Wing Shape Optimization

From optimization

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Introduction

Aerospace collectively represents one of the most sophisticated technological endeavors and largest markets in the world. Coming with substantial costs, nearly every aspect of the industry, from aircraft design to material selection to operation, has been optimized in at least one way. A critical design consideration in any aircraft is its three-dimensional wing shape; this will be explored as an example of optimization's role and importance in industry. In designing the wing of an airplane, several fluid dynamic concepts come into play. The two most prominent ones are lift and drag, which correspond to a wing's ability to fly the airplane against gravity without wasting energy by moving forward. The behavior of an airfoil, or cross-section, of wings in air is thoroughly studied to make efficient airplanes. Secondly, the overall shape of the wing can be optimized as well. Considerations generally include lift and drag, as with the airfoil, but also noise and stability.

Background

Some of the earliest wing optimization attempts can be traced back to the late 1500s. Newton applied variational calculus to drag minimization problems. The next leap in the field, however, didn't come until the 1960s when numerical solutions became available with advances in hardware. The 70s saw the application of gradient-based models for aerodynamic shapes, solving for up to 11 different variables in a three-dimensional design space. Later methods used the Stokes flow equations with Euluer's incompressible medium equations to solve for sing airfoils (or cross-sections). When iterative computational fluid dynamics (CFD) became feasible for small-mesh models, the solutions became even more robust[1] (http://www.sciencedirect.com/science/article/pii/S1018363913000159) .

Wing Types

Because of the limited types of aircraft missions (military and commercial), two main wing platforms dominate the design space.

Besides being iconic, the Boeing 787's wing shape makes it incredibly efficient.

Traditional

Most commercial and military aircraft adopt this wing style, where wings are separate from and extend out of the fuselage. This wing orientation, unlike the blended wing body, typically allows for faster speeds (which improve lift) at the expense of smaller wings (which typically hinder lift).

Blended Wing Body

Blended wing body aircraft are difficult to manufacture, but benefit from having a 100% lifting body configuration.

The blended wing body configuration is highly fuel efficient as it maximizes the lift of the entire airplane while also improving cabin size. Therefore, the commercial implications of this design are drastic. The military has already begun using this platform in the iconic B-2 bombers, along with several smaller drones.

Alternatives

A popular enhancement on traditional wing design is the 'winglet.' When normally optimizing wing dimensions for a particular load and operating at some cruising speed, there are constraints set on wing length that relate to average gate sizes. Since extending the length of the wing allows for significantly more area and improves stability, this constraint is almost always active. To continue getting more wing "length" while allowing planes to fit in the airport gates, winglets were quickly introduced.

Winglets have recently become a mainstay in wing design, as they increase the functional area of a wing without exceeding the maximum width constraints imparted by airport gate sizes.

Objectives

Even when meeting the numerous technical requirements to make an operable wing, there are several ways of gauging performance.

The lift coefficient, C_{lift}, corresponds to the wings' ability to keep a given weight, W , at a horizontal cruising speed , V_s, called the stall speed. For commercial flights, higher lift coefficients of rouhgly 3.0-3.3 are desirable for lifting heavy loads at low speeds during takeoff and ensuring longer periods of safe flight during emergencies. Military aircraft, by contrast, are willing to sacrifice their lift coefficients, going as low as 1.1, in order to have very low drag coefficients. Unlike commercial airplanes, military pilots usually have an eject seat so losing power is non-fatal making maximum speed more valuable than reliable flight.

Another performance consideration is stability. Unlike the lift coefficient, which solely pertains to vertical lift, some wing coefficients are more prone to sudden jerks than others. Again, commercial wings are designed to have significant stability which improves passenger comfort. Military ones, on the other hand, rely on instability to quickly maneuver in the air. Without sophisticated algorithms to keep the plane level, manually flying a modern military jet would be nearly impossible because of its designed tendency to quickly turn or dive at any perturbation.

Depending on the company and program, the most important quality of the wings may be the cost. Some airfoils, like highly cambered or supersonic double wedge sections, are substantially more difficult to manufacture. Therefore, it requires more skilled laborers and may result in a lower production efficiency. In light of that, many aerospace engineers must consider the financial burden of making a particular shape and weigh it against the marginal increase in performance.

Since there are so many potential objectives when designing a wing, most companies use multi-objective optimization. Therefore, the wing design chosen may not represent the "best" in any single trait, but it likely has the least net trade-offs. A common approach is a simple weighted sum or ratio between the several objectives into some macro-objective, e.g. the lift to drag ratio, C_{lift}/C_{drag}. Although the computational requirements are substantial, the largest problem with the method are actually associated with scaling each sub-objective.

