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Endogenous Information Acquisition: Essays in Applied Game Theory

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ABSTRACT

Endogenous Information Acquisition: Essays in Applied Game Theory

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This dissertation endogenizes information acquisition in two-player games across three different settings. The first chapter explores when moral hazard in a principal-agent contract can lead to pareto improvements when it is preceeded by information gathering. The second chapter studies how product differentiation affects the amount of market research done by firms that compete on price. The third chapter examines the role of costly risk in a production process when there is a debt contract between an investor and an entrepreneur.

In Chapter 1, I consider an agent who designs an experiment that reveals information about a state to a principal. The principal subsequently decides whether or not to implement a project. If she does, then she offers a limited-liability contract to motivate the agent to exert effort, which together with the state stochastically determines the project's output. If effort is contractible, the contract "holds up" the agent so that conditional on implementation his payoff is independent of the principal's beliefs. In equilibrium, he provides only enough information to maximize the probability that the principal implements the project. In contrast, if effort is non-contractible then the principal must promise the agent rent to motivate effort. Since the promised rent varies across beliefs, the agent may

provide more precise information. Thus, although the non-contractibility of effort lowers the principal's payoff at a given belief, it can improve welfare by mitigating hold-up and encouraging information provision.

In Chapter 2, I apply the main result in Persico (2000), that decision-makers acquire more information when their payoffs are more risk-sensitive, to a duopoly model of Bertrand competition with uncertain demand following Vives (1984) in order to show how the amount of private market research firms undertake depends on competition, measured as the level of product differentiation. I decompose the relative marginal return of research across competition levels into two effects, a competitive profit effect and a coordination effect, and show how each of these depends on competition. When the cost of market research is sufficiently high, the amount firms invest in market research is decreasing in the level of competition. In contrast, when the cost of market research is sufficiently low, firms perform the most market research at an intermediate level of competition. I partially extend this result to a public market research setting.

In Chapter 3, I extend a simple model of debt between a liquidity-constrained entrepreneur and an investor to allow one of the players, according to the governance structure, to choose either risky or safe production at time 1. Risky production causes capital to depreciate, lowering the value of collateral and production at time 2. When the entrepreneur is tempted to choose risky production in order to foreclose more often in the low state and less often in the high state, he must offer more collateral to the investor. In this way, his inability to commit to safe production can lower his expected value from the project in equilibrium. I provide necessary and sufficient conditions such that the entrepreneur strictly prefers for the investor to have governance over time 1 production in order to overcome this commitment problem.

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Table of Contents

ABSTRACT 3

Acknowledgments 5

List of Tables 8

List of Figures 9

Chapter 1. Persuasion, Hold-Up, and Incentive Contracts 10

1.1. Introduction 10

1.2. Model 17

1.3. Multiplicative-Quadratic Example 21

1.4. General Results 40

1.5. Binary Effort 50

1.6. Conclusion 56

Chapter 2. Market Research and Differentiated Bertrand Competition (joint with Rafayal Ahmed) 58

2.1. Introduction 58

2.2. Model 62

2.3. Returns to Market Research 64

2.4. Conclusion 73

Chapter 3. Governance, Depreciation, and Debt (Joint with Alexander Limonov) 75

3.1. Introduction 75

3.2. Model 81

3.3. Equilibrium and Optimal Governance 84

3.4. Conclusion 93

References 95

Appendix A. Omitted Proofs: Chapter 1 99

A.1. Proof of Theorem 3 99

A.2. Proof of Proposition 6 99

A.3. Proof of Proposition 7 100

Appendix B. Omitted Proofs: Chapter 2 101

B.1. Proof of Lemma 2 101

B.2. Derivation of Equation 2.1 102

B.3. Proof of Lemma 3 103

B.4. Proof of Lemma 4 104

Appendix C. Omitted Proof: Chapter 3 105

C.1. Proof of Proposition 10 105

List of Tables

1.1 Welfare Comparison 33

List of Figures

1.1	Receiver's Continuation Payoff	23
1.2	Sender's Continuation and Ex Ante Payoffs	24
1.3	Optimal Persuasion Characterization	27
1.4	Two Cases of Optimal Persuasion	28
1.5	Receiver Payoff Comparison: Cutoff vs. Full Revelation	31
1.6	Sender Payoff Comparisons	33
1.7	Decomposed Receiver Payoffs	39
1.8	Decomposed Sender Payoffs	39
1.9	Persuasion Beyond the Cutoff with Binary Effort	52
1.10	Persuasion with Early Effort	57
2.1	Competitive Profit Effect and Coordination Effect	69
2.2	Total Effect at Low Accuracy and High Accuracy	70

CHAPTER 1

Persuasion, Hold-Up, and Incentive Contracts

1.1. Introduction

Firms use information not only to determine whether or not to implement a project but also as the basis for the incentive contracts they offer. When deciding how much information to provide, an agent will anticipate both the firm's implementation decision and the value he can capture in the ensuing contract, which depends on the contracting environment. In particular, the existence of contracting frictions partially determines the agent's anticipated value from the contract and consequently affects how much information he initially provides.

Consider for example a consultant performing exploratory analysis to discover whether or not an opportunity is profitable for a prospective client. That analysis will be the basis of a contract with the client, including the work required of the consultant and his compensation. On one hand the consultant is incentivized to provide information to increase the likelihood that the project goes forward (perhaps so that he can establish a reputation or make beneficial contacts), but on the other hand he must consider how he will be compensated once his analysis has been shared. Providing too much information puts him at risk of revealing that the project is not profitable, preventing it from going forward. The client, for her part, prefers the analysis to be as informative as possible.

Similarly, consider a pharmaceutical firm deciding whether or not to buy a drug from a biotech company. The biotech can conduct initial research to test how effective the drug is. The results determine both whether the pharmaceutical firm makes the purchase as well as the nature of the ongoing interaction between the two parties. If the biotech is able to recoup some of the value of the information they provide then they will conduct more precise research. If instead the pharmaceutical can contract away all of the value of information then the biotech will conduct less precise research. Thus, while the pharmaceutical would like to capture as much value as possible, their ability to do so may reduce their profits if it leads to less precise initial information.

This paper studies a setting in which an agent engages in Bayesian persuasion[1] to convince a principal to implement a project, after which the principal offers the agent a spot contract to motivate him to exert effort. I compare information provision and payoffs across two contracting environments: the *benchmark setting* and the *moral hazard setting*. In the benchmark setting the principal can extract all the value of information, i.e. she holds up the agent. Before the information is shared, she cannot credibly promise to implement the project or to share the gains of information conditional on implementation. After the results, she offers a contract on effort and thereby retains all of the surplus created. Therefore, the agent's incentive to provide information is restricted to how it affects the implementation decision, and he may underprovide information. By contrast, in the moral hazard setting the principal must promise rent to motivate the agent because effort is non-contractible. Moral hazard functions as a commitment device that alleviates the hold-up problem. The contract is inefficient; at any given posterior belief, conditional on implementation both the principal's payoff and total surplus are lower relative to the benchmark setting. This results in a further cost: the principal implements the project more rarely. However, since the agent's rent from the contract may depend on beliefs, moral hazard potentially leads to

[1]See Kamenica & Gentzkow (2011).

more information provision. I provide conditions under which the existence of moral hazard leads to ex ante gains for both players.

Formally, I consider a game with two players: a sender (the agent, "he") and a receiver (the principal, "she"). The game has two distinct stages, a *persuasion stage* and a *contracting stage*. In the persuasion stage, the sender chooses an experiment, the design and results of which are verifiable, to test some underlying binary state. Based on the outcome, the receiver chooses whether or not to pursue the project at some fixed cost. In the subsequent contracting stage, the receiver offers a limited-liability contract to the sender. In the benchmark setting, the contract is a schedule of payments conditional on both effort and output. In the moral hazard setting it is a schedule of payments conditional on output only. If the sender accepts the contract, he chooses effort that, jointly with the state, stochastically determines the output of the project. The receiver gets the value of the output net of the implementation cost and any payment to the sender. The sender gets not only any payment from the receiver net of his cost of effort, but also a direct benefit if the project is implemented.[2]

I find the unique equilibrium in the benchmark contracting setting and show that while the contract is efficient at any given belief, the hold-up problem leads to the underprovision of information. In the benchmark setting the receiver captures all the value from the project conditional on implementation because she can condition transfers perfectly on effort. Consequently, the sender earns only his direct benefit from implementation.[3] Since the sender's payoff is solely determined by whether or not the receiver implements the project, he will design an experiment to maximize the probability that she does so. I show that the

[2]This can be interpreted as reputational concerns, ego, state-independent monetary compensation, etc.
[3]The receiver is unable to contract this away from him due to limited liability.

receiver's payoff is increasing in her beliefs, so that her implementation rule is a cutoff. The sender maximizes the probability of implementation in equilibrium. That is, he chooses an experiment that maximizes the probability that posterior beliefs equal the implementation cutoff, in which case the receiver gets no value from the project on average.[4] The sender will not risk providing more precise information since that would decrease the probability of implementation.

I partially characterize the sender's equilibrium persuasion in the moral hazard setting. The main focus of this paper is the dual effects of moral hazard, namely the effect on contracting payoffs and the effect on information provision. Moral hazard leads to inefficient spot contracts and reduces the probability of implementation. However, it also forces the receiver to share some of the surplus with the sender. Most importantly, because the receiver can only condition payments on output and not on effort directly, the amount of rent the receiver promises the sender varies in beliefs. Therefore, the sender faces a tradeoff when choosing an experiment. If he chooses a more informative experiment, the probability that the receiver implements the project decreases, but conditional on implementation the rent the sender gets from the contract may increase. I show that, as in the benchmark setting, the hold-up problem sometimes leads to the underprovision of information in the moral hazard setting. However, I find conditions such that the sender does not underprovide information, i.e. such that moral hazard mitigates the hold-up problem. I also show that in some settings moral hazard mitigates the hold-up problem only when the receiver's fixed cost of implementation is high. This implies that a firm would sometimes prefer both to be

[4]See the leading example in Kamenica & Gentzkow (2011) for a detailed discussion of this particular persuasion behavior.

unable to write perfect contracts and to face higher fixed costs, rather than have lower fixed costs or be able to write complete contracts.

I compare the players' payoffs across contracting settings. Since moral hazard leads to more information provision at the persuasion stage but lower total surplus in the contracting stage after any given experiment, either player may be better or worse off from moral hazard relative to the benchmark setting. Both players are subject to an information effect (the change in the experiment) and a contracting effect (the change in expected payoff from the contract). From the sender's perspective, the information effect is negative because the probability of implementation has decreased, but the contracting effect is positive because the receiver gives him positive rent. From the receiver's perspective, the information effect is positive because more information allows her to better tailor her implementation decision and the contract she offers to the state, but the contracting effect is negative both because the contract is inefficient and because she does not capture all of the surplus from the contract. For both players, the relative magnitude of these effects depends on the prior belief on the state. I show that the sender is better off from moral hazard when the prior is high, where the contracting effect dominates the information effect, while the receiver is better off from moral hazard when the prior is low, where the information effect dominates the contracting effect. I show that these regions sometimes overlap, such that at some priors moral hazard leads to a strict pareto improvement ex ante in spite of interim inefficiencies in the spot contract.

The rest of the paper is organized as follows. Section 1.2 gives the timing of the model and the equilibrium concept. Section 1.3 restricts the model to a particular class of multiplicative-quadratic functional forms. This class has convenient features that allow me to find equilibrium in closed form and make detailed payoff comparisons across contracting settings. I give comparative statics on the parameters and discuss the intuition behind the tradeoffs of moral hazard. Section 1.4 extends the results to a general model in which the probability of high output exhibits increasing differences in effort and the state, and the cost of effort is convex. Section 1.5 contains an extension in which I restrict effort to be binary. This allows me to closely examine the role of the production function in determining whether or not moral hazard mitigates the hold-up problem. While in the general continuous effort model I focus on the role of the parameters when the production function exhibits increasing differences, in the binary model I show that moral hazard mitigates the hold-up problem when production satisfies either (sufficiently) increasing differences or decreasing differences.[5] I discuss the different mechanisms by which these two types of production function affect the sender's tradeoff in information provision. Section 1.6 concludes.

To my knowledge, this is the first model to ground a sender's payoff in a persuasion context as the result of a limited liability contract, as well as to compare that payoff across different contracting environments. There are, however, several papers that take up some combination of these elements. Hörner & Skrzypacz (2016) study the problem of information design to induce a receiver to move forward with a project. Their model focuses on multiple stages of information transmission and a sender who knows the underlying state and gets flow payments, rather than benefitting from the project moving forward *per se*. Similar to my model, Boleslavsky & Kim (2017) consider the interrelation between persuasion and moral hazard, but in their model information design itself is used both to motivate an agent's hidden effort and to persuade a principal to take a preferred action. Effort takes place before, and partially determines, signal realizations.

[5] "Sufficiently" increasing differences refers in part to the rate of increasing differences relative to the cost of effort.

This work is related more broadly to the burgeoning literature on Bayesian persuasion, largely incited by the seminal papers Kamenica & Gentzkow (2011), hereafter "KG", and Rayo & Segal (2010) using mathematical tools developed extensively in Aumann & Maschler (1995). Many papers extend or amend the theoretical results of KG to other environments, for instance by including multiple receivers [Wang (2013)] or multiple senders [Gentzkow & Kamenica (2016, 2017)], considering heterogeneous priors [Alonso & Câmara (2016)], allowing the receiver to have private information [Kolotilin et al. (2017)], or considering a dynamic environment [Ely (2017)]. This literature often focuses on the effectiveness of persuasion from the sender's perspective, whereas I am interested in the determinants of the specific shape of the sender's persuasion policy and how they affect both the sender's and the receiver's payoff.

Moral hazard in the presence of limited liability has been studied extensively since Innes (1990). Pitchford (1998), which considers the effects of bargaining power in such an environment, provides several examples of production and cost functions and the resulting shape of the agent's payoffs. Poblete & Spulber (2012) characterize optimal contracts in terms of a simple condition when both the agent and the principal are risk-neutral. Their results are extended to include costly, private information acquisition in Su (2016) . There, a principal sometimes offers a contract that deters information acquisition while simultaneously motivating hidden state-independent effort, thereby implementing moral hazard. My work focuses in particular on how the sender's value in the receiver's (principal's) optimal moral hazard contract changes in the underlying distribution on the state, which is not something about which these papers are primarily concerned. It is important here because it determines the sender's optimal persuasion policy.

This work shares several similarities with the literature on hold-up, such as Gul (2001) and Che & Sákovics (2004). I focus in particular on a hold-up problem in information provision, rather than in investment or effort. As pointed out in Arrow (1962), there is a sense in which hold-up is intrinsic to selling information in that the value of information is unknown until that information is revealed. This is an idea taken up in the literature on intellectual property rights. Anton & Yao (2002) and Aghion & Tirole (1994) explore this friction under different contracting assumptions. Like my model, their models include both information transmission and effort, but in very different frameworks.

At its core this paper is an example of the theory of second best as detailed in Lipsey & Lancaster (1956): while one source of contracting incompleteness can reduce payoffs and lead to inefficiencies, an additional source of contracting incompleteness may alleviate, rather than exacerbate, this loss.

1.2. Model

This section describes the model. Subsection 1.2.1 contains timing and payoffs. Subection 1.2.2 gives the equilibrium concept and key definitions.

1.2.1. Timing and Payoffs

Consider a game between a receiver ("she") and a sender ("he"). The sender designs a signal structure mapping from a binary state to signals. This is a Bayesian persuasion game as in KG, meaning the sender publically commits to a signal structure before the state is realized. The receiver observes both this signal structure and its realization.

The signal is commonly observed, after which the receiver decides whether or not to implement a project. If she chooses not to implement the project, the game ends. If she chooses to implement the project, she pays a fixed cost β and the sender gets a strictly positive fixed benefit α that need not equal β. The receiver makes a take-it-or-leave-it offer of an incentive contract to the sender. The sender chooses (costly) effort, which together with the state determines the distribution over output.

Formally, the timing of the model is as follows:

(1) The sender chooses a signal structure $S(\cdot|\theta)$, which is a conditional distribution on signals given θ. The receiver observes the signal structure.

(2) Nature chooses $\theta \in \{0, 1\}$ according to the common prior $p_0 \in (0, 1)$ on $\theta = 1$, i.e. $\mathbb{E}(\theta) = p_0$.

(3) Signal s is realized according to $S(\cdot|\theta)$. Both the sender and receiver observe s.

(4) The receiver chooses $d \in \{0, 1\}$. If $d = 0$, the game ends and the sender and receiver get payoff 0. If $d = 1$, the game continues.

(5) The receiver offers the sender a contract specifying $t(e, y)$, with $t(\cdot, \cdot) \geq 0$, which the sender accepts or rejects. If the sender rejects, the game ends. If the sender accepts, the game continues.

(6) The sender chooses effort $e \in [0, \infty)$.

(7) Output $y \in \{0, 1\}$ is realized. $P(y = 1|e, \theta) = f(e, \theta)$ for some function $f(\cdot, \cdot) \in [0, 1]$.

If $d = 1$, the receiver's payoff $v(e, y)$ and the sender's utility $u(e, y)$ are as follows:[6]

[6] If $d = 0$ then $v(e, y) = u(e, y) = 0$.

$$v(e, y) = y - \beta - t(e, y).$$

$$u(e, y) = \alpha - c(e) + t(e, y).$$

The receiver captures the value of the output, but must pay a fixed cost β along with any transfer promised to the sender. The sender gets a fixed benefit α from the project[7] along with any promised transfer, but incurs a cost of effort $c(e)$.

I consider two contracting settings. In the benchmark setting the receiver can condition transfers on both effort and output. For ease of notation, I restrict attention to transfers that depend only on effort and not on output, i.e. $t(\cdot, 1) = t(\cdot, 0)$, which is without loss of generality for my results. I suppress the second argument in that setting and write $t(e)$. In the moral hazard setting the receiver is restricted to transfers that depend only on output, i.e. $t(e, 0) = t(e', 0)$ and $t(e, 1) = t(e', 1)\ \forall e, e'$. In that setting I suppress the first argument and write $t(y)$.

1.2.2. Equilibrium Concept

The solution concept in this model is Perfect Bayesian Equilibrium (PBE). In some cases there may be multiple equilibria. I restrict attention to the class of PBEs that maximize the receiver's ex ante payoff. In the benchmark setting I write this PBE*, which at a given prior is a tuple:

[7]For simplicity I treat α as exogenous and independent of β, but one can think of α as the purchase price of the project and β as this price plus any additional up-front investments the receiver must make. This interpretation fits the model if the sender cannot recoup any value of the project if the receiver does not make a purchase offer, but conditional on the offer can shop the project around and obtain a price α for it, in other words if his outside option depends on the receiver's continuation decision but not directly on beliefs.

$$\{L^*, H^*, d^*(p), t^*(e|p), e^*(p, t(e|p))\}$$

.

I write p to denote the (common) posterior belief on $\theta = 1$ after the signal is realized. Effort $e^*(p, t(e|p))$ is the sender's optimal effort at posterior p for a given transfer rule $t(e|p)$. The optimal transfer rule $t^*(e|p)$ takes this effort choice into account and maximizes the receiver's expected payoff at posterior p. The equilibrium continuation rule is such that $d^*(p) = 1$ whenever the receiver's expected value at a given posterior is positive given continuation behavior $e^*(p, t(e|p))$ and $t^*(e|p)$. The sender's signal structure for some prior p_0 maximizes the sender's ex-ante expected utility. As shown by KG, I can model optimal signal structures as a choice over at most two posteriors without loss of generality in this setting. The posteriors L^* and H^* are the two (possibly identical) optimal posteriors chosen by the sender in the persuasion stage.

I write PBE_N^* to denote an equilibrium that maximizes the receiver's ex ante payoff in the moral hazard setting. It is defined analogously to the benchmark setting:

$$\{L_N^*, H_N^*, d_N^*(p), t_N^*(y|p), e_N^*(p, t(y|p))\}.$$

In both contracting settings, I assume for simplicity that the receiver breaks ties in favor of implentation, and the sender breaks ties in favor of accepting a contract. This is without loss of generality in my setting.

Some results include equilibrium uniqueness. I define equilibrium to be *unique* if all equilibria in a particular setting are identical on-path.

It is useful for the analysis to distinguish player's payoffs at different stages for a given set of strategies. I write the receiver's *contracting payoff* as $\Pi(p) \equiv \mathbb{E}_\theta[v(e,y) + \beta | p]$ and the sender's *contracting rent* as $R(p) \equiv \mathbb{E}_\theta[u(e,y) - \alpha | p]$. These are the players' payoffs from a contract alone, assuming the receiver implements at some posterior p and suppressing direct cost (to the receiver) and benefit (to the sender) from implementation. I write the receiver's *continuation payoff* as $V(p) \equiv d(p)(\mathbb{E}_\theta[v(e,y)|p])$ and the sender's *continuation payoff* as $U(p) \equiv d(p)(\mathbb{E}_\theta[u(e,y)|p])$. These are the players' expected payoffs at any given posterior, taking into account the implementation rule in addition to effort and transfers. Finally I write the receiver's and sender's *ex ante payoffs* as $\bar{V}(p_0) \equiv \mathbb{E}_{S(\cdot|\theta)}[d(p(s))(\mathbb{E}_\theta[v(e,y)|p(s)])|p_0]$ and $\bar{U}(p_0) \equiv \mathbb{E}_{S(\cdot|\theta)}[d(p(s))(\mathbb{E}_\theta[u(e,y)|p])|p_0]$, respectively.

