 RESEARCH
REPORT

## OLIGOPOLISTIC NONLINEAR PRICING -

A STUDY OF TRADING STAMPS AND AIRLINE FREQUENT FLYER PROGRAMS

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL IN PARTIAL FULFILIMENT OF THE REQUIREMENTS<br>for the degree<br>DOCTOR OF PHILOSOPHY

Field of Managerial Economics and Decision Sciences

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EVANSTON ILLINOIS
August 1986
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#### Abstract

This study analyzes the effects product bundling and nonlinear pricing between different products can have upon retail competition. In particular, two models are developed, one of airline frequent flyer programs, the other of trading stamp promotions. In both models, a nonlinear rebate schedule creates incentives for the consumer to become brand loyal to a particular merchant or "team" of stores. Competition between firms is therefore shifted toward becoming the sole supplier for the consumers, and away from competing on an individual product basis.

In the airline model, since each airline has a different route network, consumers no longer consider differing airline's flights perfect substitutes. Frequent flyer programs allow higher prices to be supported in equilibrium.

Trading stamps can create a brand loyalty to a "team" of stores. Each merchant by raising or lowering his prices will influence how many customers purchase all their demanded products from his "team." It is shown that price changes in this environment have less of an impact upon the merchant's demand function than when neither of the competing stores participate in a trading stamp program.

It is shown that the two programs studied have entry deterring properties. The welfare effects of the programs are also examined.


## Acknowledgments

I would like to thank Professors Kenneth Judd, Milton Harris, Steven Matthews, Roger Myerson, and especially Professor Morton Kamien under whose direction and guidance this research was conducted. The many helpful comments made to me by my classmates, Jacob Glazer, Elazar Berkovitch, Linda Salchenberger, Tom Gresik, and David Shimko, were greatly appreciated.

I would also like to take this opportunity to express my deep gratitude to The Transportation Center, Northwestern University for providing the funding to complete this research.

Finally, I would like to thank Nora Lynch, who has made this period of time the happiest of my life.

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## Chapter 1. Introduction and Survey of the Literature

### 1.1 Introduction

Nonlinear pricing and product bundling have always been observed. Almost no industry where the products are not easily resalable seems to be immune to this phenomena. Many examples such as the traditional "baker's dozen" or declining block tariffs charged by utilities come to mind. Surely, within just a few moments, the reader could construct his own list many pages long.

Quite often, these nonlinearities can be found over different products, often even bridging over different firms. Purchasing one product can sometimes affect the price that needs to be paid for purchasing the other. Coupons and rebate programs are the best examples of this. Here the price paid for a jar of instant coffee, could very well depend upon whether the consumer purchased the brand of breakfast cereal that had a coupon attached to it. Product bundling is the most forceful of these nonlinearities, since, here the consumer is often not given the choice of buying an individual item.

Nonlinearities occur in many more subtle ways. The location of two stores can influence the effective price the consumer pays for the
items. Quotes like "When you go to the supermarket, can you pick up some medicine from the drug store"? can be heard daily in almost every household. The rise in popularity of shopping centers is a tribute to this fact.

Much literature has been written in the field of nonlinear pricing and product bundling, but until recently, this has almost always been in a monopoly setting. A large percentage, if not most, of the industries where nonlinear pricing is found is either oligopolistic or competitive in nature.

In this dissertation, I am interested how nonlinear prices affect the competitive structure of the industry. I show that it is exactly these effects that make the existence of some of these nonuniform price schemes appealing. By tieing the price of one good with that of others, the firms create a reason for the consumers to be "brand loyal" to a particular manufacturer, store, or "team" of merchants.

In particular, I develop a model of the frequent flyer programs that have existed in the U.S. airline industry since 1981 , and a model of the trading stamp companies that have their origins in the $19^{\text {th }}$ century. They were chosen for study since both had well defined discount schedules for repeat custömers. The frequent flyer programs are for the most part firm specific and run by the individual airlines. The trading stamps in contrast were organized and
administered by an outside firm and were issued by many small independently owned and operated stores.

The result of the frequent flyer programs having a convex rebate schedule is to make it in each passenger's best interest to be loyal to one of the airlines. Although the consumers may be indifferent between carriers to fly on any particular flight, they are not indifferent between what airline they choose to be loyal to. Since United Airline flies to most cities from Chicago, someone living in Chicago would most likely prefer belonging to United Airline's frequent flyer program over Western Airlines'. A consumer living in Salt Lake City, would most likely belong to Western's program for the same reason, although neither one of them may actually have an intrinsic preference between the two carriers.

The trading stamps programs operate under a similar structure. Here, the trading stamp companies offer the merchants the rights to issue their particular brand stamp. The small town is split into two "teams" of stores, the "green" stamp and the "yellow" stamp stores. The trading stamp companies offer the consumer a redemption schedule for the stamps that is, just as in the airline model, convex. This again insures that the consumer wifl want to purchase all the items from the same "team." Competition is again shifted from the individual product markets to competition for the total product bundle. Since the consumers' preferences over the two bundles differ
more than over one specific product, I will show that the firms in equilibrium are able to extract higher prices.

In both models, nonlinear pricing is a "defensive" strategy. By bundling all of the products together, the sales become less susceptible to being lost due to a competitor's price reduction. Since this is also true of the other merchants, all prices are allowed to increase.

The welfare effects that these programs impose is twofold. First, consumer surplus is lost and transferred to the merchants in the form of higher prices. Secondly, since consumers now choose which bundle to purchase, and not each individual item separately. some consumers purchase products they consider inferior to the competitive team's product. The new equilibrium may therefore not be allocatively efficient. Entry into the industry on a "small scale" also becomes more difficult, so the price equilibrium is in some sense stable.

In 1.2, I survey the literature, with the trading stamp literature survey supplemented in section 4.2. Chapter 2. provides information on the airlines and the structure of the frequent flyer programs observed. My frequent flyer model is presented and solved in chapter 3. Trading stamps are introduced and modeled in chapters 4. and 5. respectively.

### 1.2 Survey of the Literature

There has been relatively little written recently in the economic journals about trading stamps and nothing regarding the frequent flyer programs. Nonlinear pricing and product bundling in general have, however, generated much interest and study.

In the late 1950's and 1960's there were quite a few articles and books written about trading stamps. Interest in the subject can be found as early as 1905, when Rubinow published a note in the Journal of Political Economy describing the use of trading stamps in New York City. Duncan (1916) published a survey article on the prevailing thoughts about trading stamps and the legal controversy and court cases leading up to and including a 1916 Supreme Court decision allowing the states to impose special taxes on the programs. * Many of the arguments Duncan reported, both pro and con, continued to be advanced over fifty years later. These include:
... The Trading Stamp, profit sharing coupon, etc., do not create new business, but at best only switch from one brand or merchant to another (Literary Digest, June 5,1915,p.1363).

Trading stamps related litigation can be traced back to 1888 and continued up to the mid 1970's.

It has been argued that the giving of premiums is nothing else than a disguised form of price cutting. (Journal of Commerce, October 14, 1915)

Besides, it is argued, the system defeats its own end in that when one merchant or one manufacturer in a certain line of goods takes it up, his competitors are compelled to follow suit, and thus they are on a common footing again, with an incubus of added cost.

On the positive side:

```
... Its aim is not only to gain, but to retain the purchaser. (Rubinow, Journal of Political Economy, XIII, p574)
```

```
The premium system encourages thrift in purchasers by enabling them to secure certain articles ... In effect this amounts to a savings fund. ( H.T. Graham, Letter to C.S. Duncan)
```

Much of the marketing literature; Pickering (1973), Fulop (1964), Vredenburg (1956), and others, focus upon the ability the programs had to generate huge increases in market share. * Usually this leads to
*
Why a small reduction in price is not simply used is seldom satisfactorily addressed. Often it is attributed to a physiological desire for gifts. Fox (1968), for example, writes the shopper "may regard stamps as her 'reward' but price reductions as her 'due.'"
the oligopolistic 'lock-in' arguments presented in Tauber (1970). In this argument, the merchants become locked in a prisoner's dilemma game in which all firms issue stamps because if any one were to abandon the stamps unilaterally they would lose significant market share. Yet, everyone would be better off if no stores issued the stamps.

Sherman (1968) modeled trading stamps by comparing and analyzing the consumers' budget constraints. He concluded that a customer could be harmed whenever the consumer did not place enough value on the goods offered as premiums that he would purchase any of them if they were not bundled with the other purchases. The large trading stamp programs offered thousands of different products that they could be redeemed against, so in most cases the likelihood of the consumer finding something that he would purchase seems quite high.

Trading stamps and frequent flyer programs have a particular structure in that they are offered over a variety of products and even by different retailers. None of the models offer a good explanation as to this structure and no one predicted, or can explain why the trading stamp's popularity decreased so suddenly in the early 1970's. I believe my model offers satisfactory explanations of this.

Cremer (1983) presents a model of repeat purchases where future products are purchased at a discount. Consumers in his paper do not have complete information regarding the attributes of the product. A
discount in future purchases will make consumer's more willing to try the good. By lowering the future price closer to marginal cost, additional sales and consumer surplus is generated. This is in effect a two-part tariff. His model is quite different from what I am presenting in this paper. In my model, there is perfect information about each product and consumers only wish to purchase one of each good.

Nonlinear pricing has been found to be an effective method for firms to extract consumer surplus and to discriminate between consumers with different demand characteristics. Oi (1971) followed by Feldstein (1972) and Ng and Weisser (1974) analyzed the optimal two-part tariffs. Oi uses as an example the amusement park which can charge an admission price as well as ride tickets. Goldman, Leland and Sibley (1984), and Willig (1978) and Roberts (1979) continue the analysis for general nonlinear schedules and develop the necessary conditions that need to be satisfied.

All of the papers listed above were set in the context of a monopolist selling a single good. Often, however, nonlinear prices are found in industries which are not monopolistic, but rather, competitive or oligopolistic. Recently, this void has begun to fill. In particular, Oren, Smith, and Wilson (1983) examines the sustainability of nonuniform pricing in a multifirm environment. This differs from my work in that, in their paper, Cournot competition is
assumed. I model competition as a price game. Panzar and Postlewaite (1984) analyzes nonlinear pricing occurring as a response to the credible threat large consumers can exert to enter the industry and produce products for their own consumption. For smaller customers, this threat is less credible. The result is a nonlinear price schedule with price reductions for the biggest customers. Holmes (1985) and Borenstein (1984) consider models where each firm produces spacially differentiated products and can price discriminate between classes of consumers in this competitive environment. Spulber (1986) shows that nonlinear pricing is a Bertrand-Nash equilibrium when the products offered and the customers are spacially distributed.

