is to consider the departure time interval inconsistency in the gap-based formulation of the dynamic user equilibrium problem for the daily chain of activity-trips. Furthermore, the traffic simulator utilized here considers the daily trip chains, and loads the network based on the trip sequences and activity durations. Also, user heterogeneity is considered by applying a bi-criterion dynamic user equilibrium formulation. In addition, different scenarios in terms of considering the travel time reliability measure are tested. Last but not least, the proposed formulation and solution algorithm are modified for large-scale network implementations. For this end, a continuous simulation horizon, along with the segmented route assignment intervals, are considered. In each of these segments, different least generalized cost time interval lengths are used to improve the computational complexity.

The numerical results section develops the devised algorithm on a test sub-area network, and then implements it on a large-scale network of Chicago. The results demonstrate successful application of the methodology for the dynamic user equilibrium problem with a daily chain of activity-trips. This application shows that, by considering the inconsistency in departure time intervals, there is noticeable improvement in the convergence of the proposed algorithm. The developed solution algorithm for the dynamic user equilibrium provides an appropriate dynamic traffic assignment tool to be used in ABM-DTA integrated systems, which are considered only by few planning agencies nowadays.

7 CONCLUSION

In this dissertation, the author aimed at providing a robust mathematical framework for the equilibration of activity trip chains. For this purpose, a framework was proposed that provides an inner adjustment platform for an ABM-DTA integrated model (surrogate gap). The proposed surrogate measure captures individuals' activity scheduling and route choices in a dynamic network equilibrium framework. Such a surrogate measure is, in particular, beneficial to planning agencies because it helps overcome the high implementation time and large memory requirements associated with applications of a fully-integrated system of ABM and DTA to large-scale networks.

First, a user equilibrium framework for the path and schedule choices of individual travelers was defined, followed by the presentation of a variational inequality formulation as the variant formulation of the equilibrium problem. It was shown that the solution to the VI formulation meets the user equilibrium conditions. The equivalent gap function for the VI formulation was defined accordingly to provide a gap-based solution approach. In addition, the solution properties of the proposed fixed-point problem formulation were explored through the analysis of continuity and monotonocity of some of the involved functions. It was shown that there is no guarantee that a solution exists for the problem, due to discontinuity of the functions. Moreover, the non-monotonocity of the functions was shown through examples, which means that there might be multiple solutions for the problem, given that a solution exists.

Given the problem size and computational expenses involved with real-world networks, a heuristic solution approach, which involves an MSA based gap minimization procedure, was proposed. Convergence criteria are defined, and strategies for selecting households that adjust their activity schedules are discussed. The proposed solution approach is successfully implemented and tested on a small-scale and a large-scale network, and the numerical results illustrate the convergence of the algorithm in terms of the defined gap function. In other words, despite the fact that solution existence and uniqueness cannot be guaranteed analytically, the efficient convergence of the proposed solution algorithm is demonstrated through numerical results.

Furthermore, the proposed fixed-point equilibrium model formulation is extended to incorporate the cancellation of activity/trips. Strategies for selecting households to have cancelled activities as well as activity selection strategies for cancellation, are discussed.

Lastly, the dynamic user equilibrium problem for the daily activity-trip chains is addressed. Spatial and temporal dependencies of subsequent trips necessitate the time- and memory-consuming calculations and the storage of node-to-node time-dependent least generalized cost path trees, which is not feasible given the size of actual networks and today's technology. The proposed algorithm circumvents the need to store memory-intensive node-to-node time-dependent shortest path trees for large-scale networks by implementing a destination-based time-dependent least generalized cost path finding algorithm, while maintaining the spatial and temporal dependencies of subsequent trips.

Future work on this topic can include incorporation of activity reordering, as well as mode choice in the model development and application.

Other contributions of this dissertation are as follows:

- In the existing ABMs, certain steps have been made to ensure a partial consistency between departure and arrival times, as well as duration at the entire-tour level. The proposed approach in this study, however, allows one to include trip details, and control for feasibility of travel times within the tour framework. Certain attempts to incorporate trip departure time choice in a framework of trip chains have been made within existing DTA models. However, these attempts were limited to a tour level only, and also required a simplified representation of activity duration profiles. This constraint was specifically addressed in the course of the current study by developing a schedule-adjustment algorithm and software module.

- Physical principles, such as conservation of vehicles at nodes, which apply to network loading and flow propagation aspects in DTA procedures, are adhered to strictly (e.g. no vehicles should simply be lost or otherwise disappear from the system). Thus, travel times that are used to equilibrate the schedule are fully consistent with the DTA network state.
- Travel times between activities in the schedule, generated by the demand model, correspond to realistic network travel times for the corresponding origin, destination, departure time, and route generated by the traffic simulation model with the given demand. While most of the ABMs include a certain level of demand-supply equilibration, they are limited to achieving stability in terms of average travel times. There is no control for consistency within the individual's daily schedule. The challenge is to couple this constraint with the previous one, i.e. ensure individual schedule continuity with equilibrium travel times. This is addressed in the current study by monitoring schedule inconsistency in the equilibration.
- The rescheduling process of individuals obeys utility-maximization rules over the entire schedule and is not modeled by simplified procedures that adjust departure time for each trip separately. None of the existing operational ABMs explicitly control for activity

durations, although some of them control for entire-tour durations; or the duration of the activity at the primary destination. DTA models that incorporate departure time choice have been bound to a simplified representation of temporal utilities, and moreover limited to trip chains in order to operate within a feasible dimensionality of the associated choices when combined with the dynamic route choice. This constraint expresses consistency between activity start and end times as controlled by the schedule adjustment module.

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