To distinguish equilibrium payoffs at various stages in an equilibrium PBE* I use superscript "*", for example $R^*(p)$. In an equilibrium PBE$^*_N$ I also use a subsript "$_N$", for example $\bar{V}^*_N(p_0)$. In any PBE*, $\bar{U}^*(p_0)$ is the concavication of $U^*(p)$, and in any PBE$^*_N$, $\bar{U}^*_N(p_0)$ is the concavication of $U^*_N(p)$.

1.3. Multiplicative-Quadratic Example

In this section I restrict attention to a particular example, the *multiplicative-quadratic* or "m-q" game. This example has useful properties that make the analysis straightforward and clearly illustrates the effects of moral hazard and the role of the parameters. The intuition for the results in this section extends to the more general setting analyzed in Section 1.4.

Definition 1. *A Multiplicative-Quadratic game is such that $f(e,\theta) = e\theta$ and $c(e) = \frac{k}{2}e^2$, for some $k > 1$ s.t. $2\sqrt{k\beta} \in (0,1)$.*[8]

[8] The parametric assumptions guarantee an interior solution in effort and an interior implementation cutoff in both the benchmark setting and the moral hazard setting.

In Subection 1.3.1 I find the unique PBE* in the benchmark setting. In Subsection 1.3.2 I find the unique PBE$^*_N$ in the moral hazard setting and characterize when the sender provides full information in equilibrium. Subsection 1.3.3 compares payoffs between the two settings. I first restrict attention to a particular (low) region of priors. In that region I characterize when moral hazard leads to a strict pareto improvement and provide comparative statics on the parameters. I then discuss the role of the prior. I show that the receiver is better off in the moral hazard setting when the prior is low, while the sender is better off in the moral hazard setting when the prior is high.

1.3.1. Benchmark Equilibrium

I find the unique PBE* in the benchmark setting using backward induction. I show that while the spot contract is efficient, ex ante payoffs are less than first best. From the receiver's perspective, the sender underprovides information.

Suppose at some posterior p the recevier has implemented the project. Her optimal incentive contract in the benchmark setting solves the following maximization problem.

$$
\begin{aligned}
\Pi^*(p) = & \max_{e,t(\cdot)} \quad ep - t(e) - \beta & \\
& \text{s.t.} \quad t(\cdot) \geq 0 & \text{LL} \\
& \qquad t(e) - \frac{k}{2}e^2 \geq 0 & \text{IR} \\
& \qquad t(e) - \frac{k}{2}e^2 \geq t(e') - \frac{k}{2}e'^2 \ \forall e' & \text{IC}
\end{aligned}
$$

To implement some e, the principal can set $t(e') = 0$ for $e' \neq e$. Then IR will bind without violating IC by setting $t(e) = \frac{k}{2}e^2$. Therefore, her maximization problem simplifies to the following.[9]

$$\max_e \quad ep - \frac{k}{2}e^2$$

By standard first order approach the solution is first best effort $e^*(p) = \frac{p}{k}$ and transfer $t^*(e^*(p)) = \frac{p^2}{2k}$. The receiver's contracting payoff at a given posterior is $\Pi^*(p) = \frac{p^2}{2k}$.

The receiver implements the project whenever her contracting payoff covers her fixed cost of implementation. Since her contracting payoff is strictly increasing in the posterior, her optimal implementation policy is a cutoff, which I call $\hat{p}$. In equilibrium, it must be that $d^*(p) = 1$ *iff* $p \geq \hat{p}$. In the m-q game, $\hat{p} = \sqrt{2k\beta}$. This is the point at which $\Pi^*(p) = \beta$. Figure 1.1 depicts the receiver's continuation payoff in the posterior.

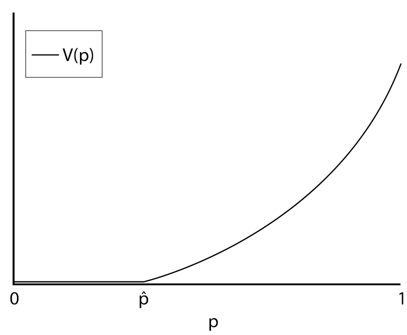

Figure 1.1. Receiver's Continuation Payoff

Given this implementation policy, the sender's continuation utility is $U^*(p) = \alpha$ at $p \geq \hat{p}$ and $U^*(p) = 0$ otherwise. This is depicted in the left-hand graph in Figure 1.2.[10] His ex ante expected utility given optimal persuasion is the concavication of his continuation utility,

[9]Note that this is the same as maximizing total surplus conditional on implementation.

[10]The sender's persuasion problem is essentially identical to the motivating example in KG precisely because the sender gets no rent from the incentive contract beyond his implementation benefit α.

shown in the right-hand graph of Figure 1.2. At low priors $p_0 < \hat{p}$, i.e. priors such that the receiver would not continue without additional information, he achieves this by designing a signal structure leading to posteriors $L^* = 0$ and $H^* = \sqrt{2k\beta}$. This is his persuasion strategy in the unique PBE at low priors. At higher priors, any signal structure that never leads to a posterior $p < \hat{p}$ is optimal; thus, there are multiple PBEs. Since the receiver's continuation payoff in that region is convex, the sender's persuasion strategy in the unique PBE* at priors $p_0 > \hat{p}$ is the most informative one: $L^* = \sqrt{2k\beta}$ and $H^* = 1$.[11]

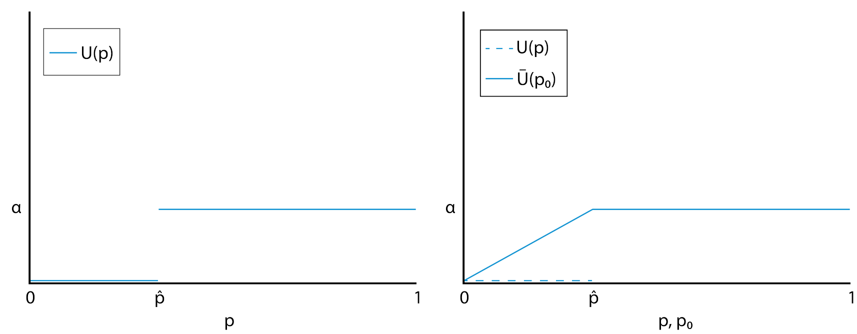

Figure 1.2. Sender's Continuation and Ex Ante Payoffs

Proposition 1 fully characterizes the unique PBE* at every prior in the benchmark setting of the m-q game. I defer discussion of equilibrium payoffs to Subsection 1.3.3

Proposition 1. *In the m-q game, at priors (i)* $p_0 \in (0, \sqrt{2k\beta})$*, (ii)* $p_0 = \sqrt{2k\beta}$*, (iii)* $p_0 \in (\sqrt{2k\beta}, 1)$ *there exists a unique PBE* with strategies:*

(1) *(i)* $L^* = 0,\ H^* = \sqrt{2k\beta}$*, (ii)* $L^* = H^* = \sqrt{2k\beta}$*, (iii)* $L^* = \sqrt{2k\beta},\ H^* = 1$

(2) $d^*(p) = 1$ *iff* $p \geq \sqrt{2k\beta}$

(3) $t^*(e^*(p)|p) = \frac{p^2}{2k}$ *and* $t^*(e|p) = 0\ \forall e \neq e^*(p)$

(4) $e^*(p) = \frac{p}{k}$.

[11]At $p_0 = \hat{p}$, the unique optimal persuasion strategy is an uninformative signal.

Even though effort is first best, this equilibrium is inefficient. The receiver does not implement at some posteriors where implementation maximizes total surplus. The sender does not provide full information, but instead persuades in such a way to maximize the probability that the posterior belief is equal to the implementation cutoff. [12]

1.3.2. Moral Hazard Equilibrium

I find the unique equilibrium in the moral hazard setting in the same manner as in the benchmark setting. Effort is inefficient at the contracting stage, and the receiver's implementation cutoff is consequently higher than in the benchmark setting. The key difference between equilibrium in the moral hazard setting and equilibrium in the benchmark setting is that in the moral hazard setting the sender's continuation payoff depends on beliefs to the extent that he sometimes reveals more information than would maximize the probability of implementation. The sender's contracting rent is convex in the posterior, so his persuasion strategy is either to maximize the probability of implementation (as in the benchmark setting) or to fully reveal the state. I characterize which of these strategies obtains in equilibrium.

The receiver's maximization problem at posterior p is as follows.

$$
\begin{aligned}
\Pi_N^*(p) = \max_{e,t(y)} \quad & ep(1-t(1)) - (1-ep)t(0) \\
\text{s.t.} \quad & t(\cdot) \geq 0 && \text{LL} \\
& ept(1) + (1-ep)t(0) - \frac{k}{2}e^2 \geq 0 && \text{IR} \\
& e \in \operatorname*{argmax}_{e'} \{e'pt(1) + (1-e'p)t(0) - \frac{k}{2}e'^2\} && \text{IC}
\end{aligned}
$$

[12]Without limited liability, full revelation is first best. With limited liability, full revelation is first best if α is sufficiently low, while if α is sufficiently high the sender's equilibrium persuasion strategy is first best given the receiver's equilibrium contracting and implementation behavior.

The sender's IC constraint pins down $t(1)$ as a function of $t(0)$ for any given e: $t(1) = t(0) + \frac{ke}{p}$. Since $t(1)$ is increasing in $t(0)$ the best the receiver can do without violating LL is set $t(0) = 0$ and $t(1) = \frac{ke}{p}$. Her simplified maximization problem is:

$$\max_{e} \quad ep(1 - \frac{ke}{p})$$

The unique solution is $e_N^*(p) = \frac{p}{2k}$, $t_N^*(0) = 0$, $t_N^*(1) = \frac{1}{2}$.[13] Effort is less than first best (by exactly half).

The receiver's contracting payoff is $\Pi_N^*(p) = \frac{p^2}{4k}$, which is lower pointwise in the posterior relative to the benchmark setting both because total surplus has decreased and because she is forced to give some of the surplus as rent to the sender to motivate effort. As in the benchmark setting, the optimal implementation policy in the moral hazard setting is a cutoff: $\hat{p}_N = 2\sqrt{k\beta} = \sqrt{2}\hat{p}$.

Unlike in the effort-contract case, here the sender's continuation utility is no longer constant in the posterior conditional on implementation. Instead, it is strictly increasing and convex.

$$U_N^*(p) = \begin{cases} 0, & \text{if } p < \hat{p}_N \\ \alpha + \frac{p^2}{8k}, & \text{otherwise} \end{cases}$$

As in the benchmark setting, the sender's ex ante utility given optimal persuasion is the concavication of his continuation utility. Because there is a discrete jump in his continuation utility at the implementation cutoff, after which it is increasing and strictly convex, one of two persuasion strategies is optimal. The first candidate is *cutoff persuasion*, in which

[13]The fact that the transfer in the high state does not depend on the posterior is a feature of the particular multiplicative-quadratic game that does not hold in general settings.

depending on the prior either $L_N^* = \hat{p}_N$ or $H_N^* = \hat{p}_N$.[14] This maximizes the probability of implementation, much like his equilibrium persuasion strategy in the benchmark setting. The second candidate is *full revelation* in which the signal perfectly reveals the state. In that case he trades off some probability of implementation for higher rent conditional on implementation. Figure 1.3 characterizes which persuasion strategy is optimal. The solid green line is the sender's continuation payoff assuming the receiver always implements the project. The dashed black line connecting the origin to his continuation payoff at posterior $p = 1$ is his ex ante payoff if he fully reveals the state. I call the point at which these intersect $\bar{p}$. If $\hat{p}_N > \bar{p}$ then full revelation is optimal. If $\hat{p}_N < \bar{p}$ then cutoff persuasion is optimal.[15]

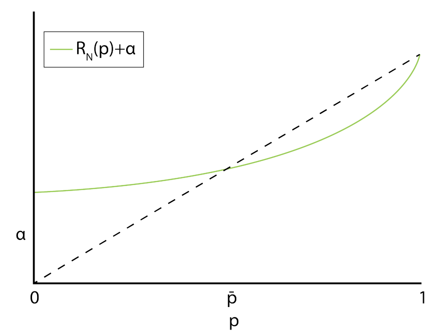

Figure 1.3. Optimal Persuasion Characterization

Figure 1.4 shows an example in which full revelation is optimal in the left-hand graph and an example in which cutoff persuasion in the right-hand graph. As in Figure 1.3, the dashed black line in the right-hand graph of Figure 1.4 represents the sender's ex ante utility if he (suboptimally) fully reveals the state.

[14]At priors $p_0 < \hat{p}_N$, $L_N^* = 0$ and $H_N^* = \hat{p}_N$, while at priors $p_0 > \hat{p}_N$, $L_N^* = \hat{p}_N$ and $H_N^* = 1$.
[15]If $\bar{p} = \hat{p}_N$ then both persuasion strategies are optimal, but full revelation is the persuasion strategy in the unique PBE_N^*.

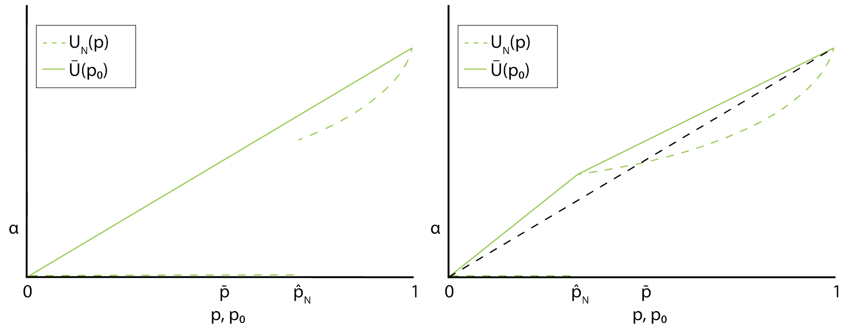

Figure 1.4. Two Cases of Optimal Persuasion

All that remains to characterize the unique PBE_N^* is to find $\bar{p}$, which solves the following equation:

$$R(p) + \alpha = p(R(1) + \alpha) \iff \alpha + \frac{p^2}{8k} = p(\alpha + \frac{1}{8k})$$

This has two solutions in p: $p = 1$ (by construction) and $p = 8k\alpha$. Thus $\bar{p} = 8k\alpha$, which is interior by the parametric assumptions in the definition of the m-q game. Full revelation is optimal whenever $\bar{p} \leq \hat{p}_N$, which is equivalent to the condition below. Cutoff persuasion is optimal otherwise.

(1.1) $$\alpha \leq \sqrt{\frac{\beta}{16k}}$$

Proposition 2 fully characterizes the unique PBE_N^* at every prior in the moral hazard setting of the m-q game. As in the benchmark setting, I defer discussion of equilibrium payoffs to Subsection 1.3.3.

Proposition 2. *In the unique PBE_N^* of the m-q game, if $\alpha \leq \sqrt{\frac{\beta}{16k}}$ then at priors $p_0 \in (0,1)$ the equilibrium persuasion strategy is $L_N^* = 0$, $H_N^* = 1$. If $\alpha > \sqrt{\frac{\beta}{16k}}$ then at priors (i) $p_0 \in (0, 2\sqrt{k\beta})$, (ii) $p_0 = 2\sqrt{k\beta}$, (iii) $p_0 \in (2\sqrt{k\beta}, 1)$ the equilibrium persuasion* *strategy is (i)* $L_N^* = 0$*,* $H_N^* = 2\sqrt{k\beta}$*, (ii)* $L_N^* = H_N^* = 2\sqrt{k\beta}$*, (iii)* $L_N^* = 2\sqrt{k\beta}$*,* $H_N^* = 1$*. In either case, equilibrium implementation, transfers, and effort are as follows:*

(1) $d_N^*(p) = 1$ *iff* $p \geq 2\sqrt{k\beta}$

(2) $t_N^*(1|p) = \frac{1}{2}$ *and* $t_N^*(0|p) = 0$

(3) $e_N^*(p) = \frac{p}{2k}$.

Even if the sender provides full information, this equilibrium is always inefficient, for somewhat different reasons than in the benchmark setting. In the moral hazard setting, effort is less than first best conditional on implementation. As in the benchmark setting, the receiver does not implement at some posteriors where implementation maximizes total surplus conditional on equilibrium contracting behavior. The sender's persuasion strategy is efficient when he fully reveals.[16]

1.3.3. Welfare Effects of Moral Hazard

This section compares both players' ex ante equilibrium payoffs across contracting settings and discusses how the comparison extends to more general settings. I first focus on equilibrium payoffs when the prior is equal to the benchmark setting implementation cutoff: $p_0 = \hat{p}$. All of the results apply to all lower priors $p_0 \in (0, \hat{p}]$. Moral hazard sometimes benefits only one player, sometimes both players, and sometimes neither player. I characterize each case. Next, I discuss the role of the parameters α, β, and k in the characterization. These parameters affect the relative continuation payoffs and implementation rules in both settings, which in turn affect equilibrium persuasion and relative ex ante payoffs. I give comparative statics in each parameter. Finally, I examine the role of the prior itself and show that the

[16]If he chooses cutoff persuasion, that maximizes total surplus given the receiver's equilibrium behavior only if α is sufficiently high (as in the benchmark setting).

receiver is better off in the moral hazard setting when the prior is low, while the sender is better off in the moral hazard setting when the prior is high.

I discuss first the receiver's and then the sender's ex ante payoffs in each setting when $p_0 = \hat{p}$. As this is the implementation cutoff, by definition the receiver's continuation payoff in the benchmark setting is $V^*(\hat{p}) = 0$. As shown in Proposition 1, in the unique PBE* the sender guarantees implementation by providing no information. The receiver's ex ante equilibrium payoff in the benchmark setting is $\bar{V}^*(\hat{p}) = V^*(\hat{p}) = 0$.

In the moral hazard setting, the receiver's ex ante equilibrium payoff depends on whether the sender engages in cutoff persuasion or fully reveals the state. In the first case $\bar{V}^*_N(\hat{p}) = \bar{V}^*(\hat{p}) = 0$. If instead the sender fully reveals the state, then $\bar{V}^*_N(\hat{p}) = \hat{p}V^*_N(1) > 0$.

Thus, at the prior $p_0 = \hat{p}$, the receiver is strictly better off in the moral hazard setting than in the benchmark setting exactly when the sender fully reveals. She is otherwise indifferent between the two settings. Define $X \equiv \sqrt{\frac{\beta}{16k}}$. From Equation 1.1, the sender fully reveals, and thus the receiver is strictly better off in the moral hazard setting than in the benchmark setting, whenever $\alpha \leq X$. This is further illustrated in Figure 1.5, which shows the receiver's equilibrium payoffs in both the benchmark and moral hazard equilibrium. In the left-hand graph the sender's equilibrium persuasion strategy in the moral hazard setting is cutoff persuasion, while in the right-hand graph he fully reveals in the moral hazard setting. It is clear from the figure that at priors $p_0 \in (0, \hat{p}]$ the receiver is strictly better off in a PBE$^*_N$ than in a PBE* if the sender fully reveals in the moral hazard setting, but that she is indifferent across settings if the sender chooses cutoff persuasion in the moral hazard setting.

Now consider the sender's equilibrium payoffs in each contracting setting at the same prior $p_0 = \hat{p}$. In the benchmark setting, he guarantees implementation but does not get any rent, so $\bar{U}^*(\hat{p}) = \alpha$. We can compare this to his expected payoff in the moral hazard

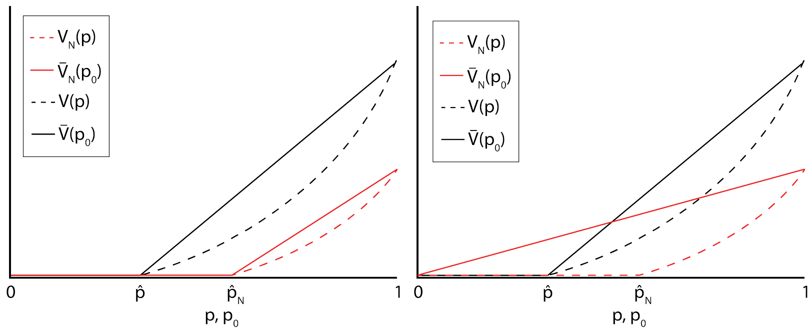

Figure 1.5. Receiver Payoff Comparison: Cutoff vs. Full Revelation

setting by examining his payoff when he chooses cutoff persuasion and his payoff when he fully reveals the state. Whichever type of persuasion the sender chooses in the moral hazard setting, his continuation payoff conditional on implementation increases relative to the benchmark setting, but the probability of implementation decreases. Proposition 3 characterizes when the sender's equilibrium payoff is higher in the benchmark setting than his equilibrium payoff in the moral hazard setting, which is the maximum of his payoff in the moral hazard setting from cutoff persuasion and from full revelation. The expression Y captures this comparison in the full revelation case, and the expression Z captures the comparison in the cutoff persuasion case.

Proposition 3. *In the multiplicative-quadratic game, at* $p_0 = \sqrt{2k\beta}$, $\bar{U}^*(p_0) > \bar{U}_N^*(p_0)$ *iff* $\alpha > \max\{Y, Z\}$.

Where $Y \equiv \frac{\beta}{2(\sqrt{2}-1)}$ and $Z \equiv \frac{\sqrt{2k\beta}}{8k(1-\sqrt{2k\beta})}$.