In their classic paper, Adams and Yellen (1976) demonstrate the situations when product bundling can significantly increase the profits to a monopolist. By selling two items together, the firm can potentially extract more consumer surplus than would be possible with individual prices. This is particularly true if the consumer's reservation prices are negatively correlated. Consider, for example, the situation where half of the residents of a small town are willing to pay $\$ .60$ for an apple and $\$ .30$ for an orange, and the other half is willing to pay $\$ .30$ and $\$ .60$ respectively. A monopolist could sell a bundled fruit basket of an apple and an orange to everyone for \$ .90. If they were priced separately, at most $\$ .60$ per person
could be received. Again this analysis is in the context of a monopolistic, and not a competitive environment.

The literature on "tie-in" sales at least goes half way. Here we have the situation where $a$ firm is the monopolistic supplier of a particular item, but sells it only when the consumer agrees to purchase another item for which the firm is not the sole supplier. Scherer (1970) offers six possible explanations. They range from simply a crude attempt to gain monopoly power in the second market, (usually the same result could be achieved by further raising the price of the monopoly good and selling the other at the competitive price) to using the tied product to discriminate between heavy and light users. Scherer uses a copying machine example where a heavy user needs to buy 10,000 sheets of paper, and a light user only 3,000. This requires the heavy user to pay a higher "effective" price than the light user. Liebowitz (1983) adds an additional "risk reduction" argument. If some of a product's purchase price is shifted from the machine to a supply good, which is correlated with the fortunes of the buyer, then the effective price of the copier is higher in prosperous years of heavy use and less in lean years. In essence, the copier is sold with a" small insurance contract attached.

In both my models, the products are "tied" together by a nonlinear rebate schedule or by outright bundling of the products. There are no "monopoly products" per se, nor, is the focus to
discriminate between customers based upon their demand elasticities. Rather, in the frequent flyer model, the discrimination achieved is more akin to dividing and allocating the market shares based upon rules that are "fair" to each company, yet remains a Nash equilibrium to the noncooperative price game. The trading stamp model has a similar flavor, in that the division of the consumers is based upon the total preference of the two teams, rather than just the preferences and prices over that particular item.

The models in this paper are unique to the literature. They show that it is because of the competitive environments that some forms of nonlinear pricing appear.

## Chapter 2. Description of US Airline industry

This chapter contains primarily a description of the important aspects of the United States Airline industry in the early 1980's, both in route structure and in pricing. Those readers already familiar with the industry may wish to skip to chapter 3 where the analytical part of my model begins.

After the deregulation of the industry in 1978 , the airlines were given the freedom and responsibility to compete both in price and choice of markets to serve. The latter is addressed first, and pricing is discussed in 2.2 .

### 2.1 Route Structures

One of the first lessons of deregulation was the great importance and power of a "Hub and Spoke" route system. In such an operation, all flights arrive in the hub city within a short period of time. The passengers who wish to make connections to any of the other aircraft do so, and then all of the aircraft again depart for their final destinations. The advantage of such flight scheduling is clear. With 10 aircraft one can serve 100 city pairs very efficiently through the use of the hub.

## ORIGINS

DESTINATIONS


Figure 1.1.

This traffic is in addition to the local traffic that is handled nonstop to and from the hub city. In general, the hub is the largest city in the region (Chicago, St.Louis, Dallas, Atlanta, etc.), so a large percentage of the traffic is usually local, to (from) this hub city.

It is interesting to note that even before deregulation, the major carriers had route authorities that looked very similar to the above. United Airlines, for example, had many flights in and out of

Chicago. The airlines, however, didn't seem to recognize how powerful the hubs had become with coordinated schedules until Federal Express Co. developed an effective hub and spoke system, for overnight package delivery. Federal Express flew ALL packages into Memphis early in the night, sorted them in a short "window" and then flew them to their destinations. The airlines' cargo of passengers lends itself even more to such an operation since the passengers can sort themselves. Having discovered the hub and spoke, most airlines spent their resources to strengthen their hubs by adding additional spokes to the hubs they had inherited, and diverting the aircraft away from direct flight between other airports. New hubs were created in under-served airports either by existing or new entrant carriers.

Below is the current route structure of a few of the major carriers, from which one can note the hub and spoke structure.


Figure 1.2a


Figure 1.2b


The net result is that almost every major U.S. city now has an airline conducting a "Hub and Spoke" operation out of that airport. This airline, due to the nature of their operation has a substantial market presence in most routes into and out of that city. It is this local dominance that allows me to get the major results of this dissertation.

### 2.2 Airline Pricing

There are two types of airline customers, those who want to fly and those who have to fly. They are usually referred to as discretionary and business travelers respectively.

The discretionary passengers tend to be very price sensitive and place a low value on time. (For example, they do not mind flying on connecting flights if that will save them money.) The business travelers' demands are relatively inelastic, and they will not consider a connecting flight if a nonstop is available.

Ideally. the airlines would like to differentiate between the two groups and offer different fares to each. Indeed they can! By requiring that the passenger stay at the destination at least one Saturday night, or make travel plans 30 days in advance, the airline can discriminate between the two groups. Almost all discretionary travelers wish to stay over at least part of the weekend at their destinations. For the business traveler this would be expensive and very inconvenient.

Due to the elasticity of the discretionary traveler, and the willingness to make connections, the prices charged to them are very competitive. I do not attempt to model this market, but rather look at the competitive structure for the "frequent business traveler." Most of the airlines' revenues are generated by these customers, since
the fares charged to them can be as high as twice or three times the comparable excursion fare. These airfares are slightly offset by rebates given in the form of Frequent Flyer Programs.

### 2.3 Frequent Flyer Programs

Frequent flyer programs were designed to reward customer brand loyalty. They issue rewards, usually free or discounted travel, based upon the number of flights or the mileage the customer has flown. Some of the features that all the programs have had since American Airlines introduced them in 1981 include:

1. Rewards are given after set number of miles.
2. Mileage is not transferable between passengers
3. Reward schedules are convex. (The more miles that you have accumulated in a program, the more valuable they become.)

Below are the redemption schedules of some of the major airlines.

## REPUBLIC AIRLINES

Miles
10,000
20,000
40,000
60,000
100,000

## Reward

First class upgrade
Free U.S. ticket
2 U.S tickets or one to Europe
3 " " " two " "
7 " " " five " "

Figure 1.3a

## UNITED AIRLINES

Miles
10,000
20,000
50,000
75,000

Reward
First class upgrade 25 \% Discount on U.S. ticket Free first class U.S. ticket 2 first class U.S. tickets or one international ticket.

Figure 1.3b

## T.W.A.

Miles
10,000
20,000
50,000
60,000
100,000

## Reward

First class upgrade
25 \% discount on future travel Free international ticket
Two free international tickets Two "around-the-world" tickets

Figure 1.3c

Although they are not identical, they do have essentially the same reward structure. Some carriers such as Republic, have lower mileage requirements, but may not fly to exotic locations such as Hawaii or the Orient.

In addition to offering credit for millage flown, many airlines have joined in partnerships in which hotels, car rental companies and other airlines (usually international or commuter airlines) give credit when purchasing their partners' products. By doing so, the programs begin to resemble the consumer trading stamps that used to exist. These trading stamp programs are the topic of the later chapters of this dissertation.

The explanation offered for the existence of frequent flyer programs most often is that the passenger makes the travel
arrangements, and gets the frequent flier benefits, but his employer pays for the trip. The airlines in effect offer a kickback to the passenger and create a moral hazard problem for the employee who is asked to replace his responsibility to keep his firm's travel expenses down, and instead concentrates on accumulating his own rewards. Indeed a lawsuit to that effect has recently been filed by such an employer.

A more sophisticated argument is that since to date no income taxes are collected on the frequent flyer awards, these programs, when considered part of the employees benefits by his employer, can be provided less expensively than through a more direct, but taxable method.

In any event, the airlines themselves are almost unanimously proud of their frequent flyer programs. * Their satisfaction could come from three sources: increases in market share, additional business travel generated, or changes in the competitive structure of the industry. It is impossible for all airlines to gain market share simultaneously, and the increase in business related travel is probably negligible. This leaves the last premise as the most likely explanation, and that is what $I$ explore in this dissertation.

[^0]The next chapter develops a microeconomic model of how these schemes influence competition. . In particular, I show that by allowing firms with overlapping product spectra to bundle products or offer nonlinear pricing schedules, equilibria other than the standard Bertrand equilibria are obtainable.

## Chapter 3. Frequent Flyer Program Model

The model consists of $N$ cities and $N$ firms, $N>3$. One airline is headquartered in each city and serves the $N-1$ routes from that hub to all other destinations. Each airline therefore competes with every other carrier in one and only one market, and every city-pair is served by at least two airlines. Figure 3.1 demonstrates this structure for $\mathrm{N}=5$.

## CITIES



Figure 3.1

I make the following assumptions:
(A) Fixed and variable costs are assumed to be zero for all firms.
(B) Consumers's hometowns are evenly distributed between all N cities. Within each city, the residents differ in the value they place upon flying. It is assumed that each passenger has a potential demand to fly to all other N-1 cities, and that he places the same value on visiting each city. This reservation value is chosen from a known uniform distribution [a,b] with $b<$ 2a. It should be emphasized that the reservation values over the flights are correlated, but the products are neither complements nor substitutes. The marginal utility of a second trip to any city is zero.
(C) The values for a and b are sufficiently different so that: (N-1)a $<(N-2) b$.
(D) The reservation value of a bundle of two or more tickets is the sum of the separate reservation values.
(E) Tickets or ticket bundles arennot transferable.
(F) The firms have perfect information as to the distribution from which the reservation values were drawn.
(G) Consumers purchase items at the lowest price available, or when weighing the option to buy a bundle, maximize consumer surplus.
(H) Firms are required to post prices of all their products or bundles they wish to sell. Firms may not discriminate between customers and must allow everyone to make any purchases at any posted price.
(I) I assume symmetry. Each firm has the same number of "natural" consumers (customers residing in their hub city). Every city's distribution of consumer reservation prices are identical.
(J) Entry into markets not presently being served, either by existing companies or by new entrants, are not permitted. In a later section, I relax this and allow "small scale" entry.