Proof. Fix $p_0 = \sqrt{2k\beta}$. Then $\bar{U}^*(p_0) = \alpha$. At any p such that the receiver implements the project, $U_N^*(p) = \alpha + \frac{p^2}{8k}$.

Case 1: $X \geq \alpha$. Then $\bar{U}_N^*(p_0) = \sqrt{2k\beta}(\alpha + \frac{1}{8k})$, and

$$\bar{U}^*(p_0) > \bar{U}^*_N(p_0) \iff \alpha > Y.$$

Case 2: $X < \alpha$. Then $\bar{U}^*_N(p_0) = \frac{1}{\sqrt{2}}(\alpha + \frac{\beta}{2})$, and

$$\bar{U}^*(p_0) > \bar{U}^*_N(p_0) \iff \alpha > Z.$$

Finally, $X \geq \alpha \iff Y \geq Z$. □

Figure 1.6 shows examples of the sender's relative equilibrium payoffs in both contracting settings, analogously to Figure 1.5 for the receiver.[17] Restricting attention to priors $p_0 \in (0, \hat{p}]$, in the top left graph the sender fully reveals in the moral hazard setting and is better off in the benchmark setting. In the top right graph the sender persuades to the cutoff in the moral hazard setting and is better off in the benchmark setting. In the bottom left graph, the sender fully reveals and is better off in the moral hazard setting, while in the bottom right graph he persuades to the cutoff and is better off in the moral hazard setting.

Table 1.1 summarizes both players' relative payoffs across contracting settings at prior $p_0 = \hat{p}$. I suppress the argument (p_0) in the payoff functions for neatness.[18] The four cases correspond to the four graphs in Figure 1.6, left to right and top to bottom. In Case 3, moral hazard leads to a strict pareto improvement.

I now discuss the channels through which each of the parameters α, β, and k determines whether or not moral hazard increases equilibrium payoffs, namely (1) their effects on the continuation payoffs conditional on implementation, in both settings, and (2) their effects on the implementation rules in both settings. These two interim effects result in (3) changes in

[17]Since the sender may be better or worse off in either setting whether he fully reveals or persuades to the cutoff, there are four cases to consider rather than two.

[18]I ignore corner cases in which any of the conditions are satisfied with equality. Those cases are straightforward.

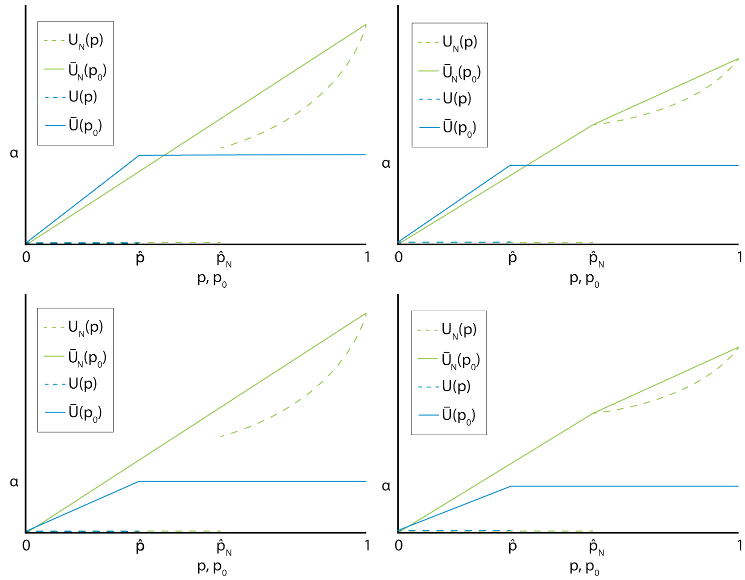

Figure 1.6. Sender Payoff Comparisons

Case	Conditions	Persuasion	Receiver	Sender
1	$Y < \alpha < X$	full	$\bar{V}_N^* > \bar{V}^*$	$\bar{U}_N^* < \bar{U}^*$
2	$\max\{Z, X\} < \alpha$	cutoff	$\bar{V}_N^* = \bar{V}^*$	$\bar{U}_N^* < \bar{U}^*$
3	$\alpha <\min\{X, Y\}$	full	$\bar{V}_N^* > \bar{V}^*$	$\bar{U}_N^* > \bar{U}^*$
4	$X < \alpha < Z$	cutoff	$\bar{V}_N^* = \bar{V}^*$	$\bar{U}_N^* > \bar{U}^*$

Table 1.1. Welfare Comparison

equilibrium persuasion.[19] The discussion of these channels extends across all priors as well as to more general settings, but for the purpose of the results I continue to restrict attention to low priors $p_0 \in (0, \hat{p}]$. I consider each of the parameters in turn.

The sender's direct implementation benefit α has the most straightforward effect. As α decreases, both the receiver and the sender are better off in the moral hazard setting relative to the benchmark setting. A change in α does not change either the implementation rule or

[19]Algebraically, the comparative statics for each parameter follow immediately from X, Y, and Z, but I focus on the effects of each parameter on the different channels to get the general intuition across.

the receiver's payoff conditional on implementation in either setting, so the effects of α are unambiguous and extend to the general model, as stated formally in Subsection 1.4.2.

Changes in α directly affect the sender's continuation payoffs; specifically, decreasing α decreases the sender's continuation payoff linearly in both settings. Fixing the sender's moral hazard persuasion strategy to be either cutoff or full revelation, the probability of implementation is higher in the benchmark setting than in the moral hazard setting. Therefore his expected payoff changes more steeply in α in the benchmark setting than in the moral hazard setting. Thus, if at some α' the sender is better off in the moral hazard setting, the same is true at all $\alpha \in (0, \alpha')$. Additionally, changes in α may change the sender's persuasion strategy. As α decreases, the sender cares less about implementation *per se* relative to the rent he receives conditional on implementation. For low enough α he fully reveals the state rather than persuading to the cutoff. In that case the receiver is better off in the moral hazard setting than in the benchmark setting.

The receiver's fixed cost of implementation β has competing effects on the players' relative payoffs across contracting settings. In the m-q game, both the sender and the receiver are better off from moral hazard when β is high. Because of the competing effects, this does not fully extend to the general setting. In settings that are similar to the m-q game, the receiver is better off from moral hazard when persuasion is high. I discuss the general result in Subsection 1.4.2.

Increasing β leads to increases in the implementation cutoff in both settings because it directly decreases the receiver's continuation payoffs in both settings. Since the cutoff is increasing, at fixed low p_0, if the sender chooses cutoff persuasion in both settings then he must provide more precise information. The probability of implementation decreases in both settings, which hurts the sender. In the m-q game the effect is stronger in the benchmark setting than in the moral hazard setting, so that the sender's payoff in the moral hazard setting increases relative to his payoff in the benchmark setting as β increases. Once β is high enough, the sender switches to full revelation in the moral hazard setting, at which point any further increase in β has no effect on his expected payoff.

When β is low enough that the sender persuades to the cutoff, the receiver has expected payoff 0 in both settings, so changes in β do not affect her. Once β is high enough that the sender fully reveals, the receiver is better off in the moral hazard setting than in the benchmark setting. The sender switches from cutoff persuasion to full revelation as β increases because his marginal rent from more information at the cutoff is increasing as the cutoff increases, i.e. because the sender's rent is convex in the posterior. In the general setting, the sender's rent need not be convex, so increases in β do not necessarily lead to more information provision.

The comparative static on β implies that if the receiver had the opportunity ex ante to reduce her fixed cost and directly contract on effort, she would choose to do neither in some settings, even for free. In other words, if the game were extended to allow the receiver both to choose fixed cost from a pair $\{\beta_1, \beta_2\}$ with $\beta_2 > \beta_1$ and to choose whether or not the contract was subject to moral hazard, there exist parameter values such that she would choose β_2 and moral hazard. A firm will sometimes willingly forego ex ante investments in cost reduction and monitoring or contracting technologies, even at very low cost and even though it would be beneficial in the interim, because the high fixed cost and contracting friction jointly act as a commitment device to sufficiently condition on beliefs the value that an information provider receives from a contract, thereby encouraging more precise information provision.

I briefly discuss the effects on the cost parameter k in order to illustrate the multiple channels through which the cost function affects equilibrium persuasion. In moral hazard problems in general, the relative curvatures of the cost and production functions jointly determine both players' expected payoffs in an optimal contract. The complexity of the IC constraint is such that very little can be said about costs in the general setting without additional restrictions.

Like β, increasing k lowers the receiver's payoff conditional on implementation in both contracting settings and therefore increases the implementation cutoffs in both settings. Unlike β, an increase in k also directly reduces the sender's rent in both settings. This effect is non-linear; changes in k affect the sensitivity of the sender's continuation payoff in the state. In the m-q game, these additional effects on the sender's rent are small enough that an increase in k has the same impact as an increase in β. As k increases, the sender prefers the moral hazard setting to the benchmark setting when k is high enough. He also switches from cutoff persuasion to full revelation in the moral hazard setting, so that the receiver also prefers the moral hazard setting to the benchmark setting when k is high enough.

I now discuss the role of prior beliefs in determining the players' relative payoffs across contracting settings. The prior affects equilibrium payoffs in a way that is fundamentally different from the other parameters. It has no effect on implementation rules or continuation payoffs, but directly affects the probability distribution over continuation payoffs for a given persuasion strategy.

I show the receiver is better off from moral hazard only at low priors, while the sender is better off from moral hazard only at high priors. After stating the result formally in Proposition 4, I explain the result by decomposing the change in equilibrium payoff from moral hazard into two effects, the information effect and the contracting effect.

Proposition 4. *In the m-q game (i)* $\exists p_0^R \in [0,1]$ *s.t.:*

(1) *If* $p_0 \geq p_0^R$ *then for any PBE* and* PBE^*_N, $\bar{V}^*(p_0) \geq \bar{V}^*_N(p_0)$.

(2) *If* $p_0 < p_0^R$ *then for any PBE* and* PBE^*_N, $\bar{V}^*(p_0) \leq \bar{V}^*_N(p_0)$.

And (ii) $\exists p_0^S \in [0,1]$ *s.t.:*

(1) *If* $p_0 > p_0^S$ *then for any PBE* and* PBE^*_N, $\bar{U}^*(p_0) \leq \bar{U}^*_N(p_0)$.

(2) *If* $p_0 \leq p_0^S$ *then for any PBE* and* PBE^*_N, $\bar{U}^*(p_0) \geq \bar{U}^*_N(p_0)$.

Furthermore, if $\exists p_0'$ *s.t.* $\bar{V}^*(p_0') < \bar{V}^*_N(p_0')$ *then for any* $p_0 \in (0, p_0')$, $\bar{V}^*(p_0) < \bar{V}^*_N(p_0)$ *and if* $\exists p_0''$ *s.t.* $\bar{U}^*(p_0'') < \bar{U}^*_N(p_0'')$ *then for any* $p_0 \in (p_0'', 1)$, $\bar{U}^*(p_0) < \bar{U}^*_N(p_0)$.

Proof. This is a special case of Theorem 3 in Subsection 1.4.2. □

The proposition follows by inspection of Figure 1.5 and Figure 1.6. First, consider the receiver (Figure 1.5). If the sender chooses cutoff persuasion in the moral hazard setting, the receiver is better off in the benchmark setting at all priors. If the sender chooses full revelation, the receiver is better off in the moral hazard setting at low priors and the benchmark setting at high priors. Now consider the sender. In both the cutoff persuasion case and the full revelation case, the sender is either better off in the moral hazard setting at every prior, or is better off in the moral hazard setting at high priors and in the benchmark setting at low priors. The result extends to the general model.

The ranking of p_0^R and p_0^S depends on the parameters. Whenever $p_0^R > p_0^S$, at every prior $p_0 \in (p_0^S, p_0^R)$, moral hazard leads to a pareto improvement.[20]

To further illustrate the result, I separate the difference in payoffs across settings into two effects, the *information effect* and the *contracting effect*, and discuss how these change in the prior. I restrict attention to the case in which the sender fully reveals in the moral hazard setting. The same intuition applies more generally.

[20]Case 3 in Table 1.1 provides a specific example of this, in which $p_0^R > \hat{p} > p_0^S$ so that moral hazard leads to a strict pareto improvement at the prior $p_0 = \hat{p}$.

First, consider the receiver. Define $V_F(p_0)$ to be the receiver's ex ante expected payoff if the sender fully reveals the state and play is otherwise as in a PBE*. I define the (receiver's) information effect at a given prior $V_F(p_0) - \bar{V}^*(p_0)$, i.e. the change in the receiver's ex ante expected payoff from an increase in information from the level provided in the benchmark setting to the level provided in the moral hazard setting. I define the (receiver's) contracting effect at a given prior is $\bar{V}^*_N(p_0) - V_F(p_0)$. Fixing the information level to be as in the moral hazard setting, the contracting effect is the change in the receiver's expected payoff from the benchmark setting to the moral hazard setting through the change in continuation payoffs. The sum of these two effects is the total difference between what the receiver gets in a PBE$^*_N$ and what she gets in a PBE*, $\bar{V}^*_N(p_0) - \bar{V}^*(p_0)$.

As was shown above, at low priors $p_0 \in (0, \hat{p}]$ whenever the sender fully reveals the state the receiver is better off in the moral hazard setting. In that region, the information effect, which is positive, is always greater in absolute terms than the contracting effect, which is negative. At priors $p_0 > \hat{p}$ the information effect is decreasing in the prior. As the receiver becomes more certain that the state is high, additional information is less valuable to her because it is less likely to have a substantial impact on her future behavior. On the other hand, the contracting effect, which is negative, is increasing in the prior in absolute terms. As the probability of her posterior being $p = 1$ increases, the discrepancy in continuation payoffs at that posterior becomes more important. Both effects are illustrated in Figure 1.7.

Now consider the sender. Define $U_F(p_0)$ analogously to $V_F(p_0)$, and the sender's information effect and contracting effect analogously to the receiver's. Unlike the receiver, the sender's information effect is always negative, and the sender's contracting effect is always positive. This is because additional information lowers the probability of implementation,

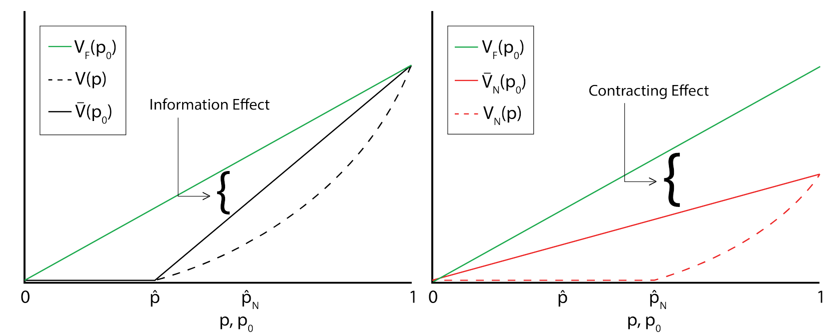

Figure 1.7. Decomposed Receiver Payoffs

but moral hazard allows the sender to capture additional rent. At $p_0 = 0$, the information effect and the contracting effect are zero. The information effect increases as the prior increases up to the benchmark cutoff $p_0 = \hat{p}$, then decreases at higher priors. This is because at priors below the cutoff the probability of implementation increase to one quickly in the prior in the benchmark setting, while under full revelation the probability of implementation increases more slowly up to the degenerate prior $p_0 = 1$. The contracting effect increases linearly in the prior. Like the receiver, the continuation payoff at posterior $p = 1$ becomes more salient for the sender as the prior approaches 1. At $p_0 = 1$, the information effect is zero and the contracting effect is strictly positive. Both effects for the sender are illustrated in Figure 1.8.

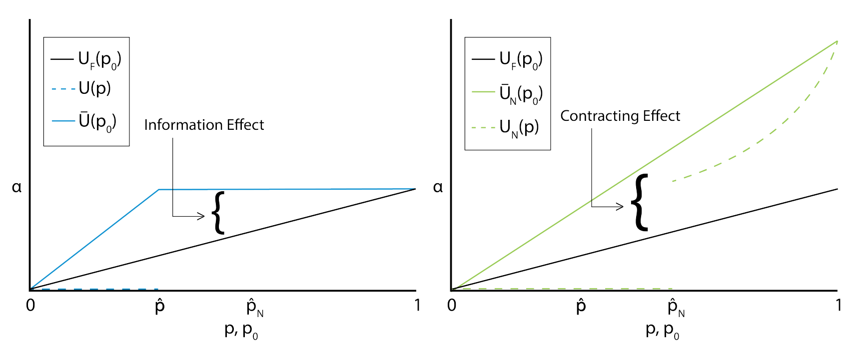

Figure 1.8. Decomposed Sender Payoffs

If the sum of the two effects is positive at $p_0 = \hat{p}$ then the sender is better off from moral hazard at every interior prior (cf. the right-hand graphs in Figure 1.6). If he is worse off from moral hazard at $p_0 = \hat{p}$, then he is better off from moral hazard only at a high range of priors (cf. the left-hand graphs in Figure 1.6).

The results on the prior demonstrate why it can be useful to restrict attention to the benchmark continuation cutoff. To answer the questions, "Does there exist a prior such that $\bar{V}^*_N(p_0) > \bar{V}^*(p_0)$?" and, "Does there exist a prior such that $\bar{U}^*_N(p_0) < \bar{U}^*(p_0)$?" one need only check the equilibrium payoffs at the prior $p_0 = \hat{p}$. This extends to the general setting whenever implementation cutoffs are interior.[21]

1.4. General Results

This section generalizes the results from the m-q game. In Subsection 1.4.1 I characterize equilibrium in both the benchmark contracting setting and the moral hazard contracting setting. In Subsection 1.4.2 I discuss welfare effects and show that, as in the m-q setting, the receiver is better off in the moral hazard setting at low priors, and the sender is better off in the moral hazard setting at high priors. I then give comparative statics on α and β.

1.4.1. Equilibrium Characterization

As in the m-q game, in the general benchmark setting equilibrium effort is efficient, there is an implementation cutoff, and information is underprovided. The sender gets no rent other than his benefit from implementation, so he maximizes the probability of implementation. In the moral hazard setting, equilibrium effort is inefficient, the implementation cutoff is higher

[21] Restricting attention to the benchmark prior is not sufficient to determine whether or not there exists a prior such that moral hazard leads to a strict pareto improvement. In general it may be that $p_0^R > p_0^S > \hat{p}$.

than in the benchmark setting, and information is sometimes, but not always, underprovided. The sender gets rent in addition to his benefit from implementation. For this reason, the sender sometimes provides more information than would maximize the probability of implementation.

In the general setting, the receiver-maximal equilibrium may not be unique. I focus on the "most-informative" equilibrium. Define a persuasion strategy inducing posteriors L, H to be *more informative* than a persuasion strategy L', H' if $L \leq L'$ and $H \geq H'$. I define a PBE* at a particular prior to be *most-informative* if the sender's equilibrium persuasion strategy in that PBE* is more informative than his persuasion strategy in every other receiver-maximal equilibrium at that prior. I define a most-informative PBE$^*_N$ analogously.

I make two assumptions that guarantee a cutoff implementation rule and a unique interior optimal effort in the benchmark contract at every posterior.

Assumption 1. *$c(0) = c'(0) = 0$, $c'(e) \geq 0$, $c''(e) > 0$, $f(e,1) \geq f(e,0)$, and $f_e(e',\theta) \geq 0\ \forall e'$. Furthermore, if $f(e,0) < 1$ then $f(e,1) > f(e,0)$, and if at any e' $f(e',\theta) < 1$ then $f_e(e',\theta) > 0$.*

This assumption states that $f(e,\theta)$, the likelihood of the project leading to high output as a function of the sender's effort and the underlying state, is strictly increasing in both arguments. I assume the sender's cost of effort is increasing and convex, with zero cost at zero effort.

Assumption 2. *$f(c^{-1}(1),1) < 1$*

This assumption guarantees that effort is not so cheap that the receiver simply pays the sender to provide high enough effort to guarantee high output. There is always some probability of low output in equilibrium.

I find the unique most-informative PBE* in the general setting using the same approach as in the m-q game. I write the probability of high output given effort level e and posterior p as $g(e,p) \equiv pf(e,1)+(1-p)f(e,0)$. At any posterior p, the receiver's maximization problem in the benchmark setting is as follows.

$$\begin{aligned} \Pi^*(p) = &\max_{e,t(\cdot)} \quad g(e,p) - t(e) & \\ &\text{s.t.} \quad t(\cdot) \geq 0 & \text{LL} \\ &\qquad t(e) - c(e) \geq 0 & \text{IR} \\ &\qquad t(e) - c(e) \geq t(e') - c(e') \ \forall e' & \text{IC} \end{aligned}$$

The solution is straightforward and can be solved using the first order approach. For a given effort, an optimal transfer rule is $t^*(e^*(p)) = c(e^*(p))$ and $t^*(e \neq e^*(p)) = 0$. At this transfer rule the IR constraint binds at the optimal effort level $e^*(p)$, and IC and LL are satisfied. Given this, the receiver can capture all of the surplus from output net of α so she optimally chooses first best effort, which (uniquely) solves $\frac{\partial}{\partial e}[g(e,p)] = c'(e)$.

Given these optimal effort and transfers, $\Pi^*(p)$ is continuous, strictly increasing, and (weakly) convex in p by Assumptions 1-2.[22] The receiver's implementation rule is a cutoff $\hat{p}$, i.e. $d^*(p) = 1$ *iff* $p \geq \hat{p}$.[23] She will continue ($d = 1$) whenever $\Pi^*(p) \geq \beta$ and will not continue ($d = 0$) otherwise.