The model consists of three stages. In the first, the airlines post the prices of all their individual flights and bundles of flights they plan to offer. Each firm reacts to the others' offerings and the expected consumer reactions based upon these offerings by adjusting their prices accordingly. Once this game reaches a Nash equilibrium,
the consumers select and purchase the flight tickets or bundles they desire. Finally the flights are taken.

For the economy to be in equilibrium, I require that the stage one price game be in a Nash equilibrium, the consumers to make purchases that maximize their consumer surplus, and that the firms' expectations of the consumer behavior be rational and correct.

To begin my analysis, I will first explore what the equilibrium would be if each carrier had monopoly power over its "natural customers," those who live in the airline's hub city. I show that monopoly profits can either be generated by bundling the flights and selling them as a package, or by selling each flight separately. I then place these firms in a competitive framework, and ask if the "monopoly" bundling and pricing remain supportable. Given that all other firms continue their monopoly behavior, I show that the best response is indeed for the airline to offer only its bundle at monopoly prices as well. The consumers are not captive to the hometown airline and may purchase from the other carriers, but in equilibrium, no airline will entice him to do so. I begin by studying the optimum monopoly behavior.

### 3.1 The Monopoly Problem

A monopolist in this model selling to his "natural", "internal", or "hometown" customers, has several options. The first is to announce that all flights are to be sold individually. and to announce the prices associated with them, $P_{S}$. The demand function for each market is:
(3.1)

$$
Q=\left(b-P_{S}\right)\left(\frac{1}{b-a}\right)
$$

$$
\text { If } a \leq P_{S} \leq b
$$

Profits for this strategy will be maximized when $P_{s}=a$, since $\mathrm{a}>\mathrm{b} / 2$, yielding:
(3.2) $\pi_{\text {internal }}=(N-1) a$

The second option is to sell all N-1 flights as a package. A sale is made if and only if the price is less than $\mathrm{N}-1$ times the consumer's reservation value. Therefore:
(3.3) $\quad Q_{b u n d l e}=\left(b-\frac{P_{B}}{N-1}\right)\left(\frac{1}{b-a}\right)$

Again the maximum profits are obtained when a total of ( $N-1$ ) a is paid. The profits are:
(3.4) $\quad \pi_{\text {bundle }}=(N-1) a$

The third strategy is to sell several singleton tickets and a package of all other flights. This again yields the same result. There is, therefore, no advantage to bundling as per Adams and Yellen (1976). However, there is no disadvantage either. Bundling will serve a defensive purpose in the next section and we will thus assume that a monopolist would follow the bundling strategy.

### 3.2 Firms in an Oligopolistic Environment

I now relax the assumption that the firms are monopolists over their "natural" customers and allow the passengers to purchase from
any source. Since each market is served by two firms it is possible that a consumer may fly to all N-1 cities without ever using his hometown airline.

If it is expected that all other carriers will continue to sell bundles as if they were monopolists, is it in any airline's best interest to change its pricing strategy to gain additional sales? The answer, (I think surprisingly) is no! If every other airline sells only bundles of their flights to their "natural" customers, the firm's best response is to do the same. Having all firms sell their "monopoly" bundles is a Nash equilibrium to the new competitive price game. The intuition for this result is that any incentive created to steal customers away from the competition also works to cannibalize the firm's own monopoly bundle sales (for a small single product sale). The costs to protect the firm's bundle sales outweighs the additional profits generated by the "outside" sales. Equally important is that it becomes difficult to attract additional "outside" customers because they are being asked to forego the purchase of all the other products that they would have otherwise purchased in the bundle.

Theorem 3.1: If we are given $N$ firms, each selling $N-1$ products, with no two firms selling more than one common good and
assumptions (A) - (J) hold, then having each firm sell only a bundle of his goods at price $P_{B}=(N-1)$ a is a Nash equilibrium. *

Proof: To show this, I analyze how a typical consumer would react if one and only one product were priced separately, say at price $P_{1}$. Possible strategies are:
I. Buy bundle from major supplier.
II. Buy singleton product and skip buying bundle.
III. Buy both.

The consumer surplus associated with the first strategy is the difference between $(N-1) \cdot R$ and $P_{B}=(N-1) a$. The surplus associated with the purchase of the singleton (option II) is $R-P_{1}$. The third option is obviously dominated by option I since a second trip to a destination has no value. In general, if $R$ is low, the customer might easily be tempted to buy only the singleton. If $R$ is high, the passenger is extracting high consumer surplus and thus it may be almost impossible to convince him to buy just a singleton product.

[^1]No firm has a captive market, so, if a firm wishes to attract some of another's customers, it may attempt to do so by offering that overlapping good. Would that pay?

Let us assume that the firm chooses to sell one of its products at price $P_{1}$. An "outside" customer has the choice described above. He will buy the single product for price $P_{1}$ if, and only if, his surplus ( $R-P_{1}$ ) is larger than the surplus received from buying the monopoly bundle. (N-1)(R-a). This is true whenever R satisfies:
(3.5)

$$
\begin{aligned}
& (\mathrm{R} \quad-\mathrm{P})>(\mathrm{N}-1)(\mathrm{R}-\mathrm{a}) \\
& \quad\left[\begin{array}{c}
1
\end{array} \quad \begin{array}{l}
\text { Consumer surplus from bundle. } \\
\text { Consumer surplus from single product. }
\end{array}\right.
\end{aligned}
$$

Or when,
(3.6) $R \geq R^{*}<\frac{(N-1) a-P_{1}}{N-2}$

The "outside" demand function is:

$$
Q_{\text {outside }}=\frac{a P_{1}}{(N-2)(b-a)} \quad \text { If } P_{1} \leq a
$$

(3.7)

Profits generated by selling to "outside" customers are:

$$
\begin{equation*}
\pi_{\text {outside }}=P_{1}\left[\frac{a-P_{1}}{(N-2)(b-a)}\right] \quad \text { If } P_{1} \leq a \tag{3.8}
\end{equation*}
$$

The firm realizes that this has an effect upon its existing customers, some of whom will prefer to buy only the singleton good rather than the bundle previously purchased. By selling each of the other products at price $P_{m}=a$, at least the monopoly profits from those goods can be retained. Later, I will show that the firm can not do better by lowering the price of the large bundle when it insists upon selling the one good at $\mathrm{P}_{1}$.

If the price for the other goods stays at $P_{m}=a$, the "internal" demand for good 1 is a function of $P_{1}$ and is given by:

$$
\text { Q1 } \text { internal }=\left(\frac{\mathrm{b}-\mathrm{P}_{1}}{\mathrm{~b}-\mathrm{a}}\right) \quad \text { If } \mathrm{P}_{1} \geq a
$$

(3.9)
$=1$.
otherwise

The internal profit from good 1 is:

$$
\pi_{\text {internal }}=P_{1}\left(Q_{\text {internal }}\right) \quad \text { If } P_{1} \geq a
$$

(3.10)

$$
=P_{1} \quad \text { otherwise }
$$

Internal profits from all other markets are:
(3.11)

$$
\pi_{\text {other }}=(\mathrm{N}-2) \mathrm{a} .
$$

Adding the profit functions from all three sources; 3.8, 3.10, 3.11, together yields:

$$
\pi_{\text {total }}=P_{1}+P_{1}\left(\frac{1}{b-a}\right)\left(\frac{a-P_{1}}{N-2}\right) a+(N-2) a \text { If } P_{1} \leq a
$$

(3.12)

$$
=P_{1}+(N-2) a \quad \text { otherwise } .
$$

With first and second derivatives:
(3.13)

$$
\frac{\partial \pi}{\partial P_{1}}=1+\left(\frac{1}{b-a}\right)\left(\frac{1}{N-2}\right) a-\left(\frac{2 P_{1}}{N-2}\right)\left(\frac{1}{b-a}\right) \text { If } P_{1} \leq a
$$

(3.14)

$$
\frac{\partial^{2} \pi}{\partial P_{1}^{2}}=\frac{-2}{(b-a)(N-2)}<0
$$

Therefore, if
(3.15) $\frac{\partial \pi(\mathrm{a})}{\partial \mathrm{P}_{1}}>0$, then $\frac{\partial \pi\left(\mathrm{P}_{1}\right)}{\partial \mathrm{P}_{1}}>0$
$\forall \mathrm{P}_{1}<\mathrm{a}$

This happens whenever:
(3.16) $1+\left(\frac{1}{b-a}\right)\left(\frac{1}{N-2}\right) a>2 a\left(\frac{1}{b-a}\right)\left(\frac{1}{N-2}\right)$

Or simply when $\quad b>\left(\frac{N-1}{N-2}\right) a$.

This is of interest since, if the derivative is positive over the whole region, profits are maximized at the $P_{1}=a$ level. Note that at that price, all profits are from equations (3.10) and (3.11) and no outside revenue is generated.

Recall from assumption (B) that $2 \mathrm{a}>\mathrm{b}$. When $\mathrm{N}=3$ the only case where both (B) and (3.16) hold is when $.5 b \leq a \leq .5 b$. When $N=4$ they hold when $.5 \mathrm{~b} \leq \mathrm{a} \leq(2 / 3) \mathrm{b}$.

It was just shown that profits can not be increased by charging $P_{1}$ for the first good and the monopoly single prices for the remaining products. To guarantee that the airlines can not do better by charging $P_{1}$ for the first good and lowering the bundle price, as mentioned earlier, we need the following lemma.

Lemma 3.2: If a firm has committed itself to selling $n$ single products at price $P_{1}$, the maximum profits that can be generated by
offering $P_{1}$ plus a bundled price is never greater than what can be achieved by pricing all ( $\mathrm{N}-1-\mathrm{n}$ ) goods separately.

Proof: Let $P_{B}$ be the profit maximizing bundled price. If $\mathrm{P}_{\mathrm{B}} \leq \mathrm{nP}_{1}+(\mathrm{N}-1-\mathrm{n}) \mathrm{a}$, you are done. This was the internal profit if every other good was sold at price a. Suppose therefore that $P_{B}>n P_{1}+(N-1-n) a$. The cutoff reservation value at which customers are indifferent between buying the stores bundles and only buying the single product, $\mathrm{R}^{* *}$, is:

$$
\begin{equation*}
n\left(R^{* *}-P_{1}\right)=\left((N-1) R^{* *}-P_{B}\right) \tag{3.17}
\end{equation*}
$$

or,

$$
\begin{equation*}
R^{* *}=\frac{P_{B}}{N-1-n} \quad-\frac{n P_{1}}{N-1-n} \tag{3.18}
\end{equation*}
$$

Since there are no customers with.reservation values higher than $b$, the actual cutoff reservation can be considered to be:
(3.19) $\quad \overline{\mathrm{R}}^{* *}=\operatorname{Min}\left(\mathrm{R}^{* *}, \mathrm{~b}\right)$.

Those internal customers with reservation values R greater than $\mathrm{R}^{* *}$ buy the bundle, everyone else buys only the single product. The internal profits therefore become:
(3.20) $\pi_{\text {internal }}\left(P_{1}, P_{B}\right)=\frac{P_{B}\left(b-\bar{R}^{* *}\right)}{b-a}+\frac{n P_{1}\left(\bar{R}^{* *}-a\right)}{b-a}$

If we consider the case when $P_{1}$ is charged for the single products $1-n$, and single prices of $P_{i}=R^{* *}$ for $i>n$, the internal profits become:

$$
\begin{equation*}
\pi_{\text {internal }}\left(\mathrm{P}_{\mathrm{j}}=\mathrm{P}_{1}, \mathrm{P}_{\mathrm{i}}=\mathrm{R}^{* *}\right)=\mathrm{nP}_{1}+(\mathrm{N}-1-\mathrm{n}) \mathrm{R}^{* *}\left(\frac{\mathrm{~b}-\overline{\mathrm{R}}^{* *}}{\mathrm{~b}-\mathrm{a}}\right) \tag{3.21}
\end{equation*}
$$

From equation (3.18) we can see that the internal profits (3.20) and (3.21) are equal. In both cases the "outside" customers are offered the same overlapping good at price $P_{1}$. For any pricing strategy which bundles the remaining products, an individual pricing
strategy can be created with profits at least as large, so to find the optimum pricing policy, bundled strategies need not be considered.
$=$
To rule out the possibility of small bundles being optimal, note that we can look at the smaller problem consisting only of those products and apply the above lemma to that. * This completes the proof of the theorem.

### 3.3 Graphical Analysis of Firms' Demand Functions

Lemma 3.2 allows us to look only at the individual product demands. In the section I graphically illustrate what is happening. Looking at the hometown demand function for one of the products yields figure (3.1).
*
If a firm issues many different bundles for sale, this can be duplicated using single product prices. Start by duplicating the purchases of the lowest type consumer (a), and add additional products priced at the reservation values for which the behavior changes.


Figure 3.1
is
If the firm owner faces the nontruncated demand curve, he would obviously wish to price its product at $P_{1}=b / 2$. Since, however, it is truncated the best he can do is at $P_{1}-a$.

The demand curve from the outside customers is flatter with slope:
(3.22) $m-\frac{-1}{(N-2)(b-a)}$
passing through point ( $\left.P_{1}-\mathrm{a}, \mathrm{Q}-0\right)$. This is shown in the next figure.


Figure 3.2

Superimposing the two, we get the total demand for the product.


Figure 3.3
From this we see that monopoly pricing $\left(P_{1}-a\right)$ is optimal whenever $a>b / 2$ and $a<d / 2$.

### 3.4 Multiple Bundle Sizes

The purpose of this chapter is to develop a model of the airline frequent flyer programs. So far, we have just shown that selling systemwide passes or bundles of the flights may be competitive equilibria. The frequent flyer programs have a schedule of rewards with several different reward levels. In this section, $I$ extend the model of 3.1 by allowing for the possibility that not all customers are identical in that they wish or need to travel to all $\mathrm{N}-1$ cities.

In such an environment, product bundling can still remain an equilibrium strategy if those who travel less frequently place a higher value on travel when they do fly. This equilibrium will have each firm offering bundles of various sizes to the market, the passengers choose what bundle size to buy. This now becomes similar to the frequent flyer programs which allows firms to charge high prices, but offers "kickbacks" when purchases of various the bundle sizes are completed.

For this revised model, assume that each city contains not only those passengers who wish to travel to all N-1 cities with R $\mathrm{U}\left[\mathrm{a}_{1}, \mathrm{~b}\right]$, but also an equal number of passengers who wish to fly to only $N-2$ of the cities with $R-U\left[a_{2}, b\right], a_{2}>a_{1}$. A monopolist would ideally like to extract monopoly rents from each consumer type.

Fortunately for him this is possible if bundles of N-1 and N-2 flights are offered at:

$$
P_{B 1}=(N-1) a_{1}
$$

and,

$$
P_{B 2}=(N-2) a_{2}
$$

The two type of customers will segregate themselves as long as the smaller bundle is priced lower than the larger. A monopoly airline would be able to extract monopoly profits from both types of
customers. Define $\mathrm{P}_{0} \equiv \mathrm{P}_{\mathrm{B} 1}-\mathrm{P}_{\mathrm{B} 2}$ to be the difference between the bundle prices. Since $a_{2}>a_{1}$ and $P_{B 1}>P_{B 2}, 0<P_{0}<a_{2}$.

I again ask whether this remains an equilibrium in the competitive environment where travel on the other carriers is permitted. To show that it can be, I continue to assume that all other airlines are selling only the two bundle sizes to their customers at the prices $P_{B 1}$ and $P_{B 2}$ and confirm that a firm's best response remains to also bundle its products in these two sizes and sell at the same price.

For convenience, let me for the moment assume that the firm is able to perfectly distinguish between the two frequency types, but not their hometowns. In this environment, the firm will be able to earn at least as much profit as when no such discrimination is possible, since it could always ignore the information. Even so, I will show that the best response to the competitors' bundles remain bunding the products also. Since this is optimal in this situation, and is also implementable when the customers types are not observable, it must be the optimal response when consumers' types are unknown.

For those passengers who have a demand to fly on only N-2 of the routes, the analysis remains exactly the same as in the previous section, as long as equation (3.16) holds for $N^{\prime}-N \quad 1$. The optimal strategy is to offer them the N-2 product bundle for (N-2) $a_{2}$, and not to attempt to lure other airlines' natural customers away.

For those passengers who have a demand to fly to all N-1 cities, the analysis is a little more difficult: If the firm chooses to offer an individual price to an "outside" customer, the customer now has three options:
I. Continue to purchase the "hometown" airline's $\mathrm{N}-1$ bundle.
II. Purchase the singleton product and the "hometown" airline's N-2 product bundle.
III. Purchase the singleton product only.

Which of the three alternatives the customer chooses will depend upon the consumers reservation value, the individual price being offered $\left(P_{1}\right)$, and the price difference $\left(P_{0}\right)$ between the two different bundles.

Lemma 3.3: If $P_{1}<P_{0}$, then all the "outside" customers will choose to purchase the singleton product and possibly the smaller bundle from their hometown airline.

Proof: Since $a_{1}>b / 2$ all outside customers will have consumer surplus associated with the purchase of the N-1 product bundle. Purchasing the smaller bundle and the individual single product will
yield the same products at a price $P_{B 2}+P_{1}<P_{B 2}+P_{0}=P_{B 1}$. Any customer who would have chosen to purchase the $\mathrm{N}-1$ product bundle will purchase the individual flight instead, and will purchase the smaller "hometown" bundle is $\mathrm{R}>\mathrm{a}_{2}$.

Lemma 3.4: If $P_{1}>P_{0}$, then all "outside" customers will either choose options I or III.

Proof: Any customer who chooses to purchase both the singleton product and the smaller bundle can reduce his cost, and thus increase his consumer surplus by buying the larger bundle from his "hometown" firm since,

$$
P_{B 1}=P_{B 2}+P_{0}<P_{B 2}+P_{1} .
$$

Lemma 3.5: If $P_{1}>P_{0}$ all "outside" customers with reservation values less than,
(3.24) $\mathrm{R}^{*}=\frac{(\mathrm{N}-1) \mathrm{a}_{1}-\mathrm{P}_{1}}{\mathrm{~N}^{-2}}$.
will purchase the single flight.

Proof: By the above lemma we know that the "outside" customer will either purchase the bundle on $\mathrm{N}-1$ goods or buy only the singleton. This is exactly the situation studied in section 3.2. and all the results from there apply.

By combining the above lemma the "outside" demand function is constructed. It is graphed below:


Figure 3.4

The external demand function is: *


Just as before, this is superimposed upon the "internal" demand, yielding the total demand function from those who wish to fly to $\mathrm{N}-1$ cities of:


Figure 3.5

This is identical to what was found earlier, with the exception of the jump at $P_{0}$. Profits are either maximized at $P_{1}=a_{1}$, as in the case with only one sized bundle, or at $\mathrm{P}_{1}=\mathrm{P}_{0}$. Clearly, whenever $a_{1}>$ $2 \mathrm{P}_{0}$, a2 is the optimal price. At this price, no effort is being made to attract any "outside" customers. Since no effort is made to attract any of the "outside" customers who demand only N-2 tickets, the optimal strategy is to charge monopoly bundle prices to each. This result was derived assuming that the firm could identify how many total flights every passenger demands. In equilibrium this information is not necessary since the two types will segregate themselves. Since the firm is not able to do better with less information, and the optimal prices are implementable without that knowledge, we are guaranteed that this is a best response.

If there are more than two types of customers, the same procedures apply.

## 3.4 "Small Scale" Entry Deterrence

Whenever an industry is makin ${ }^{*}$ abnormally high profits, other firms naturally become interested in entering the industry. In this section, I evaluated the prospects of a firm attempting to enter one of the markets. If the entering firm assumes that the incumbent firms
will continue to sell only monopoly bundles and will not react to him, then the entrant's demand from each city will be exactly the same as that of an existing airline trying to steal "outside" customers away. The total revenue therefore is twice that of equation 3.8 , or
(3.26) $\pi_{\text {entrant }}=2 \mathrm{P}_{1}\left(\frac{1}{\mathrm{~b}-\mathrm{a}}\right)\left(\frac{\mathrm{a}-\mathrm{P}_{1}}{\mathrm{~N}-2}\right)$ If $P_{1}<a$
$=0$
otherwise.