[22]Fixing any e, $\Pi(p|e)$ increases linearly in p.

[23]It is possible that $\hat{p} > 1$, in which case the receiver never implements. It can also be that $\Pi^*(p) > \beta\ \forall p$, in which case she always implements, i.e. $\hat{p} = 0$. In both of these cases the sender is indifferent across all persuasion strategies. I restrict attention to the interesting case in which $\hat{p}$ is interior.

The sender's continuation utility, $U(p)$, is the same in the general model as in the m-q game. The sender gets α whenever the receiver implements. His optimal persuasion strategy is to induce implementation as often as possible. There are multiple persuasion strategies that guarantee implementation at priors above the implementation cutoff. Since the receiver's continuation payoff is convex provided she implements, an optimal persuasion strategy in that region that maximizes the receiver's ex ante payoff is the most informative one, namely he persuades to $\hat{p}$ and to 1, as in the m-q game.[24]

The following theorem formally summarizes the benchmark equilibrium in the general model.

Theorem 1. *Under Assumptions 1 and 2, $\exists \hat{p}$ s.t. if (i) $p_0 < \hat{p}$, (ii) $p_0 = \hat{p}$, (iii) $p_0 > \hat{p}$, the following is a unique most-informative PBE*:*

(1) *(i) $L^* = 0,\ H^* = \hat{p}$, (ii) $L^* = H^* = \hat{p}$, (iii) $L^* = \hat{p},\ H^* = 1$*

(2) *$d^*(p) = 1$ iff $p \geq \hat{p}$*

(3) *$t^*(e^*(p)) = c(e^*(p))$ and $t(e) = 0\ \forall e \neq e^*(p)\ \forall p$*

(4) *$e^*(p)$ solves $\frac{\partial}{\partial e} g(e, p) = c'(e)$.*

In the benchmark setting the only distinction between the general case and the m-q game is that in the general case, equilibrium is only unique up to changes in persuasion rules over which both players are indifferent. The most-informative PBE* is unique in general.

I now partially characterize equilibrium in the general moral hazard setting. The receiver's contracting problem is complex in that the optimal contract depends on the relative curvatures of the cost function and the production function.[25] Assumptions 1 and 2 alone

[24] When the receiver's continuation payoff is not strictly convex, there exist other optimal strategies in that region of priors that maximize the receiver's ex ante payoff, in which case there are multiple distinct PBE*s.

[25] See Section 1.5 for a more detailed discussion.

do not guarantee that standard solution techniques are valid. The following assumption is a technical restriction that guarantees the first-order approach is valid for finding optimal contracts in the moral hazard setting.

Assumption 3. *$\frac{\partial^2}{\partial e^2}[f(e,\theta)] < 0$ and $\frac{\partial^2}{\partial e^2}[g(e,p)(1-\frac{c'(e)}{g_e(e,p)})] < 0$ $\forall e \in [0, c^{-1}(1)], \forall p \in [0,1]$.*

In the moral hazard setting, the receiver's maximization problem given that she has chosen to implement at some posterior p is as follows:

$$\begin{aligned}
\Pi_N^*(p) = &\max_{e,t(y)} \quad g(e,p)(1-t(1)) - (1-g(e,p))t(0) \\
&\text{s.t.} \quad t(0), t(1) \geq 0 && \text{LL} \\
&\qquad g(e,p)t(1) + (1-g(e,p))t(0) - c(e) \geq 0 && \text{IR} \\
&\qquad e \in \operatorname*{argmax}_{e'}\{g(e,p)t(1) + (1-g(e,p))t(0) - c(e')\} && \text{IC}
\end{aligned}$$

The sender's IC constraint pins down the optimal transfer rule for any effort the receiver wants to induce. The IC constraint is easier to satisfy as $t(0)$ decreases, so $t_N^*(0|p) = 0$. At high output, the receiver must balance the sender's marginal cost of effort against the marginal productivity of effort to motivate the sender. The transfer $t(1)$ satisfies the following for a given effort.

$$t(1) = \frac{c'(e)}{\frac{\partial}{\partial e}[g(e,p)]}$$

Given the optimal transfer, the receiver's maximization problem is:

$$\Pi_N^*(p) = \max_{e,t(y)} \quad g(e,p)\left(1 - \frac{c'(e)}{\frac{\partial}{\partial e}[g(e,p)]}\right)$$

By Assumptions 1-3, this has a unique interior solution $e_N^*(p)$, which is less than first best.

In general, the receiver's maximized contracting payoff need not be increasing in the posterior. As the probability of high output increases, the receiver has to pay the sender a positive transfer with higher probability, and the sender's expected return to effort is not necessarily increasing. However, if $f(\cdot,\cdot)$ satisfies increasing differences as formalized in the assumption below, the receiver's payoff conditional on implementation in the moral hazard setting is increasing in the posterior.[26] This guarantees that her equilibrium implementation rule is a cutoff, $\hat{p}_N$. Lemma 1 formalizes this.

Assumption 4. $\frac{\partial}{\partial e}[f(e,1)] \geq \frac{\partial}{\partial e}[f(e,0)]\ \forall e.$

Lemma 1. *Under Assumptions 1-4,* $\frac{\partial}{\partial p}[\Pi_N^*(p)] \geq 0.$

Proof. First, $\Pi_N(p)$ increases in p for fixed e with optimal transfers $t(0) = 0$ and $t(1) = \frac{c'(e)}{\frac{\partial}{\partial e}[g(e,p)]}$. Fixing e, the derivative of the receiver's contracting payoff in p is as follows:

$$\begin{aligned}
&\frac{\partial}{\partial p}[g(e,p)(1 - \frac{c'(e)}{\frac{\partial}{\partial e}[g(e,p)]})] \\
&= (f(e,1) - f(e,0)) * \left(1 - \frac{c'(e)}{\frac{\partial}{\partial e}[g(e,p)]}\right) + g(e,p)\frac{c'(e)\frac{\partial^2}{\partial e \partial p}[g(e,p)]}{(\frac{\partial}{\partial e}[g(e,p)])^2} \\
&> 0.
\end{aligned}$$

Therefore when effort is optimal it must be that $\Pi_N(p)$ increases in p. □

At any fixed posterior the receiver is worse off in the moral hazard setting than in the benchmark setting, since any effort she can induce in this problem she could induce more cheaply in the the benchmark problem. This implies that $\hat{p}_N \geq \hat{p}$.

[26]Note that the m-q game satisfies Assumptions 1-4.

As in every setting, the sender's ex ante utility in the prior is a concavication of his continuation payoffs in the posterior. The persuasion strategies that induce this concavication need not be unique. In the benchmark setting, it was straightforward to argue for the existence of the unique most-informative PBE*, since it can be found directly due to the weak convexity of the receiver's continuation payoff conditional on implementation. However, in the moral hazard setting the receiver's continuation payoff need not be weakly convex in general. The most-informative equilibrium at priors $p_0 > \hat{p}_N$ is not necessarily $L = \hat{p}_N$, $H = 1$, and the equilibrium persuasion strategy at high priors cannot be found in closed form.

Although the most-informative PBE$^*_N$ cannot be directly found at all priors, it does exist and is unique because of a convenient feature of concavication in one dimension. If at any prior both the receiver and the sender are indifferent over a pair of persuasion strategies $\{L_1, H_1\}$ and $\{L_2, H_2\}$, then their ex ante equilibrium payoffs across priors are linear in the region $p_0 \in (\min\{L_1, L_2\}, \max\{H_1, H_2\})$. At any prior, let the sender choose the minimum L and the maximum H from all persuasion strategies over which both players are indifferent. This strategy must satisfy a type of monotonicity in the prior, stated formally in the following remark.

Remark 1. *If at some prior p_0 the most-informative equilibrium persuasion strategy is L, H, then at any prior $p_0 \geq H$, the most-informative equilibrium strategy L', H' will satisfy $L' \geq H$ and at any prior $p_0 \leq L$, the most-informative equilibrium strategy L'', H'' will satisfy $H'' \leq L$.*

This implies that the most-informative equilibrium in the moral hazard setting is uniquely pinned down as stated in Theorem 2.

Theorem 2. *Under Assumptions 1-4, $\exists \hat{p}_N, H^{**}$ with $H^{**} \geq \hat{p}_N \geq \hat{p}$ s.t. if (i) $p_0 < H^{**}$, (ii) $p_0 \geq H^{**}$ there exists a unique most-informative PBE_N^* with the following strategies:*

(1) *(i)* $L_N^* = 0,\ H_N^* = H^{**}$, *(ii)* $H_N^* \geq L_N^* \geq H^{**}$

(2) $d_N^*(p) = 1$ *if* $p \geq \hat{p}_N$, $d_N^*(p) = 0$ *if* $p < \hat{p}_N$

(3) $t_N^*(1|p) = \frac{c'(e_N^*(p))}{g_e(e_N^*(p),p)}$ *and* $t_N^*(0|p) = 0$

(4) $e_N^*(p) = \underset{e}{argmax}\left[(g(e,p))(1 - \frac{c'(e)}{g_e(e,p)})\right] \leq e^*(p)$

The equilibrium persuasion strategy is fully characterized across an interval of low priors. The sender persuades to 0 and to some posterior H^{**}. At priors above H^{**}, the persuasion rule is only partially characterized. The existence of H^{**} is enough for the purpose of payoff comparisons in the following subsection. Whenever $H^{**} = \hat{p}_N$ the persuasion strategy resembles the cutoff strategy as in the benchmark setting. Whenever $H^{**} > \hat{p}_N$, the sender persuades *beyond the cutoff*. While in the m-q game persuasion was either cutoff or full revelation in the moral hazard setting, in the general game persuasion may be somewhere in between, with $H^{**} \in (\hat{p}_N, 1)$.

1.4.2. Relative Payoffs and Comparative Statics

This subsection extends the payoff comparisons across contracting settings from the m-q game to the general game. The key observation is that the only way for the receiver to benefit from moral hazard is if it leads the sender to persuade beyond the cutoff, i.e. to provide more information than maximizes the probability of implementation. I first extend Proposition 4 to show that the receiver is better off from moral hazard at low priors while the receiver is better off from moral hazard at high priors. I then give a general comparative static on the sender's implementation benefit, α. Both the receiver and the sender are better off from moral hazard when α is low. I give a comparative static on the receiver's fixed cost of implementation, β, for a restricted set of games that resemble the m-q game. At low priors in that setting, the receiver is better off from moral hazard when β is high.

Theorem 3 formally states the result on the prior for general games.

Theorem 3. *Under Assumptions 1-4 (i) $\exists p_0^R \in [0,1]$ s.t.:*

(1) *If $p_0 \geq p_0^R$ then for any PBE* and PBE_N^*, $\bar{V}^*(p_0) \geq \bar{V}_N^*(p_0)$.*

(2) *If $p_0 < p_0^R$ then for any PBE* and PBE_N^*, $\bar{V}^*(p_0) \leq \bar{V}_N^*(p_0)$.*

And (ii) $\exists p_0^S \in [0,1]$ s.t.:

(1) *If $p_0 > p_0^S$ then for any PBE* and PBE_N^*, $\bar{U}^*(p_0) \leq \bar{U}_N^*(p_0)$.*

(2) *If $p_0 \leq p_0^S$ then for any PBE* and PBE_N^*, $\bar{U}^*(p_0) \geq \bar{U}_N^*(p_0)$.*

Furthermore, if $\exists p_0'$ s.t. $\bar{V}^*(p_0') < \bar{V}_N^*(p_0')$ then for any $p_0 \in (0, p_0')$, $\bar{V}^*(p_0) < \bar{V}_N^*(p_0)$ and if $\exists p_0''$ s.t. $\bar{U}^*(p_0'') < \bar{U}_N^*(p_0'')$ then for any $p_0 \in (p_0'', 1)$, $\bar{U}^*(p_0) < \bar{U}_N^*(p_0)$.

Proof. See Appendix A.1. □

As in the m-q game, whether or not there exists a prior such that the receiver is better off in the moral hazard setting than in the benchmark setting is entirely dependent on whether $H^{**} > \hat{p}_N$ or $H^{**} = \hat{p}_N$. If the sender chooses cutoff persuasion in the moral hazard setting, the receiver is always better off in the benchmark setting. If instead the sender chooses to persuade to a posterior higher than the cutoff, then the receiver is strictly better off in the moral hazard setting at low priors.

Whether or not he persuades beyond the cutoff, the sender is always better off in the moral hazard setting at high enough priors that he can guarantee implementation, since his rent is higher than in the benchmark setting. At lower priors the probability of implementation is lower in the benchmark setting than the moral hazard setting, since $H^{**} \geq \hat{p}$, but his rent conditional on implementation is higher. As in the m-q game, the sender's payoff is either higher in the moral hazard setting at all priors, or is higher in the moral hazard setting at high priors and in the benchmark setting at low priors.

Comparative statics on α and β can be thought of in terms of their effects on H^{**}, in particular whether or not H^{**} is above the implementation cutoff in the moral hazard setting. As in the m-q game, changes in α have no effect on the implementation cutoff in either setting, the receiver's continuation payoff in either setting, or the sender's equilibrium persuasion in the benchmark setting. The net effects on the sender across settings are identical to the m-q game.

In the moral hazard setting, H^{**} decreases in α. Since a decrease in α linearly decreases the sender's continuation payoff conditional on implementation, the posteriors such that the ex ante payoff is the concavication of the continuation payoff must move to the right.[27] This immediately implies Proposition 5.

Proposition 5. *If $\exists \alpha, \beta$ and $\exists p_0 \leq \hat{p}$ s.t. $\bar{V}^*_N(p_0) > \bar{V}^*(p_0)$, then for any $\alpha' \in (0, \alpha)$, $\bar{V}^*_N(p_0) > \bar{V}^*(p_0)$. Similarly, if $\exists \alpha, \beta, p_0$ s.t. $\bar{U}^*_N(p_0) > \bar{U}^*(p_0)$, then for any $\alpha' \in (0, \alpha)$, $\bar{U}^*_N(p_0) > \bar{U}^*(p_0)$.*

Unlike α, changes in β have an ambiguous effect on H^{**} in general because the sender's contracting rent need not be convex. The comparative statics on β in the m-q game apply for the receiver in the restricted setting where $R^*_N(p)$ and $\Pi^*_N(p)$ are convex.[28] The effect of changing β on the sender's relative equilibrium payoff from moral hazard is ambiguous even

[27]This is immediate by geometric argument and relies on the fact that the continuation payoff does not change at priors $p_0 < \hat{p}_N$.
[28]Note that the restriction is on equilibrium behavior rather than on the primatives. The m-q game provides one example of parametric assumptions that satisfy this condition.

under this restriction, as the implementation cutoffs change in both contracting settings. However, under the restriction the sender unambiguously chooses cutoff persuasion ($H^{**} = \hat{p}_N$) at low β and full revelation ($H^{**} = 1$) at high β, so the receiver is better off from moral hazard at priors $p_0 < \hat{p}$ whenever β is high enough, as in the m=q game. Remark 2 formalizes this comparative static.

Remark 2. *If $R^*_N(p)$ and $\Pi^*_N(p)$ are convex, then if $\exists \alpha, \beta, p_0$ s.t. $\bar{V}^*_N(p_0) > \bar{V}^*(p_0)$, then for any $\beta' > \beta$ s.t. $\hat{p}_N < 1$, $\exists p'_0$ s.t. $\bar{V}^*_N(p'_0) > \bar{V}^*(p'_0)$.*

1.5. Binary Effort

In this section I restrict effort to be high or low, $e \in \{0, 1\}$, in order to find conditions on the production function such that the receiver benefits from moral hazard.[29] While in the general model I examined the payoff implications of moral hazard under the assumption of increasing differences in production, in the binary effort model I show that either (sufficiently) increasing differences or decreasing differences can lead to persuasion beyond the cutoff in the moral hazard setting, through different mechanisms. I briefly discuss how these mechanisms extend to the continuous effort model.

The timing and payoffs in the binary effort model are the same as in Section 1.2. I drop Assumptions 1-4 and assume the following.

Assumption 5. *$c(0) = 0$, $c(1) = k > 0$, $1 \geq f(e, 1) > f(e, 0)$, and $f(1, \theta) > f(0, \theta)$.*

As in the continuous effort game, in the binary effort benchmark setting the receiver extracts all the value from the contract other than the sender's implementation benefit. The

[29] This is somewhat related to the risk neutral moral hazard model in Poblete & Spulber (2012), though I emphasize the role of beliefs.

implementation rule is a cutoff, and the sender persuades to the cutoff. In the binary moral hazard setting, the contract is inefficient and the receiver is therefore worse off than in the benchmark setting at any given posterior. If there exists a prior such that the receiver is better off in a PBE^*_N than in a PBE^*, then it must be that the sender persuades beyond the cutoff in the moral hazard setting.[30]

I partially characterize the optimal contract in the moral hazard setting. The receiver's maximization problem at posterior belief p is as follows.

$$\begin{aligned}
\max_{e,t(0),t(1)} \quad & g(e,p)(1-t(1)) + (1-g(e,p))(-t(0)) & \\
\text{s.t.} \quad & t(y) \geq 0 \ \forall y & \text{LL} \\
& g(e,p)t(1) + (1-g(e,p))t(0) - c(e) \geq 0 & \text{IR} \\
& g(e,p)t(1) + (1-g(e,p))t(0) - c(e) & \text{IC} \\
& \geq g(e',p)t(1) + (1-g(e',p))t(0) - c(e') \ \forall e' &
\end{aligned}$$

As in the continuous effort model, the optimal transfer after low output is $t^*_N(0) = 0$. If the receiver wants to induce $e = 1$ then the sender's IC constraint pins down $t^*_N(1) = \frac{k}{g(1,p)-g(0,p)}$. If instead the receiver wants to induce $e = 0$ then $t^*_N(1) = t^*_N(0) = 0$.

The maximization problem can therefore be expressed:

$$\max \left\{ g(0,p) \ , \ g(1,p)\left(1 - \frac{k}{g(1,p)-g(0,p)}\right)\right\}, \tag{1.2}$$

where the first value obtains when $e^*_N = 0$ and the second value obtains when $e^*_N = 1$.

[30] When the production function does not satisfy increasing differences, it is possible that the equilibrium implementation rule in the moral hazard setting is not a cutoff. Since this does not affect the results in this section, I restrict attention to parameters such that the implementation rule is a cutoff to streamline the exposition.

The sender's continuation rent is constant at $R_N^*(p) = 0$ whenever $e(p) = 0$. When $e^*(p) = 1$, $R_N^*(p)$ is weakly positive and varies in the posterior. There are only two reasons that the sender would persuade beyond the cutoff in this setting. Either (1) equilibrium effort must increase from 0 to 1 at some posterior above the implementation cutoff, or (2) the sender's continuation payoff must be increasing and convex over a region of posteriors at which the receiver implements and induces effort $e_N^*(p) = 1$.[31] Figure 1.9 shows an example of the first type of persuasion beyond the cutoff on the left and the second type of persuasion beyond the cutoff on the right.

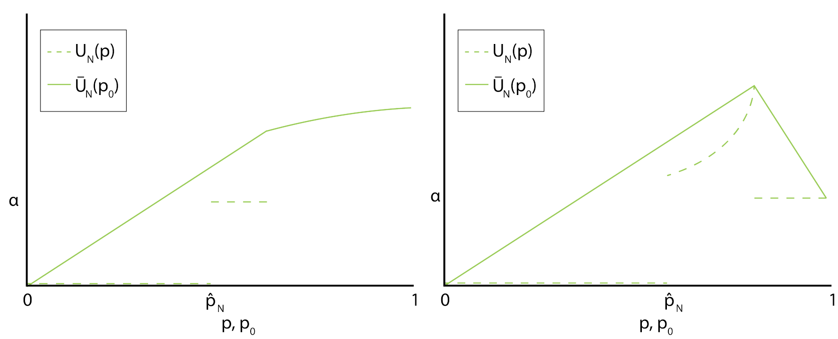

Figure 1.9. Persuasion Beyond the Cutoff with Binary Effort

The first example, in which equilibrium effort increases in the posterior, obtains when the production function exhibits sufficiently *increasing* differences. The second example, in which the sender's continuation payoff is increasing and convex, obtains when the production function exhibits *decreasing* differences. These results are stated formally in Proposition 6 and Proposition 7. Both results follow from Equation 1.2.

[31] It is immediate by geometric argument that if neither of these conditions hold the sender will persuade to the cutoff, supposing that the equilibrium implementation rule is a cutoff.

Proposition 6. *Under Assumption 5, if* $\frac{(f(1,1)-f(0,1))^2}{f(1,1)} > k > \frac{(f(1,0)-f(0,0))^2}{f(1,0)}$ *then in a* PBE_N^* *there exists a region of posteriors* $[0, p_1)$ *s.t.* $e^*(p) = 0 \ \forall p \in [0, p_1)$ *and a region of posteriors* $(p_2, 1]$ *with* $p_2 \geq p_1$ *s.t.* $e_N^*(p) = 1 \ \forall p \in (p_2, 1]$.

Proof. See Appendix A.2. □

The proposition states that if returns to effort in the high state are high enough relative to the cost of effort, which is high enough relative to returns to effort in the low state, that is sufficient for high optimal effort at high posteriors and low optimal effort at low posteriors. The condition on the production function is stronger than standard increasing differences, and it also restricts the cost of effort. This is required because the IC constraint in the receiver's maximization problem depends on the sender's cost of effort and the fact that the sender only gets compensated when output is high. This is a common feature of moral hazard problems.