Since this is a linear demand function the new entrants revenue is maximized for $P=a / 2$. The maximum obtainable revenue is:

$$
\begin{equation*}
\pi_{\text {entrant }}^{*}=\frac{a^{2}}{2(b-a)(N-2)} \tag{3.27}
\end{equation*}
$$

Recall, that from assumption (C) of the model (N-2)b $>(N-1) a$. This can be rewritten as:

$$
\begin{equation*}
b-a>\left(\frac{a}{N-2}\right) . \tag{3.28}
\end{equation*}
$$

This places an upper limit on the value of equation 3.27 of:

$$
\begin{equation*}
\pi_{\text {entrant }}^{*}=\frac{a^{2}}{2(b-a)(N-2)}<\frac{a}{2} \tag{3.29}
\end{equation*}
$$

If there was a fixed cost of providing the service of at least a/2 then no "small scale" entry is possible. Existing•carriers would continue to earn positive profits since their average revenue per route is a.

If a new entrant has the ability to enter more than one market, or an existing airline enters an outside market, it becomes easier to successfully enter, and the analysis becomes more difficult. For now, a potential customer can be offered the opportunity to fly to at least two cities on the other airline, and does not have give up flying to all other cities since this new carrier can take him to at least one other city.

I assumed that for both types of entry, the existing firm will not be able to react to the new entrant's price. In all likelihood this will not happen, and we will want to require the post-entry game also to be in Nash equilibrium. However, the moment a carrier is successful at attracting customers at a single product fare, the competitor's best response is to undercut it by $\varepsilon$. Therefore, in the post-entry equilibrium, we would have these products priced at the $\mathrm{P}=0$
level and possibly have the remaining products bundled. Entry into an industry as described above will by itself destroy the profit potential in which the newcomer was interested.

The model also predicts that it is easier for an existing airline to expand than for a new entrant to appear, and that any such new entrant will be forced operate out of $a$ hub in which it has a reasonable presence and to supply those customers with the lowest reservation values. Both of these seem to be characteristics of the successful post-deregulating airlines.

Chapter 4. The U.S. Trading Stamp Industry

Merchants in almost every industry have at some time used quantity discounts to reward those customers who place large orders and for those "good" frequent customers. Everyone is familiar with the "baker's dozen," or wouldn't be at all surprised if the local shoe store gave the children of frequent customers lollipops. Indeed, the giving of gifts and premiums to encourage sales has been going on throughout history. Even in ancient Athens the idol manufacturers offered free lamps, incense or cups whenever an idol representing a god was sold.

Since 1896, the Sperry and Hutchinson Corporation and other trading stamp companies have found a way to reward loyalty, not to just an individual firm, but to a "team" of merchants. They do this by licensing certain merchants to distribute its trading stamps to their customers, who in turn combine them with those from other merchants and redeem them for cash or merchandise directly from the trading stamp company.

These programs are interesting to an economist for several reasons. The first is the programs' longevities. Over 90 years later, Sperry and Hutchinson still issue $S \& H$ Green Stamps. (Although at an extremely diminished level since the program heyday in the 1950's and 1960's.) The next reason is the program's
interesting structure. Why don't the individual stores have their own nonlinear pricing schedules or operate their own "gift" promotions? Lastly, one would want to know why after being so popular and having become an apparently permanent fixture in the retail competitive environment did they almost completely disappear in the early 1970's.

### 4.1 Description and Method of Operation

$S \& H$ and the other stamp companies print and license individual merchants to issue stamps to their customers. The merchant agrees to give all customers the stamps at a rate which was usually 1 stamp per $\$ .10$ in the hope that this will help differentiate himself from his competitor and will increase or at least maintain his market share. To enable the merchant to so differentiate himself, the trading stamp company guarantees the merchant that he will be the only store to be allowed to issue the stamps in his market. $S \& H$ further elaborates in its SEC "10-K" filing*:

The size of the marketing area for which exclusive rights are given varies depending upon the type of business. For example, a supermarket will ordinarily have a larger exclusive area than a service station.

## *

From the 1972 Sperry and Hutchinson Co. Security and Exchange Commission form $10-\mathrm{K}$

This naturally creates a vacuum of stores not affiliated with S \& H Green Stamps. A rival trading stamp company generally fills that void and licenses these stores to issue their stamps. The result is generally an S\&H Green Stamp "team" and a regional "yellow stamp team." Of these, $S \& H$ further writes in its " $10-\mathrm{K}$ ":


#### Abstract

Another important feature of the Company's service is its "cooperative" nature. The Company endeavors to license a group of non-competing retailers within a marketing area, generally including a store which attracts a large number of customers, such as a supermarket. As a result, consumers who are attracted to one retail establishment because of their interest in obtaining $S \& H$ Green Stamps tend to become patrons of the other licensees of the Company in the area.


The structure of the redemption schedules has many similarities with those of the frequent flyer programs studied in the first half of this dissertation. In both, the consumer collects his "stamps" or "miles" with the hope to redeem them for one of the published prizes. In the airline example, the schedule was obviously convex. The trading stamps are redeemable for literally thousands of items in a thick catalog. The schedule, from the perspective of the consumer, must be convex since someone with $N$ books to redeem always has the option to "buy" two items for $\alpha N$ and $\beta N$ books respectively ( $\alpha+\beta-1$ ) if that is preferred over any of the individual items available for $N$
books. Since the value of the books is relatively small
(approximately $\$ 2.00-3.00$ per book), the transaction costs of redeeming only a few books further enhances the nonlinearity.

The convexity will be important in the model I present for the same reasons as in the frequent flyer programs. It insures that the consumer will find it in his best interest to buy all items from the same "team" of stores.

Any economic model of the trading stamp phenomenon should be able to offer an explanation for the sudden decline and almost disappearance of the trading stamps that occurred in the early 1970's. Figure 4.1 illustrates this decline in the number of merchants participating.


Figure 4.1

Two significant events took place in 1973. In September, the Federal Trade Commission issued a consent order that required the trading stamp companies to redeem the stamps for cash upon request. The rate was fixed at $\$ 2.00$ per book, subject to revision. Since consumers still retained the ability' to redeem stamps for merchandise, a nonlinear structure remained despite the linear $\$ 2.00$ / book floor. Indeed, only $3 \%$ of the consumers accepted the cash.

The more significant event appears to have been the Arab oil embargo of 1973. At that time, the service stations, which had been issuing 27-28\% of the stamps, faced regulated gas prices and shortages created lines at the gas pumps. In such an environment participating in a trading stamp program made absolutely no sense at all. Indeed, the percentage of stamps issued by service stations completely collapsed to only 4.18 in 1974.


Figure 4.2

As the percentage of stamps issued by service stations approached $4 \%$, the food store and supermarket now approached $78 \%$ of the
diminished total. After this occurred, the supermarkets began abandoning the program as well, dooming the trading stamps in that region. This is consistent with the model presented in chapter 5.0.

### 4.2 Survey of the Literature

As early as 1905 the "Journal of Political Economy" published a note written by I.M. Rubinow describing the rise of trading stamps and other "premiums" in New York City. Of the premiums he editorializes:
> ... The system influences the public to buy larger quantities of supplies than are necessary. in return for which the home of the consumer is filled to overflowing with ugly and useless articles.

He then continues to speculate that if the retail value of the stamps were established, no businessman would ever pay more for them than that, and the programs would disappear. In fact almost throughout, the trading stamps were redeemed against items in a published catalog, which remained relatively constant, yet trading stamps were being issued in great number up through sixty years later.

Since the very beginning, trading stamps and premiums have been of interest to economists and the marketing community. Although this literature dates back to at least 1905 , very few economically sound models have appeared.

Usually, after study, the authors conclude that the programs exist since the consumers "enjoy" receiving free gifts, or that housewives use trading stamps as a means of transferring resources out of the household budget and instead can use the books to purchase desired personal and household items which she would find hard to justify, but now doesn't need to since they are "free!"

Andreano (1959) analyses the effects trading stamps would have upon retail prices if they either made the firms revenue curves flatter or steeper. Yet, no satisfactory reasoning why either should be expected was provided. Davis (1959) suggests a discrimination motive. Consumers who are price sensitive collect the stamps and therefore receive a bigger rebate. Those who do not do not get this rebate. Although there is no doubt that there is some validity to this argument, the fact that $85 \%-90 \%$ of the stamps are eventually redeemed, suggests that this might be an expensive way to single out the remaining 15\%. Also, the merchant is supposed to give the stamps to all consumers, and would have to pay for them regardless of whether the consumer disposed of them or not.

The most prevalent argument, however, is that these are marketing gimmicks initially adopted to increase market share, but become "oligopolistic 'lock-ins'" when the competitors join or form their own trading stamp program. In the oligopolistic 'lock-in,' the firms are stuck with the costs but without the benefits. Unilateral withdrawal
is unreasonably expensive, since the remaining merchant would get the increased market share. It is a "prisoner's dilemma" game, where it is in both stores' best interests to have the programs, while both agree that neither belonging would be preferred.


Figure 4.3

There are two main problems I find with all of the above explanations. The first is that they do not provide a rationale for having the outside trading stamp company, instead of having their own internal program. Secondly, they don't explain the almost complete collapse of the trading stamp phenomenon since 1973.

The empirical studies in this field are equally disappointing, with some finding an increase in price level among cities with trading stamps and others not finding any. This is especially tricky since my model would predict trading stamps to occur in locations in which competition among firms is in a fierce Bertrand price cutting environment. In a less rigorous competition, or even collusion,
higher prices might be expected, even without a trading stamp program.

Chapter 5. Trading Stamp Model

This chapter develops an economic model of the trading stamp phenomenon. In particular, I show how the implementation of a nonlinear rebating scheme over a "team" of stores can enable all the equilibrium prices to rise. I also show the welfare costs the trading stamps impose, and why they have all but disappeared since 1973.

The model is structured in many ways like that of the frequent flyer programs presented earlier. Two important differences exist, however. The first is that the various products are each sold in small independently owned and operated stores, rather than by the relatively few airlines that sold tickets for a series of flights to many destinations. Secondly, I assume that the products offered are not homogeneous as was the case for the individual airline flights, but rather slightly differentiated due to the store's location, brand carried, color, or similar causes.

In 3.0 , it was primarily route structures which were differentiated, not the individual flights. The bundling, or nonlinear pricing shifted the competition to the route structure level from the individual flights. Here the trading stamps will similarly give an incentive for the customer to buy all products from the same "team." The consumers' preferences over these teams will generally be stronger and more diversified, being the sum of the consumer's
preferences of each item. This affects the store owner's demand function since the consumer now chooses between the teams and not the particular items. This, coupled with the fact that they will not be concerned with the with the externalities their prices inflict upon other team members, will allow prices to rise.

### 5.1 Store Structure

In this chapter, I model a "small town" environment by assuming that there exist a whole series of small independently owned and operated stores. For each type of store (butcher, baker, and candlestick maker) there are two competing shops. These two merchants compete against each other in a price game. If their products were homogeneous this would imply that the standard "Bertrand" equilibrium prices would prevail in any direct competition. Since the products are differentiated in this chapter, equilibrium prices will naturally be higher. As will be seen in section 5.4 , these prices and profits are further enhanced with the existence of a trading stamp company.

### 5.2 Consumer's Demand

Every resident of this town has a completely inelastic demand for one good from each type of store. He, however, has a preference between the stores. This is such that he prefers the product from store A unless he can obtain store B's product for X dollars less. X is distributed according to a density function $f(x)$ which is:

1. Continuous
2. Bounded
3. Symmetric
4. Unimodal
5. Mean equal to zero.

A negative value for X indicates a preference for $\mathrm{B}^{\prime} \mathrm{s}$ product over A's. In the example presented in $5.6 \mathrm{f}(\mathrm{x})$ will be uniformly distributed between [ $-1,1]$ although many other common distributions could have been used.

If a consumer is required to make a purchase of a bundle of two goods from store B, he will do so if and only if the price is at least $X_{1}+X_{2}$ dollars less than he could purchase those items from the "A"
stores. Similar calculations are made when deciding to purchase larger bundles.

### 5.3 Equilibrium Without Trading Stamps

For a merchant to calculate his optimum prices, he needs to take into account his competitor's price and the knowledge that if he undercuts that price he will sell more; if he offers his goods at a premium his sales will suffer. The exact number of customers lost or gained can be calculated from the distribution function $f(\cdot)$ introduced in 5.2 .
f(•)


Figure 5.la


Figure 5.1b

If his competitor is charging price $P^{c}$ the demand at his store when he charges $P^{c}+\Delta$, is given by the shaded area in figures $a$ and $b$. Note that $f^{*}(\cdot)$ is just the distribution $f(\cdot)$ centered about the competitors price $\mathrm{p}^{\mathrm{C}}$.

In equilibrium, both of the firms must be charging identical prices. This is guaranteed by the following lemma.

Lemma 5.1: Given that consumers' preferences are such that the monetary equivalent of their preferences are distributed according to a continuous unimodal and symmetric distribution centered about zero, any prices charged in equilibrium must be identical.

Proof: Suppose not, then there exists an equilibrium such that one store charges a premium of $\Delta$ over the other. This firm would capture all customers in area I below, The other firm would sell to those customers in region II, $F(\Delta)$.


Figure 5.2

Since this is supposedly in equilibrium $M R=0$ for both firms.
The marginal revenue from cutting the price by a small $\varepsilon$ is given by:

$$
\begin{align*}
& M R_{1}=P_{1} f(\Delta)-(\text { Area } I)  \tag{5.1}\\
& M R_{2}=P_{2} f(\Delta)-(\text { Area } I I)
\end{align*}
$$

$$
\text { Firm } 1
$$

Firm 2

But $P_{1}>P_{2}$, and Area $I<I I$, so if $M R_{2}-0$ then $M R_{1}>0$ which contradicts the assumption that we were at an equilibrium.

$$
(\rightarrow \leftarrow)
$$

The equilibrium price of $P^{*}$ will thus be charged by both firms and each will get $1 / 2$ of the market. For $P^{*}$ to be the equilibrium, neither firm must be able to benefit from a slight $\varepsilon$ reduction in price. $P^{*}$ must therefore satisfy the following for both firms:

$$
\begin{align*}
& 0=\operatorname{MR}(\varepsilon)^{*}=P(\varepsilon f(0))-\varepsilon(1 / 2) .  \tag{5.2}\\
& {\left[\begin{array}{l}
\text { Loss from existing customers. } \\
\text { Gain in revenue from new customers. }
\end{array}\right.}
\end{align*}
$$

If $f(\cdot)$ is the uniform distribution on ( $-1,1$ ), this would imply an equilibrium price of $\mathrm{P}^{\star}=1 / 2(1 / 2)=1$. This yields a profit of ( 1 ) $1 / 2=1 / 2$ for each store. This will be repeated in each industry.

### 5.4 Trading Stamp Company

As before, I now introduce a trading stamp company to the small town. This company offers to each merchant the opportunity to participate in one of two trading stamp programs. The structure of these include the following features:
1.) The merchant agrees to issue a stamp to a consumer whenever a sale is made.
> 2.) No competing stores will be offered participation in a program its competitor is affiliated with.
3.) Consumers collect all stamps issued to them and after all purchases are made, redeem them from the T-Stamp Co. according to a published schedule, $\mathrm{V}(\mathrm{X})$.
4.) Redemption schedules are convex (marginal value of stamp is increasing).
5.) $V(N)>2 V(N / 2)+(N)(\operatorname{Max}|X|)$
6.) Stores are charged the expected average stamp redemption value.
7.) Stamps are non-tramsferable.

For the example in this chapter, I choose the redemption schedule,
(5.4) $\quad V(X)=X^{2} / 12$.

The marginal value per stamp is given by.

$$
\begin{equation*}
M V(X)=V^{\prime}(X)=X / 6 \tag{5.5}
\end{equation*}
$$

Other redemption schedules that satisfy requirement 5.0 could also be chosen. The reader can easily verify that the above satisfy \# 1 -6 when $\mathrm{N}=36$. In particular;

$$
\begin{equation*}
\operatorname{MV}(36)=6>1 \tag{5.6}
\end{equation*}
$$

$$
\begin{equation*}
V(36)>V(X)+V(36-X)+X(1-(-1)) \tag{5.7}
\end{equation*}
$$

(5.6) guarantees that if all products except the last are purchased from one team, the last will also be purchased from that team. The fact that $M V(N)$ is substantially larger than 1 indicates he will continue to do so even if the competitor undercuts the price by up to $\$ 5.00(\$ 6.00-\$ 1.00)$. (5.7) further guarantees that if all store pairs price identically, consumers will always find it advantageous to purchase all goods from the same team. The proof of the last statement is as follows. Consider the extreme case when for $M$ products Team A's products are preferred by the maximum amount $(+1.00)$ and forvthe remaining $36-M$ products team $B$ is preferred by the maximum amount ( $-\$ 1.00$ ). For $M=1$, equation 1 guarantees that the consumer will not defect. For $M=2$ or more, we must consider the possibility of purchasing more than one product from
the rival team. Figure (5.3) below shows the total "rebate" that a consumer will receive when purchasing $m$ products from one team and $N$-m products from the other.


Figure 5.3

The non-shaded area corresponds to the loss in stamp redemption value if $m$ products are purchased from the other team. If this area is less than $m \cdot(\operatorname{Max}|X|)$, the maximum preference, the consumer will not purchase goods from separate store "teams". This is formalized in 1emma 5.2 .

Lemma 5.2: If equation 5.3 is satisfied, MV(X) > 0 , and all firms charge identical prices, then the consumers will purchase all products from firms issuing the same color stamps.

Proof: From (5.3),

$$
\mathrm{V}(\mathrm{~N})>2 \mathrm{~V}(\mathrm{~N} / 2)+\mathrm{N} / 2(\operatorname{Max}|\mathrm{X}|)
$$

or,

$$
\begin{equation*}
\int_{0}^{N} \operatorname{MV}(n) d n-2 \int_{0}^{N / 2} \operatorname{MV}(n) d n>\frac{N}{2}(\operatorname{Max}|X|) \tag{5.8}
\end{equation*}
$$

Breaking the first integral into two pieces yields:
(5.9)

$$
\int_{N / 2}^{N} \operatorname{MV}(n) d n-\int_{0}^{N / 2} \operatorname{MV}(n) d n>\sum_{2}^{N}(\operatorname{Max}|X|)
$$

Through a change in variable, this is rewritten as :
(5.10)

$$
\int_{0}^{N / 2} \operatorname{MV}(N-m)-M V(m) d m>\frac{N}{2}(\operatorname{Max}|X|)
$$

Let $z(m) \equiv M V(N-m)-M V(m)$,

This is the difference between the different marginal values of the two types of stamps when $\mathrm{N}-\mathrm{n}$ and n of each respective type is already held.

It can easily be verified that $f(m)$ has the following properties:

$$
\begin{array}{ll}
z(m) \geq 0 & \forall \\
z^{\prime}(m)<0 & \forall N / 2 \\
z(N / 2)=0 & m \leq N / 2
\end{array}
$$

Since $z(m)$ is a decreasing function :
(5.11)

$$
\int_{0}^{M} z(m) d m \quad \geq \frac{M}{N / 2} \int_{0}^{N / 2} z(m) d m \quad \forall M \leq N / 2
$$

$$
\begin{aligned}
& \geq \frac{M}{N / 2}\left[\frac{N}{2}(\operatorname{Max}|X|)\right] \\
& =M \cdot(\operatorname{Max}|X|)
\end{aligned}
$$

The lower value of stamp rebate is therefore greater than the maximum preference a customers can have over those $M$ products. The consumers therefore will always purchase all goods from one of the two "teams."

Any symmetric price equilibrium will therefore have all customers loyal to a team of stores that issue the same brand stamp. If the left hand side of equation 5.11 is sufficiently greater than the right, the above result holds even if there is an individual price difference in one of the products. This is true whenever the price difference is less than that surplus.