The condition on the production function alone is not sufficient for the sender to persuade beyond the cutoff. It must be that the implementation cutoff $\hat{p}_N$ is interior on the region of posteriors at which the optimal contract induces zero effort. Let $\hat{b}_N$ be the lowest posterior such that $e_N^*(p) = 1$. Corollary 1 gives sufficient conditions for the existence of parameter values such that the receiver is better off in the moral hazard setting than in the benchmark setting.

Corollary 1. *Suppose* $0 < \hat{p} < \hat{p}_N < \hat{b}_N < 1$. *Then, if* $\frac{(f(1,1)-f(0,1))^2}{f(1,1)} > k > \frac{(f(1,0)-f(0,0))^2}{f(1,0)}$ *then* $\exists \alpha, p_0$ *s.t.* $\bar{V}_N^*(p_0) > \bar{V}^*(p_0)$.

Sufficiently increasing differences leads to persuasion beyond the cutoff because equilibrium effort increases in the posterior. Decreasing differences leads to persuasion beyond the cutoff because of changes in the sender's continuation payoff in the posterior, holding effort constant. Proposition 7 states that the sender's equilibrium continuation payoff conditional on implementation is both increasing and convex in the posterior exactly when the production function exhibits decreasing differences.[32]

Proposition 7. *Under Assumption 5, over any region $[p', p'']$ s.t. $e(p) = 1$ $\forall p \in [p', p'']$, the following two inequalities are jointly satisfied iff $f(1,1) - f(0,1) < f(1,0) - f(0,0)$:*

(1) $\frac{\partial}{\partial p}[R(p)] > 0$

(2) $\frac{\partial^2}{\partial p^2}[R(p)] > 0.$

Proof. See Appendix C. □

Under decreasing differences, as the posterior increases the receiver must compensate the sender increasingly more to provide high effort to prevent her from shirking, because the marginal return to effort (in terms of probability of high output) is decreasing in the posterior. At the same time, the probability of high output, and therefore the probability that the sender receives the transfer, is increasing. The net effect is that the sender's rent is both increasing and convex holding effort constant at $e^*_N(p) = 1$.

These results show that increasing differences and decreasing differences lead to persuasion beyond the cutoff through two different mechanisms. Sufficiently increasing differences can lead to persuasion beyond the cutoff because of the rate at which total continuation surplus increases in the posterior. The receiver encourages more effort as the returns to effort

[32]Under increasing differences the sender's rent when $e^*_N(p) = 1$ is either increasing and concave or decreasing and convex in the posterior.

increase and compensates the sender accordingly. Decreasing differences can lead to persuasion beyond the cutoff not because of increases in total continuation surplus, but because of the rate at which the sender's share of the continuation surplus increases in the posterior.[33]

Although Proposition 6 and Proposition 7 do not hold when effort is continuous, the intuition of these results extends to the continuous effort model. The binary effort model allows for results on the production function by simplifying the effects of the marginal cost of effort. In the continuous model, both mechanisms may encourage (or discourage) persuasion beyond the cutoff, even restricting attention to production functions that exhibit increasing differences. In the continuous effort model the optimal contract must prevent local deviations in effort, unlike the binary setting in which the sender can only deviate discretely to zero or one. As the posterior increases, the receiver must consider marginal returns to effort across all effort levels, as well as the marginal cost of effort. Effort in the optimal contract may be either increasing or decreasing in the posterior. Even holding effort fixed, the transfer the receiver must pay to motivate that effort level may increase or decrease in the posterior. However, the results in the binary model illustrate the ways in which the production function affects whether or not moral hazard benefits the receiver in general.

The m-q game serves as an example of how sufficiently increasing differences can lead to persuasion beyond the cutoff in a continuous effort game. In the m-q game, the equilibrium transfer rule under moral hazard is constant in the posterior, and effort is increasing in the posterior. Interim payoffs are increasing and convex in the posterior so that the sender will sometimes persuade beyond the cutoff. The sender's continuation payoffs are increasing and convex in the posterior for two reasons. First, holding effort constant, interim payoffs

[33] Interim total surplus may be increasing or decreasing in the posterior when the production function exhibits decreasing differences.

increase linearly in beliefs. Second, because of increasing differences and linear marginal cost (because cost of effort is quadratic), equilibrium effort is increasing. Thus, the mechanism by which sufficiently increasing differences leads to persuasion beyond the cutoff is the same in both the m-q game and the binary model.[34]

1.6. Conclusion

This paper explores how moral hazard mitigates a hold-up problem in information revelation. While the results depend on the timing of the game, the hold-up problem that arises from the receiver's ability to contract on effort after implementation is salient under some alternate specifications. For example, suppose a biotech can move beyond the R&D stage of drug development prior to a pharmaceutical buying them out. Formally, suppose the model is extended to allow the sender to publicly perform some "early effort" after the signal realization in stage 3 and before the implementation decision in stage 4. The contract still holds up the sender if effort (in stage 5) is contractible. In equilibrium, the sender will provide even less information than in the benchmark setting, such that the persuasive message alone would not convince the receiver to implement, but the additional effort "makes up the difference." Figure 1.10 shows an example of the sender's continuation and ex ante payoffs in such a setting. The receiver's continuation payoffs are identical to the benchmark setting.

This paper restricts attention to a receiver who makes a go-no-go decision about a particular project. In many cases it may be that a sender persuades a receiver not only about

[34]As an example of a continuous effort game in which there are *insufficiently* increasing differences for persuasion beyond the cutoff, consider $f(e,\theta) = (1-\epsilon)(e+\theta) + \epsilon e\theta$ and $c(e) = \frac{k}{2}e^2$. For small ϵ and large k the sender's equilibrium continuation payoff is concave in the posterior because the marginal returns to effort increase slowly in the state relative to the marginal cost of effort.

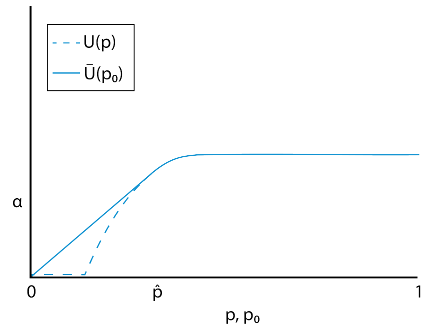

Figure 1.10. Persuasion with Early Effort

whether or not to pursue a project, but about which project to pursue.[35] Multiple senders may also send competing persuasive messages. It remains to be seen how the trade-off I identify would extend to such settings.

[35]For example, one could extend Rayo & Segal (2010) to a contract setting.

CHAPTER 2

Market Research and Differentiated Bertrand Competition (joint with Rafayal Ahmed)

2.1. Introduction

Firms learn demand in order to optimally set prices. In competitive settings, market research not only directly informs a firm about demand for its own good, but indirectly informs the firm about how its competitor will price in the face of uncertain demand for its good. Firms will only perform market research to the extent that the returns from doing so exceed the costs, and these returns may vary with the level of differentiation between one firm's product and its competitor's product.

We explore this phenomenon in the context of a standard differentiated duopoly Bertrand model with uncertain linear demand, in the style of Vives (1984). Rather than assume exogenous signals of the demand intercept, we instead allow firms to covertly choose the accuracy of their signals at some cost. We compare the level of market research in (symmetric) equilibrium across different levels of competition, as measured by how differentiated the goods are. We give sufficient conditions such that endogenous market research monotonically decreases in the level of competition, as well as sufficient conditions such that endogenous market research is non-monotonic in the level of competition.

In this model, fixing some exogenous level of market research, a firm optimally prices by setting an average price plus a linear function of its signal. The more accurate a firm's signal, the more it will condition its price on its signal. Its average price will not change, fixing the other firm's behavior. As the goods become less differentiated, competition sharpens: both firms' prices will go down for any given signal, which lowers overall profits.

Fixing the level of competition, as one firm's accuracy increases, its expected profits increase through two channels. First, it is better able to match its price to demand. Second, it is better able to coordinate its price with the other firm. Fixing average prices, one firm would rather price high when the other firm prices high, and low when the other firm prices low. A more accurate signal of demand is also a more accurate signal of the other firm's price. Because of this, if either firm's accuracy exogenously increases, both firms will condition their prices more on their signals. Otherwise, they will price conservatively in order to coordinate better. At any level of differentiation (other than perfectly homogenous goods), profits for both firms increase when either firm's accuracy increases.

The size of the marginal return to increasing accuracy varies with the amount of competition and can be broken down into two effects, which we call the *competitive profit effect* and the *coordination effect*. Both of these effects are weighted by the sensitivity of the firm's price to its signal; prices compress towards marginal cost as competition increases, so that the accuracy of a signal becomes less important fixing the other firm's behavior. The competitive profit effect is that as goods become less differentiated, so that both firms not only set prices lower on average but also condition prices less on the state, the firm cannot improve profits as much by setting high prices when the state is high and low prices when the state is low. If a firm is a monopoly, it can better align its prices with the state by increasing the accuracy of its signal. However, when the firm is forced to price conservatively because of increased competition, it cannot fully take advantage of a more accurate signal to match its price to the state.

The coordination effect has two components in addition to the sensitivity of the firm's price to its signal: the *substitution effect* and the *competitor pricing effect*. The substitution effect is that as goods become less differentiated, demand for one firm's good is more sensitive to the difference between the firms' prices. It becomes more important for a firm to coordinate its price with the other firm's price. The competitor pricing effect moves in the other direction. As competition intensifies, the firm's competitor not only lowers its price after any signal, but also compresses those prices towards marginal cost. This makes it easier to coordinate pricing, since the firms' prices are close even if their signals are very different. In the extreme case of homogenous goods, prices equal marginal cost and the competitor pricing effect is zero. At the other extreme, when goods are completely differentiated and firms function as monopolies, the substitution effect is zero. The total coordination effect is inverted U-shaped, so that it is highest at some intermediate level of competition.

We examine the competitive profit and coordination effects together. Marginally increasing accuracy always helps firms match the state better and coordinate better. However, the amount that it allows one firm to better coordinate with the other depends on the other's accuracy level. When both firms have very low accuracy, one firm marginally increasing its accuracy does not help it coordinate much with the other firm, whose price is not very correlated with demand. When both firms have high accuracy, one firm increasing its accuracy also allows it to better coordinate its price with the other firm. Thus, the relative importance of the competitive profit effect and the coordination effect depends on accuracy levels. We show that the competitive profit effect dominates when research costs are sufficiently high, so that equilibrium market research is monotonically decreasing in the level of competition. We also show that when research costs are sufficiently low, the coordination effect is large enough that equilibrium research is highest at an intermediate level of competition.

This paper is related to a wider literature on market research and competition. Building on the differentiated duopoly models of Singh & Vives (1984), Vives (1984) examines whether firms would prefer to commit to making their endogenous research public. He shows that firms prefer to pool their information in a Bertrand setting but not in a Cournot setting. Other models have endogenized market research, although they have tended to focus on Cournot rather than Bertrand competition, overt rather than covert research, and on different measures of competition than we do. For example, Hwang (1993) studies overt research in Cournot duopolies when goods are homogenous, but firms face different costs of acquiring information. Hwang (1995) also studies overt research in a Cournot setting with homogenous goods, but measures competition as the number of firms as well as a somewhat idiosyncratic "conjectural variation" model of competition. That paper finds a result qualitatively similar to ours: firms perform the least amount of research when competition is perfect, and perform the most amount of research either in an oligopoly or in a monopoly, depending on the parameters. Hauk & Hurkens (2000) study covert research in a Cournot setting, where competition is measured as the number of firms and goods are homogenous. Vives (2000) is an excellent overview of competition more broadly, and addresses some models of market research.

We utilize the central result of Persico (2000) in order to compare equilibrium market research at different levels of competition. That paper shows that when signals are ordered by accuracy, a concept first presented by Lehmann (1988), marginal returns to accuracy can be ranked according to a relatively straightforward single crossing condition. The paper then applies that ranking to compare information acquisition in first and second price auctions, building on the work of Milgrom and Weber (1982). To our knowledge, this is the first direct application of the theorem to a duopoly setting.

The paper shares some similarities to the literature on innovation, though in our setting market research hurts rather than helps consumers, since firms use the information to extract more surplus rather than to create better products.[1] Questions about the effects of competition on innovation have been raised and debated since seminal works by Schumpeter (1912, 1942). We do not address this debate, except to note that Aghion et al. (2005) find evidence of an inverted-U shape in equilibrium innovation that is qualitatively similar to our coordination effect. Goettler & Gordon (2014) also find an inverted-U shape between innovation and competition in their model of dynamic oligopoly with endogenous market structure.

The rest of the paper is organized as follows. Section 2.2 contains the model. Section 2.3 applies Persico's theorem to identify the two effects of competitiveness on returns to market research and gives the main results. Section 2.4 concludes.

2.2. Model

We give the timing and payoffs and review the relevant result of Vives (1984). Two symmetric firms indexed by i each privately choose a signal distribution indexed by $v_i \in [0, \infty)$ at differentiable cost $K(v_i)$. The state $\alpha \sim \mathscr{N}(\bar{\alpha}, V_\alpha)$ is realized. The cdf of this disribution, $G(\alpha)$, is commonly known to the firms when they choose v_i. Each firm receives a private signal realization $s_i = \alpha + \epsilon_i$, where $\epsilon_i \sim \mathscr{N}(0, v_i)$, and ϵ_1 and ϵ_2 are independent. Define $t_i = \frac{V_\alpha}{V_\alpha + v_i} \in (0, 1]$. Since for any V_α there is a one-to-one, continuous relationship between v_i and t_i, we consider firm i to be choosing t_i at cost $C(t_i)$. We assume that $C(t_i) \geq 0$ and $C'(t_i) \geq 0$.

[1] We address this further in Section 2.4.

We write the conditional distribution on α after seeing signal realization s_i as $G^{t_i}(\alpha|s_i)$. For a given α' and t_i we write the conditional distribution on all signals s_i as $F^{t_i}(s_i|\alpha')$. For a given t_i, we write the prior distribution on all signals s_i as $F^{t_i}(s_i)$.

After privately receiving signals, firms simultaneously set prices p_1 and p_2. Following Vives (1984), firm i faces the following linear inverse demand:[2]

$$p_i = \alpha - q_i - \gamma q_{-i}.$$

Direct demand is

$$q_i = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_{-i}.$$

Goods are substitutes, i.e. $\gamma \in [0, 1)$.[3] The state α, the demand intercept, captures the level of demand, while increasing γ decreases the level of differentiation between firms. When $\gamma = 0$ the firms are monopolies, while as $\gamma \to 1$ demand approaches perfect competition. We normalize the marginal cost of production to be 0 for simplicity. After privately observing a signal, each firm chooses price. Firm i earns profits $p_i q_i$.

We consider Perfect Bayesian Equilibrium of this game, with firm i's equilibrium strategy written $\{t_i^*, p_i^*(s_i|t_i)\}$. In the Bertrand competition stage, firms maximize their expected profits given their conjecture of the other firm's pricing strategy as a function of their signal. Firm i's maximization problem after receiving signal s_i when their signal structure is indexed by t_i and the conjectured signal structure of firm $-i$ is indexed by t_-i, is

$$\max_{p_i} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dG^{t_i}(\alpha|s_i).$$

[2]This is a special case of Vives (1984) with β normalized to 1, so that $\gamma \in [0, 1]$ fully characterizes the level of substitutability across firms, and with independent signals to simplify the firm's choice of t.
[3]Direct demand is undefined at $\gamma = 1$, where profits are discontinuous in price.

Equilibrium prices must be as in Vives (1984):

$$p_i^*(s_i|\gamma, t_i) = A + B_i t_i \left(s_i - \frac{\bar{\alpha}}{1+\gamma} \right)$$

Where

$$A = \frac{\bar{\alpha}(1-\gamma)}{2-\gamma}$$

$$B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}.$$

Anticipating this, firm i chooses t_i to maximize $R(t_i) - C(t_i)$, with

$$R(t_i) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i^*(s_i|\gamma, t_i) q_i(p_i^*(s_i|\gamma, t_i), p_{-i}(s_{-i}|\gamma, t_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dF^{t_i}(s_i|\alpha) dG(\alpha).$$

We call this the *market research problem* and we call t_i firm i's *accuracy level*.

Following Persico (2000), let *asymmetric marginal revenue* $AMR_\gamma(t, t')$ be firm i's marginal returns from increasing t_i from $t_i = t$ when the level of differentiation is γ and firm $-i$ plays pricing strategy $p_{-i}^*(s_i|\gamma, t_{-i} = t', t_i = t')$, i.e. when firm $-i$ has accuracy level t' and prices as if firm i also has accuracy level t'. Define marginal revenue of accuracy at level of differentiation γ as as $MR_\gamma(t) \equiv AMR_\gamma(t, t)$. Define the marginal cost of accuracy as $MC(t) \equiv C'(t)$.

We focus on symmetric equilibrium in which $t_i^* = t_{-i}^* = t^*(\gamma)$ and $p_i^*(s_i|\gamma, t^*(\gamma)) = p_{-i}^*(s_{-i}|\gamma, t^*(\gamma))\ \forall s_i = s_{-i}$. At such an equilibrium it must be that $MR_\gamma(t^*(\gamma)) = MC(t^*(\gamma))$.

2.3. Returns to Market Research

This section contains the main results of the paper. We state the relevant result from Persico (2000) in the framework of our model. Without directly solving for marginal returns to accuracy, we are able to apply the result in order to rank marginal returns to accuracy across different levels of differentiation. We decompose relative marginal revenue from accuracy into two components, the competitive profits component and the coordination component. We then give two main results: (1) when the cost of accuracy, $C(\cdot)$, is sufficiently high, market research in the unique symmetric equilibrium is decreasing in the level of competition, and (2) when $C(\cdot)$ is sufficiently low, equilibrium market research is higher at an intermediate level of competition than in either the monopoly or perfect competition setting. Finally, we show that the second result extends to a setting in which the both firms' choice of accuracy is publicly observed.[4]

Let $u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i})) \equiv \int\limits_{s_{-i}=-\infty}^{\infty} p_i^*(s_i) q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha)$. When $t_1 = t_2 = t$, denote this as $u_\gamma(\alpha, p^*(s,t))$. Given two payoff functions $u_{\gamma'}(\alpha, p_i^*(s,t))$ and $u_{\gamma''}(\alpha, p_i^*(s,t))$, we write $u_{\gamma'} \succeq u_{\gamma''}$ if $u_{\gamma'} - u_{\gamma''}$ has the single-crossing property, i.e. if $\frac{\partial u_{\gamma'}(\alpha,p)}{\partial p}$ crosses $\frac{\partial u_{\gamma''}(\alpha,p)}{\partial p}$ at most once, and from below, as α increases. We write $u_{\gamma'} \succ u_{\gamma''}$ if $u_{\gamma'} \succeq u_{\gamma''}$ and $u_{\gamma''} \not\succeq u_{\gamma'}$.

Lemma 2. *For γ' and γ'', if $u_{\gamma'}(\alpha, p^*(s,t)) \succ u_{\gamma''}(\alpha, p^*(s,t))$, then $MR_{\gamma'}(t) > MR_{\gamma''}(t)$.*

Proof. See Appendix B.1. □

The lemma states that in order to compare the marginal returns of accuracy at two different competition levels, it suffices to show that their difference satisfies single-crossing.[5]

Note that $p_i^*(s_i)$ is non-decreasing in s_i. In order to show for a given pair γ',γ'' that $MR_{\gamma''}(t_i) > MR_{\gamma'}(t_i)$, it suffices to show that

$$\frac{\partial}{\partial s_i} \left[u_{\gamma''}(\alpha, p_i^*(s_i|\gamma'', t)) - u_{\gamma'}(\alpha, p_i^*(s_i|\gamma', t)) \right]$$

[4] We do not extend the first result to the public setting.

[5] See Persico (2000) for a detailed discussion.

is increasing in α. To that end, we first examine $\frac{\partial^2}{\partial s_i \partial \alpha}\left[u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i}))\right]$ for fixed $\gamma \in [0, 1)$, which satisfies the following.[6]

$$\frac{\partial^2}{\partial s_i \partial \alpha}\left[u_\gamma(\alpha, p_i^*(s_i|, t_i))\right] = \frac{\partial q_\infty}{\partial \alpha}\frac{\partial p_i^*}{\partial s_i} + \left(\frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}\right). \tag{2.1}$$

Where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$.

From the equation we see that firm i's marginal return to accuracy has two components. Both components are weighted by the sensitivity of the firm's optimal price to their signal, and they are smaller if the firm's optimal price is not very sensitive to the signal.

The first component is the *competitive profit effect*, $CMP(t, \gamma) \equiv \frac{\partial q_\infty}{\partial \alpha}\frac{\partial p_i^*}{\partial s_i}$. This depends on the change in expected profit as the state changes evaluated when firm $-i$ sets its price at ∞. When the state increases, the quantity demanded at any price also increases. As the firm's accuracy increases, it is better able to tailor its demand to the state, α. However, if the firm's are very insensitive to their signal due to either low accuracy or high competition, then the firm cannot benefit as much from a high state. Even though this effect is evaluated when the competing firm chooses a fixed high price, we call it the "competitive" profit effect because it is dependent on the firm's ability to price high and condition its price on the state, which is determined by the level of competition.

The second component is the *coordination effect*, $CRD(t, \gamma) \equiv \frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}$. As the firm's accuracy increases, it not only learns more about the state, but also learns more about the other firm's pricing. It is able to better coordinate its pricing with the competing firm.

[6]See Appendix B.2 for a derivation of this equation, which depends on our distributional and linear demand assumptions. Arguments are suppressed for neatness.

Fixing an average price, the firm is better off pricing high when its competitor prices high, and low when its competitor prices low. The coordination effect measures this benefit.

The coordination effect has two components in addition the sensitivity of firm i's price to its signal: the sensitivity of firm i's demand to firm $-i$'s price, $\frac{\partial q_i}{\partial p_{-i}}$, i.e. the substitution effect, and the sensitivity of firm $-i$'s price to its signal, $\frac{\partial p_{-i}}{\partial s_{-i}}$, i.e. the competitor pricing effect. The substitution effect reflects that if demand is more sensitive to firm $-i$'s price, it is more important that firm i prices accordingly. As goods become less differentiated, then the quantity a firm sells is highly dependent on the difference between the two firms' prices. This is magnified by the competitor pricing effect. If firm $-i$'s price is more sensitive to its signal, then it is more important for firm i to coordinate signals with firm $-i$. A small difference in signals leads to a large difference in prices when firm $-i$'s price is very sensitive to its signal.

We now plug in equilibrium prices to Equation 2.1. For given accuracy levels t_i, t_{-i}, the equation is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i))}{\partial\alpha\partial s_i} = \frac{1}{1+\gamma} B_i t_i + B_i t_i \frac{\gamma}{1-\gamma^2} B_{-i} t_{-i}. \tag{2.2}$$

Recall that $B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}$. Since we are interested in symmetric equilibrium, suppose $t_i = t_{-i} = t$, in which case $B_i = B_{-i}$. Then Equation 2.2 is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t))}{\partial\alpha\partial s_i} = \frac{1-\gamma}{2-\gamma t} t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2} t^2 \tag{2.3}$$

We can now examine how both CMP and CRD depend on the level of competition γ.

Proposition 8. *For any $t \in (0, 1]$, the competitive profit effect $CMP(t, \gamma)$ is strictly decreasing in γ.*

Proof. For $t \in (0,1]$, $\frac{\partial}{\partial\gamma}\left[\frac{1-\gamma}{2-\gamma t}t\right] = \frac{-t(2-t)}{(2-\gamma t)^2} < 0$. □

As the environment becomes more competitive and firms price more aggressively, not only does the size of the pie effectively shrink, but the firms are less able to maximize their profits by tailoring prices to demand. The more a firm is forced to compete, the less it is able to condition its price on its signal and better match its price to the state. Accuracy becomes marginally less valuable.

Proposition 9. *For any $t \in (0,1]$, the coordination effect $CRD(t,\gamma)$ is single-peaked in γ, and $CRD(t,0) = \lim_{\gamma\to 1} CRD(t,\gamma) = 0$.*

Proof. First note that at $\gamma = 0$ and at $\gamma = 1$ the coordination effect is $\frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2 = 0$. The derivative of the coordination effect w.r.t. γ is

$$\frac{\partial}{\partial\gamma}\left[\frac{(1-\gamma^2)\,\gamma}{(2-\gamma t)^2}t^2\right] \quad = \quad \frac{-6\gamma^2 + (\gamma^3+\gamma)t + 2}{(2-\gamma t)^3}t^2.$$

This is continuous, positive at $\gamma = 0$, and negative at $\gamma = 1$. Setting it equal to zero, there is only one real-valued solution in γ, which must be interior by the intermediate value theorem. It must be the global maximum in γ on $\gamma \in [0,1)$. □

Changes in competition change the relative size of the coordination effect in two ways. First, as γ increases, firm i's profits are more dependent on firm $-i$'s price. Thus, it becomes more important to learn the state in order to learn more about firm $-i$'s price. Second, the size of this effect depends on how sensitive firm $-i$'s price is to the signal s_{-i}. Since these effects are multiplicative, the coordination effect is highest at intermediate levels of competition, where prices are sensitive enough to signals that coordinating prices requires high accuracy, and goods are similar enough that price coordination is important.

Examples of the competitive profit effect and the coordination effect as a function of γ are shown in Figure 2.1 for $t = 0.5$.

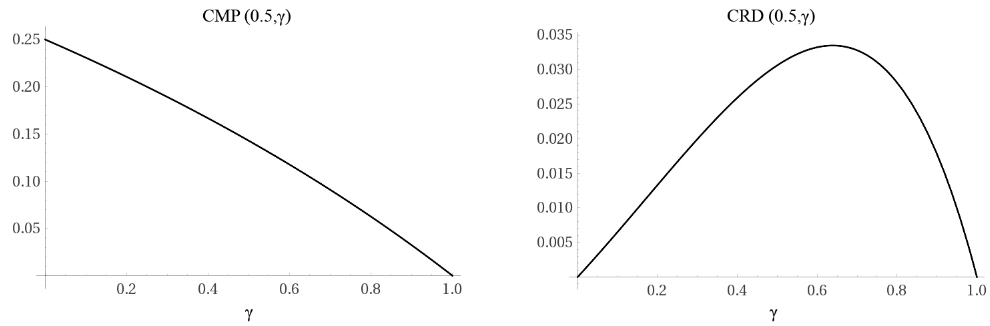

Figure 2.1. Competitive Profit Effect and Coordination Effect

Corollary 2. *For any* $t \in (0, 1]$, $MR_0(t) > \lim_{\gamma \to 1} MR_\gamma(t)$.

The competitive profit effect is positive in the monopoly case, i.e. $\gamma = 0$, where firms' profits when they price optimally are very sensitive to the state. The coordination effect is 0 in the monopoly case, since one firm's price has no impact on the other firm's demand or optimal price. In the (almost) perfect competition case, i.e. as γ approaches 1, both the competitive profit effect and the coordination effect approach 0. Each firm's equilibrium price approaches marginal cost at all signals, so there are minimal returns to better information.

The change in the total effect across competition levels, $\frac{\partial}{\partial \gamma}[CMP(t, \gamma) + CRD(t, \gamma)]$, depends on the level of accuracy, t. If both firms' signals are not very accurate, then one firm getting better accuracy does not help coordination very much, but it does help that firm better match the state. Thus, when t is low enough, the competitive profit effect is relatively more important than the coordination effect. The marginal return to accuracy is monotonically decreasing in γ in that case. When t is high enough, the coordination effect becomes relevant so that the marginal return to accuracy is no longer monotonically decreasing in the level of competition, but instead is highest at some intermediate level of competition. Examples of $CMP(t,\gamma)+CRD(t,\gamma)$ are shown in Figure 2.2 for $t = 0.5$ on the left and $t = 0.98$ on the right. In the right-hand graph, $CMP(t,\gamma)+CRD(t,\gamma)$ is maximized at an interior value of γ.

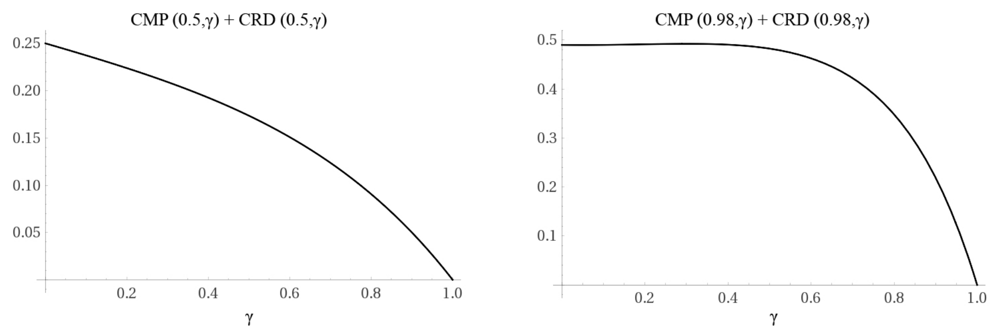

Figure 2.2. Total Effect at Low Accuracy and High Accuracy

The following lemmas formally state that the marginal return to accuracy is monotonically decreasing in γ when t is low, and that it is maximized at some interior γ when t is high.

Lemma 3. *$\exists \bar{t}$ such that for any $t \in (0, \bar{t})$, $\frac{\partial}{\partial \gamma}\left[MR_{\gamma}(t)\right] < 0$.*

Proof. See Appendix B.3. □

Lemma 4. *$\exists \underline{t}$ s.t. for any $t > \underline{t}$ $\exists \gamma' > 0$ s.t. $MR_{\gamma'}(t) > MR_0(t)$.*[7]

Proof. See Appendix B.4. □

We can compare equilibrium levels of market research across levels of competition as long as there exists a symmetric equilibrium in market research. For any pair t, γ both

[7]The lowest such $\underline{t}$ is approximately 0.96778. Note that the notation is somewhat idiosyncratic in that the minimum $\underline{t}$ satisfying Lemma 3 is larger than the maximum $\bar{t}$ satisfying Lemma 2.

the competitive profit and coordination effects are weakly positive. This is true even if $t_i \neq t_{-i}$, as in Equation 2.2. It is immediate by inspection that for any tuple $\{\gamma, t_i, t_{-i}\}$, $AMR_\gamma(t_i, t_{-t}) > 0$, i.e. firm i always benefits from more accuracy.[8]

This implies that we can find a cost function $C(t)$ such that when this is the cost of accuracy for both firms, at any γ there exists a unique symmetric equilibrium in accuracy $t^*(\gamma)$. Furthermore, we can find a cost function such that for some $\bar{t}$, $t^*(\gamma) \in (0, \bar{t})$ $\forall \gamma \in [0, 1]$. Call such a cost function $C^{\bar{t}}(t)$. We can also find a cost function such that for some $\underline{t} > \bar{t}$, $t^*(\gamma) \in (\underline{t}, 1)$ $\forall \gamma \in [0, 1]$. Call such a cost function $C_{\underline{t}}(t)$.

Finally, in order to state the main result we must formally define "higher costs" and "lower costs" of accuracy. For a given cost function $\hat{C}(t)$, let $\{\hat{C}(t)\}_L$ be the set of all cost functions $C(t)$ such that $\forall \gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0, 1]$, $C(t') \leq \hat{C}(t')$ and $C'(t') \leq \hat{C}'(t')$. Similarly, let $\{\hat{C}(t)\}_H$ be the set of all cost functions $C(t)$ such that $\forall \gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0, 1]$, $C(t') \geq \hat{C}(t')$ and $C'(t') \geq \hat{C}'(t')$.

Theorem 4. *There exist $\{\bar{t}, \underline{t}\}$ with $1 > \underline{t} > \bar{t} > 0$ such that:*
(1) $\exists C^{\bar{t}}(t)$ such that for any cost function $C(t) \in \{C^{\bar{t}}(t)\}_H$, at every $\gamma \in [0, 1)$ there exists a unique symmetric equilibrium with market research $t^*(\gamma)$ s.t. $\frac{\partial}{\partial \gamma}[t^*(\gamma)] < 0$, and
(2) $\exists C_{\underline{t}}(t)$ such that for any cost function $C(t) \in \{C_{\underline{t}}(t)\}_L$, at every $\gamma \in [0, 1)$ there exists a unique symmetric equilibrium with market research $t^*(\gamma)$ s.t. $t^*(\gamma') > t^*(0) > \lim_{\gamma \to 1} t^*(\gamma)$ for some $\gamma' \in (0, 1)$.

Proof. By Lemma 2, single crossing is sufficient for ranking marginal returns to accuracy. Existence of symmetric equilibrium is immediate from Lemmas 3 and 4. $t^*(\gamma)$ is continuous

[8] $AMR_\gamma(t_i, t_{-t})$ approaches 0 as $\gamma \to 1$.

in γ by the continuity of equilibrium prices and equilibrium payoffs in all arguments. For (1), by Lemma 3 there exist some $\bar{t}$ and $C^{\bar{t}}(t)$ such that $\frac{\partial}{\partial\gamma}[MR_\gamma(t)] \leq 0\ \forall t \in [0,\bar{t}]\ \forall\gamma$ and $t^*(\gamma) < \bar{t}\ \forall\gamma$. This is true for all higher cost functions such that there exists a unique equilibrium at every γ. For (2), by Lemma 4 there exist some $\underline{t}$ and $C_{\underline{t}}(t)$ such that $\forall t > \underline{t}\ \exists\gamma' \in (0,1)$ s.t. $MR_{\gamma'}(t) > MR_0(t)$ and $t^*(\gamma) > \underline{t}\ \forall\gamma$. In particular, for $t^*(0)\ \exists\gamma'$ s.t. $MR_{\gamma'}(t^*(0)) > MR_0(t^*(0))$. Therefore it must be that $t^*(\gamma') > t^*(0)$. This is true for all lower cost functions such that there exists a unique equilibrium at every γ. $\square$

The theorem states that, for cost functions such that there exists a unique equilibrium at all levels of competition, equilibrium private market research is decreasing in competition when accuracy costs are sufficiently high, and is maximized at some intermediate level of competition when accuracy costs are sufficiently low.

The second part of the result readily extends to the case of public market research. Suppose that after firms choose accuracy levels v_i and v_{-i}, both firms observe v_i and v_{-i} prior to choosing prices. The game is otherwise as in Section 2.2. Call this the overt game. In this setting, both accuracy and prices are strategic complements.[9] Thus, firms have weakly higher marginal returns to accuracy compared to the private market research game. However, in the monopoly case there is no strategic effect from increasing accuracy, so marginal returns are the same in both settings. Let $t^*_O(\gamma)$ denote market research in a symmetric equilibrium of the overt market research game. For any cost function such that there exists a unique symmetric equilibrium in both the private research game and the overt game at some γ, it must be that $t^*_O(\gamma) \geq t^*(\gamma)$. In the monopoly case ($\gamma = 0$), $t^*_O(0) = t^*(0)$. Furthermore,

[9]See Chapter 8 in Vives (2000) for a more thorough discussion.

returns to market research approach zero in both settings as competition approaches perfect competition: $\lim_{\gamma \to 1} t^*_O(\gamma) = \lim_{\gamma \to 1} t^*(\gamma) = 0$.

As in the private market research setting, in the overt game for any $\underline{t}$ one can find a cost function $C_{\underline{t}}(t)$ such that at every γ there exists a symmetric equilibrium in the overt game with $1 > t^*_O(\gamma) > \underline{t}$. Define $\{\hat{C}(t)\}^O_L$ in the overt game analogously to $\{\hat{C}(t)\}_L$ in the private market research game. Corollary 3 immediately follows.

Corollary 3. *$\exists \underline{t}, C_{\underline{t}}(t)$ such that for any cost function $C(t) \in \{C_{\underline{t}}(t)\}^O_L$, at every $\gamma \in [0, 1)$ there exists a unique symmetric equilibrium with market research $t^*_O(\gamma)$ s.t. $t^*_O(\gamma') > t^*_O(0) > \lim_{\gamma \to 1} t^*_O(\gamma)$ for some $\gamma' \in (0, 1)$.*

As in the private market research game, in the overt game when accuracy costs are sufficiently low, firms facing some intermediate level of competition invest more in market research than monopolistic firms.

2.4. Conclusion

This paper examines how differentiation affects equilibrium market research in a Bertrand duopoly. We conjecture that in symmetric Bertrand oligopolies with $n > 2$ firms, the results hold qualitatively, meaning there exist parameters such that firms with partially differentated goods invest more in market research than firms with completely differentiated goods.

We do not explicitly analyze consumer welfare across differentiation levels, as to do so would require finding equilibrium market research in closed form, but we can say something about it. Increased accuracy has competing effects on consumer welfare. When firms increase their accuracy, they condition their prices more on their signals and thus better align their prices with the state. This is partially beneficial for consumers, since fixing the average price, they would prefer to pay a high price when the state is high and a low price when the state is low, rather than the same price in all states. However, consumers also prefer for firms to have different prices from each other, as it allows them to substitute the cheaper good for the more expensive good. When firms increase their accuracy, their prices tend to be closer. This harms consumers. The net effect in our model is that consumer surplus decreases in the firms' accuracy.[10]

Fixing the accuracy of both firms, consumer welfare increases as goods become closer substitutes. However, as we have shown accuracy is sometimes non-monotonic in the level of differentiation. This highlights a challenge in regulating either market research or pricing behavior when market research is endogenous. For a given market research cost function, it may be that consumer welfare is sometimes higher when goods are less differentiated than when goods are more differentiated.

[10]See Proposition 6 in Vives (1984).

CHAPTER 3

Governance, Depreciation, and Debt (Joint with Alexander Limonov)

3.1. Introduction

Liquidity-constrained entrepreneurs require funding from investors. If an entrepreneur is unable to write a long-term contract with an investor, the entrepreneur will be tempted to appropriate returns instead of repaying a loan. The entrepreneur can put up collateral to partially alleviate this commitment problem. However, the value of that collateral depends on how it is used by the entrepreneur.

The entrepreneur may benefit from using capital in a state-dependent, uncertain production process, rather than a state-independent production process, because its success or failure gives him better information about the future value of capital. In this way, he forecloses more often when the expected value of the project is low and less often when it is high. On the other hand, such a production process may require experimentation or trial-and-error that damages capital so that it depreciates more, lowering its value both with respect to its future productive output for the entrepreneur and as collateral for the investor. If the entrepreneur cannot commit to a production process, then the investor will anticipate this and require more collateral to fund the project.

If the entrepreneur were not liquidity-constrained, he would always choose a safe production process rather than a state-dependent production process in order to avoid the depreciation cost, provided the expected returns from the two processes are the same. However, since in a debt contract he will foreclose when output is low, he may prefer to depreciate capital in exchange for foreclosing more often in low states and less often in high states. If the benefit from shifting foreclosure to the low state is high enough relative to the depreciation cost, then the entrepreneur will choose the risky production process, forcing him to offer more collateral to the investor. If the benefit is high enough, then this is efficient given that there is sometimes foreclosure. If the benefit is low enough, then the entrepreneur will efficiently choose the safe process. However, if the benefit is high enough to tempt the entrepreneur to choose the state-dependent process but not high enough to make up for the higher collateral he must promise, then his choice of state-dependent production is inefficient. This occurs because the entrepreneur does not internalize the lowered value of collateral from depreciation when he forecloses. In that case, the entrepreneur would be better off if the investor had decision rights over which production process is used.

To model this, we consider a one period debt contract offered by a liquidity-constrained entrepreneur to an investor with deep pockets, followed by a two period production process. The debt contract consists of a promised payment and percentage of capital to be used as collateral should the entrepreneur fail to make that payment. During the first period of production, either the investor or the entrepreneur, whoever has *governance*, decides whether to use a state-dependent production process (where the state is unknown and persistant) or a safe production process. Both processes have the same expected first period output. If after first period output is realized the entrepreneur fails to repay the contracted amount, he can renegotiate a new repayment amount, which he pays if the investor accepts this renegotiation. If the investor rejects the renegotiation, then the entrepreneur must turn over a percentage of capital as originally agreed, i.e. "foreclose." The entrepreneur then receives time 2 output in proportion to the amount of capital he has not turned over to the investor. If whoever has governance chose the state-dependent production process, then time two output is lowered by some depreciation percentage. The investor also receives some time 2 output in proportion to the capital she has received, lowered by the depreciation percentage if first period production was state-dependent. In the second period, there is only one production process, which depends on the state. Regardless of the state, we assume that for the same amount of capital, the entrepreneur's time 2 output is higher than the investor's time 2 output to capture the idea that the project is more valuable to the entrepreneur than the investor.[1]

We show that in equilibrium in this setting, the entrepreneur will always renegotiate with the investor when output is high enough to do so, so that foreclosure only occurs when first period output is low. We then provide two simple conditions that fully characterize when the entrepreneur is better off in equilibrium when production is under the investor's governance than when production is under his own governance. Since the investor always chooses the safe production process if she has governance, the first condition is that the entrepreneur is not sufficiently better off from the state-dependent process that it is worth the higher collateral he must offer in equilibrium when he has governance. In other words, *the state-dependent production process is inefficient*. The second condition is that at the low level of collateral in equilibrium under the investor's governance, the entrepreneur prefers the state-dependent production process. This condition prevents him from offering the same level of collateral when he has governance that he can offer when the investor has governance, since he cannot credibly commit to choosing the safe production process. In other words, if he has governance, *the entrepreneur will choose the state-dependent production process.*

[1]We interpret this as some non-appropriable skill that the entrepreneur has in using capital for production.

To understand when these two conditions are satisfied, consider how the production process affects the entrepreneur's payoff for some fixed debt contract. When the production process is state-dependent, the posterior on the state is higher after high output and lower after low output, so that foreclosure occurs more often in the low state than in the high state relative to when the production process is safe. After high time 1 output, the entrepreneur's expected time 2 output increases, but so does the expected value of the contracted collateral. The entrepreneur will have to pay the investor more in a renegotiation to prevent foreclosure. After low output, the entrepreneur always defaults rather than paying off the investor, so the investor's value of collateral in that state does not affect the entrepreneur's payoff. Thus, state-dependent production leads to a benefit if the entrepreneur's time 2 output is more sensitive to the state than the investor's time 2 output, i.e. the value of collateral. If instead the investor's time 2 output is more sensitive to the state than the entrepreneur's, then the entrepreneur is always better off from the safe production process, even if there is no depreciation cost. In that case, he can offer the same low collateral when he governs the production process as when the investor governs the production process, since he will credibly choose the safe production process. He is indifferent between governance structures, and our second condition is not satisfied.