I now calculate what the new equilibrium prices must be, and show that this is indeed a Nash equilibrium. The candidate for equilibrium I propose has every firm charging a price :

$$
\begin{equation*}
P=A V+P^{*} \tag{5.12}
\end{equation*}
$$

where $A V$ is the average redemption value of the trading stamps if $N$ total stamps are collected and;

$$
\begin{equation*}
P^{*}=1 / 2 f_{B}(0) \tag{5.13}
\end{equation*}
$$

where $f_{B}(\cdot)$ is the marginal distribution of $\Sigma X_{i}$.
Before continuing, the following theorem is presented to establish several key relationships between $f_{b}(\cdot)$ and the individual distributions $f(\cdot)$. Recall that $X_{i}$ is the dollar value of the preference for buying good i from store $A$ instead of $B$, that is independent of $X_{j} \quad \forall i \neq j$.

Theorem 5.3: Let $f(\cdot)$ and $g(\cdot)$ be two probability functions which are:

1. Continuous
2. Bounded
3. Unimodal
4. Symmetric about zero. $f(x)=f(-x), g(y)=g(-y)$.

Then if $X \sim f(\cdot)$ and $Y \sim g(\cdot)$ are independently distributed, the random variable $\mathrm{X}+\mathrm{Y}-\mathrm{h}(\cdot)$, where $\mathrm{h}(\cdot)$ has the properties that:
i. $h(\cdot)$ is symmetric about zero. $h(z)=h(-z)$.
ii. $h(0) \leq f(0)$ and $h(0) \leq g(0)$.

```
iii. H(Z) \geqF(Z)
* z<0
H(Z)}\geq\textrm{G}(\textrm{Z}
* z<0.
```

Proof: $\quad H(Z) \equiv \operatorname{Pr}(X+Y \leq Z)$
$=\int_{-\infty}^{\infty} F(z-y) g(y) d y$
$=\int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f(x) g(y) d x d y$

$$
=\int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f(-x) g(y) d x
$$

$$
=\int_{-\infty}^{\infty} \int_{y-z}^{\infty} f(x) g(y) d x d y
$$

$$
=\int_{-\infty}^{\infty} \int_{y-z}^{\infty} f(x) g(-y) d x d y
$$

$$
=\int_{-\infty-y}^{\infty} \int_{-\infty}^{\infty} f(x) g(y) d x d y
$$

(5.14)

$$
=\operatorname{Pr}(X+Y \geq-Z)
$$

$$
H(Z)=1-H(-Z)
$$

$$
\forall \quad z
$$

$$
\begin{aligned}
& H(Z)=\int_{-\infty}^{\infty} F(Z-y) g(y) d y \\
& h(z)=\int_{-\infty}^{\infty} f(Z-y) g(y) d y \\
& h(0)=\int_{-\infty}^{\infty} f(-y) g(y) d y
\end{aligned}
$$

Since $f(\cdot)$ is unimodal, centered about zero, $f(x) \leq f(0) \quad \forall x$.
$(5.15) \quad h(0) \leq \int_{-\infty}^{\infty} f(0) g(y) d y \quad=\quad f(0)$

$$
H(Z)=\int_{-\infty}^{\infty} F(Z-y) g(y) d y
$$

$$
=\int_{-\infty}^{\infty}\left[F(Z)-\int_{Z-y}^{Z} f(x) d x\right] g(y) d y
$$

$=F(Z)+\int_{-\infty}^{\infty} \int_{Z-y}^{Z} f(x) g(y) d x d y$
$=F(Z)+\int_{0-y}^{\infty} \int_{Z}^{Z} f(x) g(y) d x d y+\int_{-\infty}^{0} \int_{Z-y}^{Z} f(x) g(y) d x d y$
(5.16) $=F(Z)+\int_{Z-y}^{\infty} \int_{Z(x)}^{Z} g(y) d x d y+\int_{-\infty}^{0} \int_{Z-y}^{Z} f(x) g(-y) d x d y$

With a change in variable in the second integral becomes:
$H(Z)=F(Z)+\int_{0-y}^{\infty} \int_{Z(x)}^{Z} g(y) d x d y+\int_{Z+y}^{\infty} \int_{Z}^{Z} f(x) g(y) d x d y$ (5.17)

Again with a change in variable, this time in the x 's:
(5.18) $=F(Z)+\int_{0}^{\infty}\left[\int_{0}^{y} f(Z+x) d x-\int_{0}^{y} f(Z-x) d x\right] g(y) d y$

If $Z<0, f(z+x)>f(z-x) \quad \forall>0$.

Therefore, $\int_{0}^{Y} f(Z+X)-f(Z-x) d x \quad>0 . \quad \forall \quad Y>0, Z<0$
$\therefore \quad H(Z) \geq F(Z)$ for all $Z<0$.

Similarly. $\quad \underline{H}(Z) \geq G(Z)$ for all $Z<0$.
(iii)
\#
Essentially, the theorem states that the sum of the two random variables has a distribution which is a special mean preserving spread of the two base distribution functions, where mass is taken from the center and shifted towards the tails. For the sum of a larger number
of independently distributed $X_{i} s$ the theorem can be applied repeatedly. The central limit theorem tells us that as that number $N$ becomes large this is going to approach a normal distribution function, with mean zero and variance equal to the sum of the variances, $f_{b}(\cdot)-N\left(0, \Sigma \sigma_{i}{ }^{2}\right)$. This is regardless of what bounded symmetric distribution the $X_{i}$ 's came from.

With these results, we can now return to calculating what equilibrium prices will be expected if a trading stamp company existed in the small town. To do this, I begin by looking at the problem from the point of view of a shop owner. If all other firms, both his competitor and all other merchants, are charging price

$$
P=A V+P^{*}
$$

Also charging price $P=A V+P^{*}$ will yeild sales to half the population. (Those with $\Sigma \mathrm{X}_{\mathrm{i}}>0$.) By lowering the price by $\delta$, additional customers can be attracted who fall into one of the following groups:

1.) $\Sigma X_{i} \in(-\delta, 0) \quad$ Those customers who abandon the $\quad$| competitive bundle completely, and |
| :--- |
| now purchase all items from your |
| "team". |

2.) $X_{i}+\delta-\operatorname{MV}(N)>0, \quad$ Those who do not abandon their other (but do not satisfy 1.) bundle, but are offered such a low price that they can't resist buying the singleton item from this store.

If $M V(N)$ is high, the number of customers satisfying 2.) can be made arbitrarily small. Indeed, if $M V-P^{\star}+\operatorname{Max}|X|$, no customers of type (2) could ever be attracted without charging prices below cost. The MV(•) need not be that large to keep firms from attempting to woo these customers as will be shown in the example of 5.6.

The firms net profit per good sold is:

$$
\begin{equation*}
P_{N E T}-P-A V, \tag{5.18}
\end{equation*}
$$

where $A V$ is the cost of the stamp paid to the trading stamp company, or just $P^{*}$. If $M V$ is high enough to rule out type 2.), the demand can be calculated from the cumulative normal distribution obtained from the normal distribution centered about $\mu=0$. A price cut of $\delta$ will make the bundle of products from his store $\delta$ dollars cheaper and will
convince those represented by the shaded area below to purchase their entire bundle from this merchant's "team".

If all other firms are offering their products at price:

$$
\begin{equation*}
P=P^{*}+A V \tag{5.19}
\end{equation*}
$$

The first order conditions for an optimum at price $P-P^{*}+A V$ hold. Computer simulations confirm that this local optimum is indeed global.

Therefore, if all other firms are charging the above price for their products, the best response for the merchant is to also price his product at $P=P^{*}+A V$. All firms are therefore at a Nash equilibrium. Comparing the Nash equilibrium found here to that found in 5.3, we find that the effective price of the goods (after the cost for stamps is deducted) charged has increased to:

$$
\begin{equation*}
P^{*}-\frac{1}{2 f_{B}(0)} \quad \text { from } \quad P-\frac{1}{2 f(0)} . \tag{5.20}
\end{equation*}
$$

From theoren 5.3, we know that $f_{b}(\cdot)$ is a mean preserving spread of $f(\cdot)$ with $f_{b}(0) \leq f(0)$, and that

$$
P * \geq P .
$$

Through the higher prices charged, consumer surplus is transferred to the merchants who enjoy greater profits, and are made better off.

### 5.5 Example

To illustrate the effectiveness of trading stamps to reduce competitive pressures, I introduce the following example.

Consider a town with 72 small independently owned and operated shops, two of 36 different types, (again butchers, bakers, and candlestick makers) so $N=36$. Consumers have an inelastic demand for one of each 36 goods and have preferences over which stores to purchase them. These are characterized by $X_{i}$, the monetary equivalent that would make them indifferent between the two shops. $X_{i}$ is distributed uniformly from [-1,1]. $\mathrm{X}-\mathrm{U}[-1,1]$.

The price equilibrium we would see if no trading stamp companies existed would be where each store charges price $\mathrm{P}=1$. Profits would be $\pi_{i}-1(1 / 2)$, since $1 / 2$ of the customers would purchase each product at that price.

A trading stamp company now enters the town and offers half of the businesses the right to issue जreen stamps and the other half the right to offer Yellow stamps. It charges each merchant a fee of $\$ 3.00$ per stamp, the expected average redemption value (AV) that it expects
it will have to return to the customers, based upon the redemption schedule:
(5.21) $V(n)=\frac{n^{2}}{12}$,
where n is the number of stamps that a customer has collected. The marginal value, $\operatorname{MV}(\mathrm{n})$, for a stamp a function of how many others are already, or will be collected:
(5.22) $M V(n)=\frac{n}{6}$.

The distribution $\mathrm{f}_{\mathrm{b}}(=)$ of $\Sigma \mathrm{X}_{\mathrm{i}}$ for each customer is approximately normal with $\mu=0$ and $\sigma^{2}=36(2 / 3)=24$.


Figure 5.4

The equilibrium effective market price charged by store owners is therefore approximately :
(5.23) $P^{*}=\frac{\sqrt{2 \pi 24}}{2} \approx 6.14$

The total price charged is $\$ 6.14+\mathrm{AV}-\$ 9.14$.

This equilibrium was calculated based upon the $f_{b}(\cdot)$, with no customers choosing to purchase this single product from this store and all others from their normal "Team." It can easily be seen that in


#### Abstract

equilibrium, it does not pay for a firm to attempt to woo customers to purchase just a single product. Customers will not even consider such a step unless the price differential $\Delta$ plus their intrinsic preference for that product (at most $\$ 1.00$ ) is greater than $M V(N)=6$. No such sales are made unless $P<P^{*}+1-6<2$. If at that price, all customers purchase the good (Not all will), the total profits would still be only $2(1)$, which is significantly less than the profit made at price $P=P^{*}+A V$. So, indeed the candidate is an equilibrium.