Suppose that the entrepreneur's time 2 output is more sensitive to the state, so that he benefits when foreclosure shifts in probability from the high state to the low state. Then, for a fixed contract, he will prefer the state-dependent production process as long as it is sufficiently aligned with the state and the depreciation rate is sufficiently small. The more the state-dependent production process is aligned with the state, the more he can shift the probability of foreclosure towards the low state. This is enough to compensate for the cost of depreciation (which occurs in all states), as long as that cost is not too high. In this case, our second condition is satisfied.

The investor is always worse off from state-dependent production. She is held to her outside option (the value of collateral) after high output and receives collateral after low output, so ex ante she always receives the value of contracted collateral in equilibrium. When production is state-dependent, that collateral is depreciated. When our second condition is satisfied, the entrepreneur cannot credibly commit to choosing the safe production process. He must offer higher collateral under his governance than he would under the investor's governance.

Even though the entrepreneur must offer higher collateral in this case, he may still prefer his own governance to the investor's governance if time 1 output is sufficiently aligned with the state and the depreciation cost is sufficiently small. In that case, our first condition is not satisfied. The first condition is satisfied when the benefit of shifting foreclosure to the low state is sufficiently small relative to the cost of depreciation and the loss from higher collateral. Thus, the two conditions are satisfied whenever the state-dependent production process is moderately dependent on the state, and the depreciation cost is in an intermediate range, provided the entrepreneur's time 2 output is more sensitive to the state than the investor's time 2 output. In that case, state-dependent production is beneficial enough holding the contract fixed to prevent the entrepreneur from offering low collateral, but not beneficial enough to cover the loss from being forced to offer higher collateral. Then, it is better for the entrepreneur if the investor has governance.

This paper is related to the liquidation and renegotation models of Hart (1995) and Hart & Moore (1998), as well as to the residual control models of Grossman & Hart (1986), Hart & Moore (1990), and Aghion & Bolton (1992). Aghion & Bolton (1992) in particular show

how control rights might be imperfectly allocated across different states, so that governence structures are contingent and can be conditioned on a verifiable state of the world. Hart & Moore (1998) instead model debt contracts when capital ownership from foreclosure on debt is an endogenous consequence of defaulting. Capital ownership can be thought of as a type of Aghion & Bolton (1992) style governance that is different from governance in our model. Our model is a special case of Hart & Moore (1998) if we exogenously fix the production process.

We can think of governance in our model as qualitatively similar to Aghion & Bolton (1992), who show that it is optimal for the entrepreneur to have governance in states where private benefits from the action are high, while the investor should have governance in states where the entrepreneur's private benefits are low. In our model, the entrepreneur's private benefit comes from allocating time 2 capital ownership to the low state through Hart & Moore (1998) style foreclosure. We show that the investor should have governance if the entrepreneur's time 2 payoffs are more sensitive to the state than the investor's, and his private benefit from state-dependent production is in an intermediate range.

We restrict attention to debt contracts. Many papers take up the question of whether debt or equity (or some other contract) is optimal for financing under various frictions, including Hart & Moore (1998), Dewatripont & Tirole (1994), Fluck (1998), and Holmström & Tirole (1997). We do not speak to the optimality of debt, instead focusing on the optimal governance structure when the contract is restricted to be debt.

The rest of the paper is organized as follows. Section 3.2 contains the details of the model. Section 3.3 characterizes when the entrepreneur is better off under investor governance than under his own governance. Section 3.4 discusses remaining questions and concludes.

3.2. Model

A project requires capital that costs K. A liquidity constrained entrepreneur E with no wealth at time 0 offers a contract $\{r, l\}$ to an investor I in exchange for K, where r is a time 1 repayment amount and l is the percentage of capital that E puts up as collateral. Should E fail to pay r, that capital is turned over to I (the project is partially liquidated), or E can offer a renegotiated payment up to his time 1 liquidity constraint.

If the project is funded, E produces noisy time 1 output $Y \in \{0, y\}$. Output may partially depend on a hidden state θ which distributed equally on $\{\theta_L, \theta_H\}$, with $1 > \theta_H > \theta_L > 0$. Let $\bar{\theta} \equiv \mathbb{E}[\theta]$ and let $\epsilon \equiv \theta_H - \bar{\theta} = \bar{\theta} - \theta_L$.[2] There is a state-dependent production process that depends on realized θ, and a safe production process that depends only on $\bar{\theta}$. Whichever player has *governance* chooses the time 1 production process. There is a second round of production with only one possible production process that always depends on the state. The timing is as follows.

(1) Time 0: E offers a contract $\{r, l\}$ to I, with $r \geq 0$, and $l \in [0, 1]$. If I rejects, the game ends and both players receive payoff zero. If I accepts, the game continues.
(2) If governance is g_E (g_I), E (I) chooses $a \in \{0, 1\}$.[3]
(3) θ is determined. $Pr(\theta = \theta_H) = Pr(\theta = \theta_L) = \frac{1}{2}$. This is not observed.
(4) Time 1: $Y \in \{0, y\}$ is publicly realized with $Pr(Y = y) = a\theta + (1 - a)\bar{\theta}$.
(5) E chooses to default ($d = 1$) or not default ($d = 0$). E must choose $d = 1$ if $Y = 0$.
(6) E offers renegotiation $\{\hat{r}, \hat{l}\}$ with $\hat{r} \in [0, Y]$ and $\hat{l} \in [0, 1]$, which I accepts or rejects.

[2] The results hold qualitatively without assuming the states are equally likely, but this formulation allows for clear comparative statics on ϵ.
[3] This can be extended to $a \in [0, 1]$ without affecting our results.

(7) Time 2: If the renegotiation was accepted, I gets time 2 payoff $\hat{l}v_2(\theta)(1-ca)$ and E gets time 2 payoff $(1-\hat{l})u_2(\theta)(1-ca)$. Otherwise, I gets time 2 payoff $lv_2(\theta)(1-ca)$ and E gets time 2 payoff $(1-l)u_2(\theta)(1-ca)$

If output is high, E chooses whether to divert funds, thereby defaulting, or not. If output is low, E must default since he is liquidity constrained and cannot pay I the promised amount. If he does not default, he pays r to I and maintains ownership of the project. If E defaults, he must either turn over l of the project to I, or he can instead offer a renegotiated payement and collateral $\{\hat{r}, \hat{l}\}$ to I, where $\hat{r}$ cannot exceed realized time 1 output. If I does not accept this renegotiation, a proportion l of capital is turned over to I according to the original contract. If I does accept, E pays I the renegotiatied payment $\hat{r}$ and turns over proportion $\hat{l}$ of the project. At time 2, players earn payoffs that depend on θ and the amount of the project they possess at time 2.

We call $u_2(\theta)$ the *production value* of capital, and $v_2(\theta)$ the *collateral value* of capital. Aligning time 1 production with the state $(a = 1)$ "uses up" some of the capital, for example through increased wear-and-tear. We model this as a state-independent percentage loss $c \in (0,1)$ to time 2 payoffs, which we call the cost of depreciation. E's payoff for a given θ is $u(\theta) = Y - R + (1-L)u_2(\theta)(1-ca)$, and I's payoff for a given θ is $v(\theta) = R + Lv_2(\theta)(1-ca) - K$, where repayment amount R and collateral percentage L are as follows.

$$R = \begin{cases} \hat{r} & \text{if } I \text{ accepts renegotiation} \\ r(1-d) & \text{if } I \text{ rejects renegotiation} \end{cases} \qquad L = \begin{cases} \hat{l} & \text{if } I \text{ accepts renegotiation} \\ ld & \text{if } I \text{ rejects renegotiation} \end{cases}$$

Our equilibrium concept is Perfect Bayesian Equilibrium. When $a = 0$, posterior beliefs on the high state are $Pr(\theta = \theta_H | Y = y) = Pr(\theta = \theta_H | Y = 0) = \frac{1}{2}$. When $a = 1$, posterior beliefs are characterized as follows for $j \in \{L, H\}$:

$$Pr(\theta = \theta_j | Y = y) = \frac{\theta_j}{2\bar{\theta}}$$
$$Pr(\theta = \theta_j | Y = 0) = \frac{1 - \theta_j}{2(1 - \bar{\theta})}$$

When $a = 1$, time 1 output Y is a more precise signal of θ.

After observing output, we write E's interim expected payoff with posterior p on $\theta = \theta_H$ as $u_1(p, Y) = Y - R + (1-L)(pu_2(\theta_H) + (1-p)u_2(\theta_L))(1-ca)$. We write I's interim expected payoff with posterior p on $\theta = \theta_H$ as $v_1(p, Y) = R + L(pv_2(\theta_H) + (1-p)v_2(\theta_L))(1-ca) - K$. We write ex ante expected equilibrium payoffs as u (for E) and v (for I). We restrict attention to equilibria that maximize $u + v$. Since in any equilibrium the entrepreneur will offer an initial contract such that the investor's IR constraint binds, this is equivalent to equilibria that maximize u. We write $u(g_i)$ to denote equilibrium ex ante payoffs under government structure g_i.

We assume that both the production value and collateral value are higher when the state is high than when the state is low. We also assume that in both states the production value is higher than the collateral value, i.e. capital is worth more to the entrepreneur than to the investor at time 2. Furthermore, we assume high output y is sufficiently high that the entrepreneur can afford to pay the investor the value of her collateral when $Y = y$. This allows us to use backward induction to solve for equilibrium. Finally, we assume that K is small enough that the project is always funded in equilibrium.[4]

[4]The assumption on K does not qualitatively affect the results, but makes comparisons between governance structures clearer.

Assumption. $u_2(\theta_H) > u_2(\theta_L) > 0$, $v_2(\theta_H) > v_2(\theta_L) \geq 0$, $u_2(\theta) > v_2(\theta)$, $y > K\frac{(\theta_H v_2(\theta_H)+\theta_L v_2(\theta_L))}{\bar{\theta}(v_2(\theta_H)+v_2(\theta_L))}$, *and* $K < \frac{(1-c)(v_2(\theta_H)+v_2(\theta_L))}{2}$.

3.3. Equilibrium and Optimal Governance

This section characterizes the conditions such that it is strictly better for the entrepreneur when the investor has governance. We first solve for the optimal default rule and renegotiation given any initial contract $\{r, l\}$ and action a. We then find equilibrium collateral and payoffs under governance g_I. The investor always chooses $a = 0$. We provide a condition such that this makes the entrepreneur worse off than if the action a were exogenously chosen to be $a = 1$. We then provide a necessary and sufficient condition such that collateral is higher in equilibrium under governance g_E than under g_I. Finally, we show that these two conditions together characterize when the entrepreneur is better off under g_I than under g_E, and we discuss the role of the parameters.

We first find E's optimal renegotiation for some $\{r, l, a\}$ given that $Y = y$ and he has defaulted. E offers a renegotiation contract that solves the following maximization problem.

$$\begin{aligned}
\max_{\hat{r}\in[0,y],\hat{l}\in[0,1]} \quad & y - \hat{r} + (1-\hat{l})(1-ca)\left(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}u_2(\theta_H) + \left(1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}\right)u_2(\theta_L)\right) \\
\text{s.t.} \quad & \hat{r} + \hat{l}(1-ca)\left(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H) + \left(1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}\right)v_2(\theta_L)\right) \\
& \geq l(1-ca)\left(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H) + \left(1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}\right)v_2(\theta_L)\right) \qquad \text{IR}_2
\end{aligned}$$

The constraint IR_2 must be satisfied for I to accept the renegotiation, given the originally contracted collateral as her outside option. The solution is $\hat{l} = 0$ and

$$\hat{r} = l(1-ca)\left(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H) + \left(1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}\right)v_2(\theta_L)\right).$$

If the entrepreneur renegotiates, he pays off the investor rather than liquidating, since the project is more valuable under the entrepreneur's ownership than under the investor's ownership ($u_2(\theta) > v_2(\theta)$). The entrepreneur's interim expected payoff is

$$u_1(p, y) = y + (1 - ca)\left(\frac{a\epsilon + \bar{\theta}}{2\bar{\theta}}(u_2(\theta_H) - lv_2(\theta_H)) + \left(1 - \frac{a\epsilon + \bar{\theta}}{2\bar{\theta}}\right)(u_2(\theta_L) - lv_2(\theta_L))\right).$$

The investor's interim expected payoff is $v_1(p, y) = l(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H) + (1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}})v_2(\theta_L)) - K$.

If $Y = y$ and E does not default, then the initial contract will remain in effect. The entrepreneur's interim expected payoff is

$$u_1(p, y) = y - r + (1 - ca)\left(\frac{a\epsilon + \bar{\theta}}{2\bar{\theta}}u_2(\theta_H) + \left(1 - \frac{a\epsilon + \bar{\theta}}{2\bar{\theta}}\right)u_2(\theta_L)\right).$$

The investor's interim expected payoff is $v_1(p, y) = r - K$.

Based on these payoffs, E's default decision when $Y = y$ is $d = 1$ whenever $r > l(1 - ca)(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H) + (1 - \frac{a\epsilon+\bar{\theta}}{2\bar{\theta}})v_2(\theta_L))$.[5] He defaults if the originally agreed upon repayment is more expensive than paying off the investor's collateral value.

If Y=0, there is no room for renegotiation; the collateral l is turned over to I. The investor's interim expected payoff is $v_1(p, 0) = l(\frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})}v_2(\theta_H) + (1 - \frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})})v_2(\theta_L)) - K$. The entrepreneur's interim expected payoff is $u_1(p, 0) = (1-l)(\frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})}u_2(\theta_H) + (1 - \frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})})u_2(\theta_L))$.

We now solve for equilibrium payoffs under governance g_I. First, I's optimal a for any $\{r, l\}$ is $a = 0$, as is immediate from her maximization problem.

[5] The assumption that E breaks ties in favor of not defaulting does not effect our results on equilibrium total surplus.

$$\max_{a\in\{0,1\}} \quad \bar{\theta}\min\left\{l(1-ca)\left(\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}v_2(\theta_H)+\left(1-\frac{a\epsilon+\bar{\theta}}{2\bar{\theta}}\right)v_2(\theta_L)\right), r\right\}$$
$$+\,l(1-ca)(1-\bar{\theta})\left(\frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})}v_2(\theta_H)+\left(1-\frac{1-a\epsilon-\bar{\theta}}{2(1-\bar{\theta})}\right)v_2(\theta_L)\right)$$

Increasing a from 0 to 1 makes I's interim expected payoff lower after low output and higher after high output. However, her interim expected payoff after high output is capped by the intially agreed upon repayment, r. If r is low, changing from $a = 0$ to $a = 1$ hurts the investor even without considering depreciation. If r is high, then the expected value of collateral is unchanged between $a = 0$ and $a = 1$, but collateral is depreciated at a rate c when $a = 1$. Since the investor gets no benefit from high a, she will always choose $a = 0$ to avoid costly depreciation.

In the initial contracting stage under governance g_I, E offers a contract $\{r, l\}$ that solves the following maximization problem.

$$\max_{r\geq 0, l\in[0,1]} \quad \bar{\theta}y+\frac{1}{2}(u_2(\theta_H)+u_2(\theta_L))-l\left(\frac{1-\bar{\theta}}{2}\right)(u_2(\theta_H)+u_2(\theta_L))$$
$$-\min\left\{\bar{\theta}\frac{l}{2}(v_2(\theta_H)+v_2(\theta_L)),\ \bar{\theta}r\right\}$$
$$\text{s.t.} \quad \min\left\{\bar{\theta}\frac{l}{2}(v_2(\theta_H)+v_2(\theta_L)),\ \bar{\theta}r\right\}$$
$$+\,(1-\bar{\theta})\frac{l}{2}\left(v_2(\theta_H)+v_2(\theta_L)\right)\geq K \qquad \text{IR}_1$$

The solution is $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$ and $r \geq K$. The entrepreneur would like to offer as little collateral as possible. Since he has the ability to renegotiate, promised payments r in excess of the collateral value after high output are not credible. The entrepreneur therefore must offer I collateral that has expected value of at least K.[6] Lemma 5 gives E's equilibrium payoff from this contract.

Lemma 5. *Under governance g_I, the entrepreneur's ex ante expected equilibrium payoff is*

$$u(g_I) = \bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L)) - \left(\frac{K}{v_2(\theta_H) + v_2(\theta_L)}\right)\left(\bar{\theta}(v_2(\theta_H) + v_2(\theta_L)) + (1-\bar{\theta})(u_2(\theta_H) + u_2(\theta_L))\right).$$

If the entrepreneur were not liquidity constrained, then his expected utility would be $\bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L)) - K$, i.e. the expected value of the project net of its initial cost. However, in equilibrium foreclosure occurs with positive probability, which is inefficient. Thus, the cost K is scaled by a factor greater than 1.

Before solving for equilibrium payoffs under governance g_E, we find the entrepreneur's optimal initial contract and payoff holding $a = 1$ fixed, which solves the following maximization problem.

$$\begin{aligned}
\max_{r\geq 0, l\in[0,1]} \quad & \bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L))(1-c) - \frac{l}{2}(1-c)((1-\theta_H)u_2(\theta_H) + (1-\theta_L)u_2(\theta_L)) \\
& - \min\left\{(1-c)\frac{l}{2}(\theta_H v_2(\theta_H) + \theta_L v_2(\theta_L)),\ \bar{\theta}r\right\} \\
\text{s.t.} \quad & \min\left\{(1-c)\frac{l}{2}(\theta_H v_2(\theta_H) + \theta_L v_2(\theta_L)),\ \bar{\theta}r\right\} \\
& + (1-c)\frac{l}{2}\left((1-\theta_H)v_2(\theta_H) + (1-\theta_L)v_2(\theta_L)\right) \geq K \qquad \text{IR}_1
\end{aligned}$$

The solution is $l = \frac{2K}{(1-c)(v_2(\theta_H)+v_2(\theta_L))}$ and $r \geq K\frac{(\theta_H v_2(\theta_H)+\theta_L v_2(\theta_L))}{\bar{\theta}(v_2(\theta_H)+v_2(\theta_L))}$. Much like when a is fixed at $a = 0$, when $a = 1$ the entrepreneur must promise collateral with expected value to cover cost K. In this case, capital will be depreciated so that E must promise a higher l.

[6] This result is similar to Hart & Moore (1998), as for fixed a this game is a special case of their model.

E's payoff is:

$$\begin{aligned}\bar{u}(1) &= \bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L))(1-c) \\ &\quad - \left(\frac{K}{v_2(\theta_H) + v_2(\theta_L)}\right)\left(\bar{\theta}(v_2(\theta_H) + v_2(\theta_L)) + (1-\bar{\theta})(u_2(\theta_H) + u_2(\theta_L))\right) \\ &\quad + \left(\frac{K\epsilon}{v_2(\theta_H) + v_2(\theta_L)}\right)\left((u_2(\theta_H) - u_2(\theta_L)) - (v_2(\theta_H) - v_2(\theta_L))\right),\end{aligned}$$

where we write $\bar{u}(a)$ to denote E's optimal payoff when action a is exogenously fixed.

There are two distinctions between this payoff and $\bar{u}(0) = u(g_I)$. First, when $a = 1$ the entrepreneur's expected time 2 payoff decreases due to depreciation. The second term is scaled by $(1 - c)$. Second, since the entrepreneur pays off the investor and keeps the collateral when time 1 output is high and turns it over when time 1 output is low, choosing $a = 1$ shifts the probability of foreclosure to the low state from the high state relative to when $a = 0$. As when $a = 0$, the investor still sometimes receives collateral in equilibrium, which is inefficient. However, that loss of efficiency is mitigated (or exacerbated) by shifting the probability of foreclosure between states. The last term of $\bar{u}(1)$ captures the change in the entrepreneur's payoff from this shift. We subtract $\bar{u}(1)$ from $u(g_I)$ to get the following condition, which we call Condition 1.

Condition 1.

$$2K\epsilon\Big((u_2(\theta_H) - u_2(\theta_L)) - (v_2(\theta_H) - v_2(\theta_L))\Big) < c\Big((u_2(\theta_H) + u_2(\theta_L))(v_2(\theta_H) + v_2(\theta_L)\Big)$$

Lemma 6. *$\bar{u}(1) < u(g_I)$ iff Condition 1 is satisfied.*

Notice that the condition is always satisfied if $u_2(\theta_H) - u_2(\theta_L) < v_2(\theta_H) - v_2(\theta_L)$, in other words, if the collateral value $v_2(\theta)$ is more sensitive to the state than the production value $u_2(\theta)$. In that case, the entrepreneur is better off when $a = 0$ than when $a = 1$. The intuition is straightforward. Choosing $a = 1$ increases posterior beliefs on the state after high output and decreases posterior beliefs on the state after low output. This means that in the high state, the entrepreneur's continuation value increases, but the amount he must pay off the investor increases even faster. In the low state, he doesn't pay off the investor so his continuation value is unaffected by the value of collateral.[7]

If instead $u_2(\theta_H) - u_2(\theta_L) > v_2(\theta_H) - v_2(\theta_L)$, then in the high state the entrepreneur's continuation value increases more than the amount he must pay off the investor. Thus it is beneficial for him to chooose $a = 1$. If this benefit is large enough relative to the depreciation cost, then $\bar{u}(1) > \bar{u}(0)$. The distinction between these two cases highlights a fundamental aspect of liquidity constraints and renegotiation. Choosing $a = 1$ always leads to more precise information at time 1 than $a = 0$, but because of liquidity constraints, that information can only be used to shift foreclosure probability from the high state to the low state, not from the low state to the high state.

We now turn to equilibrium under governance g_E. We solve for optimal a, which we write as $a^*(r, l)$, for a given contract $\{r, l\}$. We restrict attention to $l \geq \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$ since equilibrium l can never be lower than that value. We also restrict attention to $r \geq \max\{(1-c)\frac{l}{2}(\theta_H v_2(\theta_H) + \theta_L v_2(\theta_L)),\ \frac{l}{2}\bar{\theta}(v_2(\theta_H) + v_2(\theta_L))\}$. In this range, E will always (weakly) prefer to default after high output. This restriction does not affect E's payoff in the payoff-maximizing equilibrium.

[7]In this case, Condition 1 will always be satisfied, meaning $a = 0$ is efficient, but Condition 2 will always be violated. E will optimally choose $a = 0$. We discuss this further below.

Fixing some some $\{r, l\}$ satisfying these restrictions, E's expected payoff from choosing $a = 0$ is:

$$\bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L)) - \frac{l}{2}\big((1-\bar{\theta})(u_2(\theta_H) + u_2(\theta_L)) - \bar{\theta}(v_2(\theta_H) + v_2(\theta_L))\big) \tag{3.1}$$

E's expected payoff from choosing $a = 1$ is:

$$\bar{\theta}y + \frac{1}{2}(u_2(\theta_H) + u_2(\theta_L))(1-c) - \frac{l}{2}(1-c)\big((1-\theta_H)u_2(\theta_H) + (1-\theta_L)u_2(\theta_L) + \theta_H v_2(\theta_H) + \theta_L v_2(\theta_L)\big) \tag{3.2}$$

E chooses $a = 1$ whenever Expression 3.2 is greater than Expression 3.1.[8] We subtract Expression 3.1 from Expression 3.2 at the optimal level of collateral under governance g_I, $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$, to obtain the following condition, which we call Condition 2.

Condition 2.

$$\begin{aligned}
&2K\epsilon\Big((u_2(\theta_H) - u_2(\theta_L)) - ((v_2(\theta_H) - v_2(\theta_L))\Big) > \\
&c\Big((u_2(\theta_H) + u_2(\theta_L))(v_2(\theta_H) + v_2(\theta_L)) \\
&\quad + 2K\epsilon((u_2(\theta_H) - u_2(\theta_L)) - ((v_2(\theta_H) - v_2(\theta_L))) \\
&\quad - 2K(\bar{\theta}(v_2(\theta_H) + v_2(\theta_L)) + (1-\bar{\theta})(u_2(\theta_H) + u_2(\theta_L)))\Big)
\end{aligned}$$

Lemma 7. *For $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$ and $r \geq K\frac{(\theta_H v_2(\theta_H)+\theta_L v_2(\theta_L))}{\bar{\theta}(v_2(\theta_H)+v_2(\theta_L))}$, $a^*(r, l) = 1$ iff Condition 2 is satisfied.*

Proof. For $r \geq K\frac{(\theta_H v_2(\theta_H)+\theta_L v_2(\theta_L))}{\bar{\theta}(v_2(\theta_H)+v_2(\theta_L))}$, E's optimal default choice is $d = 1$ for $a = 0$ and $a = 1$, so payoffs are as in Expression 3.1 and Expression 3.2, respectively. The result follows. $\square$

[8] We assume E breaks ties in favor of $a = 0$, which is without loss of generality for our results.

The result can be extended to higher l, but for our purposes it suffices to provide the condition such that E cannot credibly offer the same collateral under g_E as it would under g_I, which is $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$. When Condition 2 is satisfied, at the equilibrium collateral level under g_I, E prefers to choose $a = 1$. The gains from aligning output with the state offset the losses from depreciation. Notice that if Condition 2 is violated, Condition 1 must be satisfied.[9] This is because fixing some a, E's payoff is decreasing in l. If at $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$, E prefers $a = 0$ to $a = 1$, then E also prefers $a = 0$ and that collateral level to $a = 1$ and collateral $l = \frac{2K}{(1-c)(v_2(\theta_H)+v_2(\theta_L))}$, the minimum collateral I will accept when $a = 1$.

Notice that if the entrepreneur could fund the project himself, he would never choose $a = 1$. The state-dependent production process does not on average perform better than the safe production process, and it results in costly depreciation. The entrepreneur only chooses $a = 1$ because of the asymmetric effect of his time 1 liquidity constraint at different levels of output.

The following proposition states that Condition 1 and Condition 2 together characterize when the entrepreneur is better off under governance g_I than under governance g_E.

Proposition 10. *$u(g_I) > u(g_E)$ iff Condition 1 and Condition 2 are satisfied.*

Proof. See Appendix C. □

If Condition 2 is satisfied, then E cannot credibly promise action $a = 0$ at contracted collateral $l = \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$. In equilibrium under g_E, he must offer more collateral, which fixing his action makes him worse off. However, if he chooses $a = 1$ then he gains the benefit of turning over collateral more often in the low state and less often in the high state. When Condition 1 is satisfied, that benefit is not large enough to make up for the increased

[9]If instead Condition 2 is satisfied, Condition 1 may or may not be satisfied, depending on the parameters.

collateral he must offer. Thus, when both conditions are satisfied the benefit from $a = 1$ is high enough that it prevents him from playing the same equilibrium as under g_I, but not so high that it compensates for the inefficiency of higher collateral. In that case, it is better for the investor to have governance.

As discussed above, Condition 1 is satisfied if $u_2(\theta_H) - u_2(\theta_L) \leq v_2(\theta_H) - v_2(\theta_L)$. Similarly, Condition 2 is violated when this inequality holds. At all collateral levels, if the collateral value of capital is more sensitive to the state than the production value of capital, then the entrepreneur is better off not aligning output to the state. It is more efficient to turn collateral over to the investor in high states than in low states, and choosing $a = 0$ maximizes the probability of that. Both the investor and the entrepreneur prefer $a = 0$, and $\bar{u}(0) = u(g_E) = u(g_I)$.

If instead $u_2(\theta_H) - u_2(\theta_L) > v_2(\theta_H) - v_2(\theta_L)$, but $(u_2(\theta_H) - u_2(\theta_L)) - (v_2(\theta_H) - v_2(\theta_L))$ is not too large, then both conditions are satisfied at intermediate values of ϵ and c. When ϵ is very low and c is very high, the entrepreneur will choose $a = 0$ in equilibrium, so that $\bar{u}(0) = u(g_E) = u(g_I)$. When ϵ is very high and c is very low, then the gains from $a = 1$ are worth the increased equilibrium collateral, so $\bar{u}(1) = u(g_E) > u(g_I) = \bar{u}(0)$. However, at intermediate ranges of ϵ and c, the entrepreneur cannot credibly choose $a = 0$, but would prefer to be able to commit to do so rather than be forced to offer high collateral. In that region, $\bar{u}(0) = u(g_I) > u(g_E)$.

When $u_2(\theta_H) - u_2(\theta_L) > v_2(\theta_H) - v_2(\theta_L)$, both conditions are also satisfied at intermediate values of K. When the initial cost of capital is high, collateral in equilibrium must also be high to compensate the investor. When collateral is high, it becomes relatively more important to shift foreclosure to the low state. Thus, K functions similarly to ϵ. At very low values Condition 2 is violated, and at very high values Condition 1 is violated.

3.4. Conclusion

We have extended a very stylized model of debt and renegotiation to accomodate learning and governance over production. We suppress potential confounding tradeoffs in order to isolate the relationship between governance and risk-taking in production. However, the model can easily accommodate other features, such as costly a, different expected output under different capital or governance structures, and continuous output and capital.

The model could also be altered to have a slightly different interpretation. Rather than comparing payoffs between two exogenous governance structures, we could instead allow the initial contract to consist of a repayment, collateral, and governance, $\{r, l, g\}$. Then, rather than characterizing when E is better off under the investor's governance, our two conditions would characterize when g_I would obtain uniquely in equilibrium. Under this formulation, our model gives one reason why entrepreneurs might contract away more direct control over the production process to their investors: in order to get better terms on their debt contracts when they can't be trusted not to choose costly production processes.

We take as exogenous that either the entrepreneur's or the investor's continuation payoff is more sensitive to the state. An important question that we have not addressed is when each of these cases would occur. One way to answer that question is to embed our model into a larger market model. For example, suppose when the investor seizes capital from foreclosure, she faces a market of entrepreneurs who would like to purchase it (perhaps using funds raised from other investors). The value of capital to any of these entrepreneurs is lower than the value to the original entrepreneur, but the size of that difference may depend on the state. Highly effective capital (a high state) may be more "transferable" to a new entrepreneur than less effective capital (a low state), or vice versa. This would correspond to the investor's continuation payoff being more or less sensitive to the state (or to beliefs about the state) than the original entrepreneur's continuation payoff. We leave to future research the question of what types of capital exhibit which type of transferability.

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APPENDIX A

Omitted Proofs: Chapter 1

A.1. Proof of Theorem 3

Let $L_N^* = 0, H_N^* = H^{**}$ be the persuasion strategy in the most-informative PBE_N^* at priors $p_0 < \hat{p}_N$. Let L', H' be the persuasion strategy in the most-informative PBE_N^* at some prior $p_0' \geq H^{**}$.

$$\bar{V}^*(p_0') \geq \frac{H' - p_0'}{H' - L'} V^*(L') + \frac{p_0' - L'}{H' - L'} V^*(H') \geq \frac{H' - p_0'}{H' - L'} V_N^*(L') + \frac{p_0' - L'}{H' - L'} V_N^*(H') = \bar{V}_N^*(p_0')$$

At all priors less than H^{**}, $\bar{V}_N^*(p_0)$ increases linearly in the prior to $V_N^*(H^{**})$, while $\bar{V}^*(p_0) = 0$ for priors less than some $\hat{p} \leq \hat{p}_N \leq H^{**}$, then increases linearly to $V^*(H^{**}) \geq V_N^*(H^{**})$. If $V_N^*(H^{**}) > 0$ then $\bar{V}^*(p_0)$ crosses $\bar{V}_N^*(p_0)$ once from below. Call that crossing p_0^R. If instead $V_N^*(H^{**}) > 0$ then let $p_0^R = 0$

At $p_0' \geq H^{**}$, $\bar{U}_N^*(p_0') \geq U_N^*(p_0') \geq \alpha = \bar{U}^*(p_0')$. At all priors less than H^{**}, $\bar{U}_N^*(p_0)$ increases linearly in the prior to $U_N^*(H^{**})$, while $\bar{U}^*(p_0)$ increases linearly to α at priors less than some $\hat{p} \leq H^{**}$, then is constant at α at all higher priors. If $\frac{\hat{p}}{H^{**}} U_N^*(H^{**}) < \alpha$ then $\bar{U}_N^*(p_0)$ crosses $\bar{U}^*(p_0)$ once from below. Call that crossing p_0^S. If instead $\frac{\hat{p}}{H^{**}} U_N^*(H^{**}) \geq \alpha$ then let $p_0^S = 0$

A.2. Proof of Proposition 6

If the principal chooses $e = 0$ for every posterior her payoff is continuous in p. The same is true for $e = 1$. Therefore if (i) at $p = 0$ the principal strictly prefers $e = 0$, and (ii) at

$p = 1$ the principal strictly prefers $e = 1$, then there exists a region where $e = 0$ with strictly positive width to the left of a region where $e = 1$ with strictly positive width. (i) can be restated as follows:

$$\begin{aligned} & g(1,0) - g(0,0) - \frac{g(1,0)k}{g(1,0) - g(0,0)} < 0 \\ \iff & \frac{(g(1,0) - g(0,0))^2}{g(1,0)} < k \\ \iff & \frac{(f(1,0) - f(0,0))^2}{f(1,0)} < k. \end{aligned}$$

(ii) can be restated as follows:

$$\begin{aligned} & g(1,1) - g(0,1) - \frac{g(1,1)k}{g(1,1) - g(0,1)} > 0 \\ \iff & \frac{(g(1,1) - g(0,1))^2}{g(1,1)} > k \\ \iff & \frac{(f(1,1) - f(0,1))^2}{f(1,1)} > k. \end{aligned}$$

A.3. Proof of Proposition 7

Fix $e = 1$. Then $R(p) = \frac{kg(0,p)}{g(1,p)-g(0,p)}$. The signs of the derivatives are the same for any $k > 0$. Let $k = 1$. If $f(1,1) - f(0,1) < f(1,0) - f(0,0)$, then:

$$\frac{\partial}{\partial p}[R(p)] = \frac{g(1,p)(f(0,1) - f(0,0)) - g(0,p)(f(1,1) - f(1,0))}{(g(1,p) - g(0,p))^2} > 0.$$

If $\frac{\partial}{\partial p}[R(p)] > 0$, then:

$$\begin{aligned} & \frac{\partial^2}{\partial p^2}[R(p)] = 2\frac{\partial}{\partial p}[R(p)] * \frac{(f(1,0) - f(0,0)) - (f(1,1) - f(0,1))}{g(1,p) - g(0,p)} > 0 \\ & \Leftrightarrow f(1,1) - f(0,1) < f(1,0) - f(0,0). \end{aligned}$$

APPENDIX B

Omitted Proofs: Chapter 2

B.1. Proof of Lemma 2

The result is a special case of Theorem 2 in Persico (2000). It suffices to show that the market research problem satisfies the assumptions of that Theorem.

First we show that for each firm i, signal s_i is *affiliated* with α. Two random variables S and A with joint density $f(s,\alpha)$ are affiliated if for any realizations $s^{'} > s$ and $\alpha^{'} > \alpha$, $f(s^{'},\alpha^{'})f(s,\alpha) \geq f(s,\alpha^{'})f(s^{'},\alpha)$.

Using the probability density functions of normal distributions with equal variance, for any two states $\alpha^{'} > \alpha$ and any two signal realizations $s^{'} > s$, we can see that

$$\frac{f(s^{'},\alpha^{'})}{f(s,\alpha^{'})} = \frac{\exp\left(-\frac{(s^{'}-\alpha^{'})^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha^{'})^2}{2v}\right)} = \exp\left(\frac{(2\alpha^{'} - s^{'} - s)(s^{'} - s)}{2v}\right)$$

$$> \exp\left(\frac{(2\alpha - s^{'} - s)(s^{'} - s)}{2v}\right) = \frac{\exp\left(-\frac{(s^{'}-\alpha)^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha)^2}{2v}\right)} = \frac{f(s^{'},\alpha)}{f(s,\alpha)}.$$

So by definition of affiliation, s_i is affiliated with α.

Given two signals S^{t_1} and S^{t_2}, we say that S^{t_1} is more *accurate* than S^{t_2} if $F^{t_1^{-1}}(F^{t_2}(s|\alpha)|\alpha)$ is nondecreasing in α, for every s; where $F^{t_1}(\cdot|\cdot)$ and $F^{t_2}(\cdot|\cdot)$ are cumulative distibution functions for S^{t_1} and S^{t_2}, respectively. [See Lehmann (1988).] For each firm i, the accuracy of its signal s_i is increasing in t_i. [See example 4 in Section 3.2 of Persico (1996).]

By inspection, for each firm i, $u_i(\alpha, p_i) \equiv \int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha) dF^{t_{-i}}(s_{-i}|\alpha)$ is differentiable in p_i, and the optimal action $p_i^*(s_i, t_i)$ is differentiable in s_i and t_i.

Finally, the cdf of the normal distribution with variance v_i and state α is

$$F(x|\alpha, v_i) = \int_{-\infty}^{x} \left(\frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{(z-\alpha)^2}{2v_i} \right) \right) dz,$$

which is differentiable with respect to v_i and continuous in α. Now, because $v_i = \frac{V_\alpha}{t_i} - V_\alpha$ is differentiable in t_i, it follows that $F(x|\alpha, v_i)$ is differentiable in t_i.

Thus, the conditions of Theorem 2 in Persico (2000) are satisfied by the market research problem.

B.2. Derivation of Equation 2.1

At a given signal s_i and with accuracy t_i, denote firm i's optimal price p_i^* as in Vives (1984). Define p_{-i}^* similarly.

$$u_\gamma(\alpha, p_i^*) = \int_{s_{-i}=-\infty}^{\infty} p_i^* q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF(s_{-i}|\alpha)$$

Integrating by parts:

$$\begin{aligned}
u_\gamma(\alpha, p_i^*) &= p_i^* \left\{ \left[q_i(p_i^*, p_{-i}^*, \alpha, \gamma) F(s_{-i}|\alpha) \right]_{s_{-i}=-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \right\} \\
&= p_i^* \Bigg\{ \left(q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma) F(\infty|\alpha) - q_i(p_i^*, p_{-i}^*(-\infty), \alpha, \gamma) F(-\infty|\alpha) \right) \\
&\quad - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \Bigg\} \\
&= p_i^* q_\infty - p_i^* \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i}
\end{aligned}$$

Where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$. We take the derivative with respect to s_i. Note that when pricing functions are as in the equilibrium of Vives (1984), both $\frac{\partial q_i}{\partial p_{-i}}$ and $\frac{\partial p_{-i}^*}{\partial s_{-i}}$ are independent of s_{-i}.

$$\frac{\partial u_\gamma(\alpha, p_i^*)}{\partial s_i} = \left(q_\infty + p_i^* \frac{\partial q_\infty}{\partial p_i}\right) \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha)\right) ds_{-i} \right\}$$

We take the derivative with respect to α. Note that $\frac{\partial q_i(\infty)}{\partial p_i}$, p_i^*, and $\frac{\partial p_i^*}{\partial s_i}$ are independent of α, and that conditional on some realization α' of the state signals are normally distributed with mean α' and some variance that is independent of α. The derivative is

$$\begin{aligned}
\frac{\partial^2 u_\gamma(\alpha, p_i^*)}{\partial \alpha \partial s_i} &= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int_{-\infty}^{\infty} \left(F_\alpha(s_{-i}|\alpha)\right) ds_{-i} \right\} \\
&= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int_{-\infty}^{\infty} \left(-f(s_{-i}|\alpha)\right) ds_{-i} \right\} \\
&= \frac{\partial q_\infty}{\partial \alpha} \frac{\partial p_i^*}{\partial s_i} + \left(\frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \right).
\end{aligned}$$

B.3. Proof of Lemma 3

By Lemma 2, it suffices to show that $\exists t'$ s.t. $\frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0 \ \forall t < t'$.

$$\begin{aligned}
CMP(t,\gamma) + CRD(t,\gamma) &= \frac{1-\gamma}{2-\gamma t} t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2} t^2 \\
\frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] &= \frac{t\left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right)}{(2 - t\gamma)^3} \\
\therefore \frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0 &\Leftrightarrow \left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right) < 0.
\end{aligned}$$

Suppose $t \leq \frac{1}{2}$. Then $t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4$ is maximized on the domain $0 \leq \gamma < 1$ at $\gamma = 0$. At $\gamma = 0$

$$t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4 = 4t - 4 < 0.$$

The result follows.

B.4. Proof of Lemma 4

$$CMP(t,\gamma) + CRD(t,\gamma) = \frac{1-\gamma}{2-\gamma t}t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2$$

At $\gamma = 0$, $CMP(t,0) + CRD(t,0) = \frac{t}{2}$. Since $\lim_{\gamma\to 1}\left[CMP(t,\gamma) + CRD(t,0)\right] = 0$ and both $CMP(t,\gamma)$ and $CRD(t,\gamma)$ are continuous, it must be that if there exists some γ s.t. $CMP(t,\gamma) + CRD(t,\gamma) > \frac{t}{2}$, then there exist two values of γ such that $CMP(t,\gamma) + CRD(t,\gamma) = \frac{t}{2}$. There are two solutions γ^* for $CMP(t,\gamma^*) + CRD(t,\gamma^*) = \frac{t}{2}$:

$$\gamma^* = \frac{1}{2} - \frac{t}{4} \pm \frac{\sqrt{t^3 - 4t^2 + 36t - 32}}{4\sqrt{t}}$$

If $t < 1$, the solutions are real-valued and interior exactly when

$$t^3 - 4t^2 + 36t - 32 \geq 0.$$

When $t = 1$, the smaller of the two solutions is not interior, but the higher solution is interior. The left hand side of this expression is increasing in t, strictly negative at $t = 0$ and strictly positive at $t = 1$. The results follow.

APPENDIX C

Omitted Proof: Chapter 3

C.1. Proof of Proposition 10

First, Condition 1 and Condition 2 are jointly necessary for $u(g_I) > u(g_E)$. If Condition 2 fails, then by Lemma 7 E can choose the same $\{r, l\}$ under governance g_E as he does under governance g_I and receive payoff $u = u(g_I)$. Suppose Condition 2 is satisfied. Then for any $l \geq \frac{2K}{v_2(\theta_H)+v_2(\theta_L)}$ and $r \geq (1-c)\frac{l}{2}(\theta_H v_2(\theta_H) + \theta_L v_2(\theta_L))$, $a^*(l, r) = 1$. Therefore, $u(g_E) \geq \bar{u}(1)$. If Condition 1 is not satisfied, then by Lemma 6 $u(g_E) \geq \bar{u}(1) > u(g_I)$. Thus, both conditions are necessary for $u(g_I) > u(g_E)$.

Second, Condition 1 and Condition 2 are jointly sufficient for $u(g_I) > u(g_E)$. Suppose Condition 2 is satisfied. Then if $a = 0$ in equilibrium under governance g_E, it must be that $u(g_E) < u(g_I)$. Suppose that $a = 1$ in equilibrium under governance g_E. Then $u(g_E) = \bar{u}(1)$. By Lemma 6, when Condition 1 is satisfied $\bar{u}(1) < \bar{u}(0) = u(g_I)$. Thus, Condition 1 and Condition 2 are necessary and sufficient for $u(g_I) < u(g_E)$.