### 5.6 Explanation for the decline in Trading Stamps and Why K-Mart and Zayre don't issue them.

An important element in the model presented in this chapter was that each store be independently owned and operated. This is necessary since any change in price effects not only the demand for the store's good, but also his fellow team members' demand functions. If two stores are jointly owned, (or if one shop sells type types of goods, like a supermarket) this effect can not be ignored.

Consider for example, modeling the same town as in the last section, except instead of having two bakers and two butchers, let us have two supermarkets that sell both products. The structure of the town now looks like:

| X | X | X | X | X | X | X | X | X |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |$\quad$ "green" stores

Fiqure 5.5

For all the individual stores, the equilibrium effective prices will remain $\mathrm{P}^{*}$ as long as the two supermarkets charge prices identical to each other.

For all firms the equilibrium prices charged will have to satisfy the first order condition that $\mathbb{M}=0$. Let $\hat{\mathrm{P}}$ be the effective price charged by the supermarkets. For one of them to lower its price by $\Delta$ will yield additional sales revenue of:

$$
\begin{equation*}
\text { New Revenue }=2 \hat{P}\left(f_{B}(0)\right) \Delta . \tag{5.24}
\end{equation*}
$$

Lowering the price will yield less profit from the existing sales, thus:

Lost Revenue $=(1 / 2) \Delta$.

In equilibrium therefore:

$$
\begin{equation*}
\hat{P}=\frac{1}{4 f_{B}(0)}=\frac{P^{*}}{2} . \tag{5.26}
\end{equation*}
$$

In the extension of the example of 5.5 , this would yield an effective price of $\$ 3.00$ per good. This is less than the $\$ 6.00$ found earlier, but still higher than $P=\$ 1.00$ which would prevail without any trading stamp programs.

Even if one merchant were to control two stores of the same team a similar effect will take place. Suppose one firm now controls two of the stores, the town now looks like figure 5.6 below:

| X | X | x | X | X | x | X | X | X | "green" stores |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | "yellow" stores |

Figure 5.6

The firm now has a greater incentive to lower price since it gains not only sales in the producí reduced, but also sales in the other product. This lowering of price leads the "green" team to capture more customers. As this happens, both the fellow "green" team members and the "yellow" merchants react and dampen the effects. The
"yellow" merchants do so by lowering their price in an attempt to recapture some of the customers. The remaining "green" store owners attempt to capitalize on increased market by raising their prices. Neither of these can completely compensate. In equilibrium, the prices and profits move in the directions indicated by figure 5.7.

| $\mathrm{P}_{\mathrm{XX}}$ | $\downarrow$ | $\mathrm{P}_{\mathrm{X}}$ | $\uparrow$ | $\mathrm{P}_{\mathrm{O}}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\pi_{\mathrm{XX}}$ | $\downarrow$ | $\pi_{\mathrm{X}}$ | $\uparrow$ | $\pi_{0}$ |
|  | $\downarrow$ |  |  |  |

Figure 5.7

The merchant could, of course, operate the two shops as if they were independent, but this wouldn't be a Nash equilibrium since technically it is not the "best response" to the other store prices.

A better strategy for a businessman would be to own two stores, each of which participates in a different stamp program.


Figure 5.8

By doing so, the owner can easily raise prices since any customers who abandon one bundle due to the higher prices are sold products from the merchants other store. This is almost completely identical to the situation when one merchant owns both stores of the same type.

Returning to the case in which one type of store sells more than one product, we find that as the number of products sold by the same type of stores increase, the supportable effective prices decreases. The extreme case is when all products are sold by two competing department stores. Here the department stores would compete in the total price of the bundles. This is isomorphic to choosing the optimal average price. Lowering the average price by $\Delta$, will cause those customers whose average preference for the other brand is less than $\Delta$ to purchase from your store.

The equilibrium prices that would be charged if the stores insisted on bundling their products together would be:

$$
\begin{equation*}
P_{\bar{X}}=\frac{1}{2 f_{\bar{X}}^{(0)}} . \tag{5.27}
\end{equation*}
$$

The average preference however is distributed among the customers with a substantially lower variance, than the underlying individual product preferences. $f_{\bar{X}}(0)$ is therefore less than $f(0)$, and thus the
supportable effective prices are now substantially lower than what could be achieved by selling the products individually without any bundling or stamp program at all. For the example developed in this chapter, if the department stores insisted on participating in the trading stamp programs, the supportable prices would only be $\mathrm{P}_{\overline{\mathrm{X}}}=17.1$ cents compared with effective price $\$ 1.00$ per item when all items were sold separately.

The model therefore predicts why would not expect to see large department stores such as K-Mart, or Zayre offer any trading stamps or other nonlinear pricing schedules. It also explains the decline of the trading stamp programs in the U.S. trading stamp programs in the period beginning in 1973.

In 1973 the Federal Trade Commission ordered that all stamps must be redeemable for cash (\$2.- per book) as well as the merchandise that had previously been offered. This had the effect of making the redemption schedules more linear. More importantly however, 1973 was the year that the Arab oil embargo created gasoline shortages, regulated prices, and lines at the gas pumps. Almost immediately. the service stations who had accounted for approximately $28 \%$ of the stamps issued stopped participating. This left the grocery store and supermarkets issuing approximately $77 \%$ of the stamps. For the grocery store chains, who issued the majority of the stamps, the programs became ineffective for the reasons above.

### 5.7 Trading Stamps as Entry Barriers

Just as the airline frequent flyer programs provided a natural entry barrier in section 3.5 , trading stamps protect the existing merchants from new competitors emerging. This protection is however only against entry on the individual merchant level, and will not deter a department store from opening. This was true in 3.5 as well. There, no airline would attempt to serve an additional single route segment, yet there was no guarantee that large scale entry, with many additional routes, could be deterred.

The consumer's reaction to a new store not issuing stamps will depend upon the price differential between this new store and the store issuing stamps, as well as the total number of stamps that the consumer expects to collect from all other purchases. If this is the only store not participating in a stamp program, than the consumer will consider the value of receiving the trading stamp as the difference between $V(N)$ and $V(N-1)$, or just the $M V(N)$.

The new merchant will be able to sell his product if and only if his price, $P$, is lower than $\overline{\mathrm{P}}-\mathrm{MV}(\mathrm{N})$. where $\overline{\mathrm{P}}$ is the price charged by the incumbent merchant. Note that if $\widetilde{P}$ is less than $M V(N)$. the entrant will not be able to successfully enter. At that price however, the existing customers are earning positive profits, $\widetilde{P}$

AV. The firms only pay the average value of the stamps, which is less than the marginal value the consumer will place on it. For that reason, if a firm were to enter the market it would be at a severe competitive disadvantage, and would be driven out of the market, while the existing stores continue to earn money.

If more than one new store opens simultaneously (especially if the stores are coordinated, such as by having the same owner or being located at a new "shopping center") the consumer now may not use $\mathrm{MV}(\mathrm{N})$ as the value placed upon the stamp. An additional option now availilable to him is to buy from all of these independent stores. The value he places upon the stamp may, therefore, be as low as $\mathrm{MV}(\mathrm{N}-\mathrm{m})$, where m is the number of independent stores available.

### 5.9 Welfare Effects of Trading Stamp Programs

The social welfare implications are undeniably negative. Although the merchants profits have increased, this wealth was transferred dollar for dollar from the consumers. The consumers welfare is further diminished since.the new equilibrium is not even allocatively efficient. Before, in the standard pricing game, every customer received the goods they preferred. Now, with the trading stamps they only receive the "bundle" they prefer, although it may
contain individual items that they consider inferior. As mentioned earlier, salt is further rubbed into their wounds, since now they have to pay more for the goods too.

## Chapter 6. Conclusion

This paper has shown that when non-monopoly firms are given the freedom to offer products in bundles, either directly, or through a rebate scheme, additional Bertrand-Nash equilibria can appear. These new equilibria allow the firms to generate positive economic profits while they are able to retard "small scale" entry.

Two models were analyzed, a model of airline frequent flyer programs, and a model of the U.S. trading stamp industry. The key feature in each case is its ability to create incentives to purchase all items from the same source, or in the trading stamp model, from the same "team" of stores. The competition for the customers' business is now based upon the entire product line instead of at the individual product level.

In the frequent flyer model, the airlines have highly differentiated route structures, and are thus able to support high fares. The carriers providing service on an individual flight segment would be very susceptible to price wars if the competition were held at that level since the passengers as a rule do not have a strong preference for one carrier over artother.

The trading stamps force the consumer to pick between two "teams" of stores. Each individual store, by lowering or raising its individual price, can influence some customers to either buy or
abandon the team's bundle of products. Since the distribution of reservation price differences for the bundle has a higher variance than the individual product, the merchant's demand is less elastic than in an environment without trading stamps. In equilibrium therefore, all shop owners charge a higher price.

The model explains how the Arab oil embargo caused the 1973 trading stamp decline. For the model to hold, each of the small shop owners must ignore the externalities its prices have upon the other team's members. After the gas stations abandoned the program due to oil shortages, the supermarkets issued a vast majority of the stamps, and thus became very concerned about the externality it was imposing upon a team that had become essentially itself. The trading stamp programs could no longer keep the equilibrium grocery store prices high, and the grocers quickly abandoned the programs as well.

The welfare effect of the frequent flyer model presented is merely a shift in consumer surplus to producer surplus from what would be observed if no frequent flyer programs existed. For the trading stamps however, not only is there this same transfer of surplus, but the resulting equilibrium is allocatively inefficient. The consumers purchase their favorite bundles, but these may contain some items that the consumer finds inferior. Another consumer buying the other bundle may very well have preferred this item over the one she actually received. As mentioned earlier, to add insult to injury, the consumer
even had to pay more for this item than in the price equilibrium without trading stamps when each would have received the item of choice.

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## Vita

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[^0]:    One exception being Piedmont Airlines, a large regional airline operating approximately $80 \%$ of its route system in markets as a monopoly carrier.

[^1]:    * 

    It should be noted that in addition to this Nash equilibrium, the standard Bertrand equilibrium still remains. If everyone is selling their products at cost (zero), no firm can sell products at a higher price, since the entire product line can be purchased at no cost from all other suppliers.