It is not uncommon to find literature that claims to have found a Boeing Dreamliner wing with 8% less drag than the current design. While it may be true, it is exceedingly likely that the authors used a simplified objective function that neglects other important considerations such as manufacturing costs, serviceability, etc.

Design space and variables

An aircraft wing is geometrically complex and can be described by dozens of variables. Some of the most important are described here.

As already mentioned, the airfoil, or cross-section, of a wing plays a critical role in generating lift. Unlike the Wright Brothers who relied on high-area, low-efficiency bi-plane wings, modern optimization tools allow for complicated and variable airfoils to be justified. Since the airfoil is an oriented two-dimensional form, several variables describes its shape. The chord line describes the length from leading edge to trailing edge via the horizontal reference of the airfoil. The optimum angle of attack, AOA, represents the steepness of wing with respect to the horizontal motion direction. In many situations, the AOA is taken as a parameter for the entire wing, although it is possible to calculate an optimum for each airfoil section.

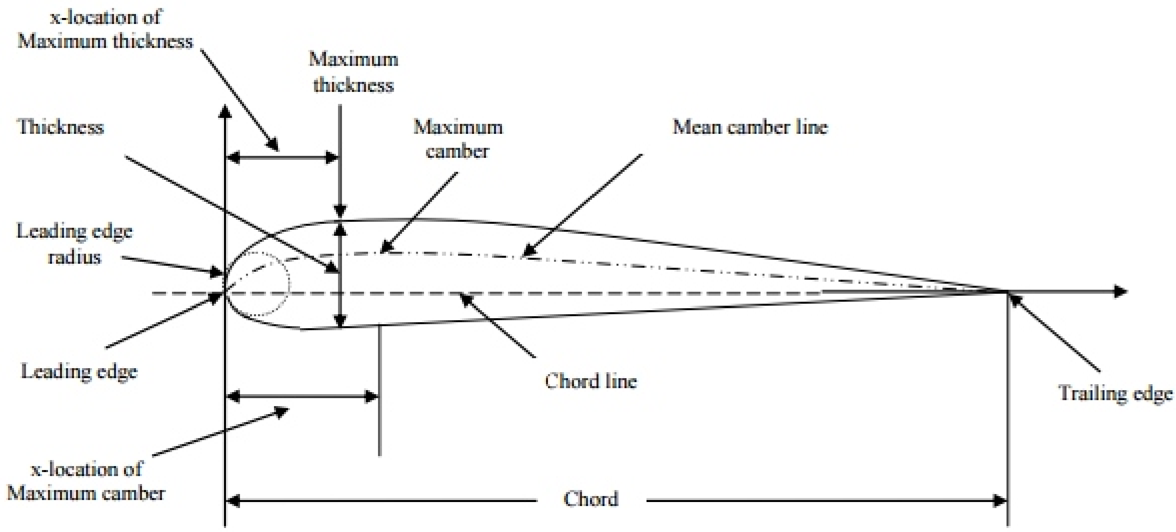
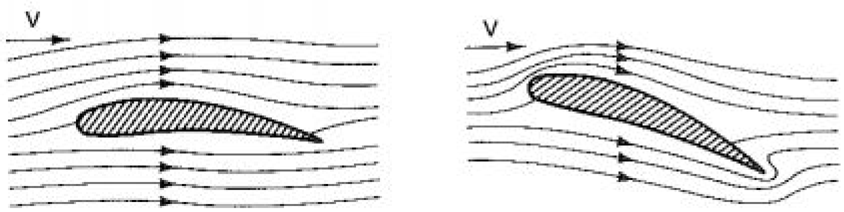


Figure 5.5. Airfoil geometric parameters



a. *Small angle of attack* b. *Large angle of attack*

Figure 5.6. Flow around an airfoil

Wings with high sweep angles are becoming more prevalent as they often impart higher fuel efficiency. However, airplane designs are limited by airport gate sizes which constrain perpendicular wing length. The efficiency effect is so tantalizing for airlines that some airplanes in development, like the Boeing 777-X, will have wing extensions that fold away when parked in the gate. A more conventional approach to building a functionally longer wing (while still fitting in the gate) requires sweeping the wing away from the wing axis. Angling the wing forward does this, but comes with unacceptable instability. Therefore, most commercial planes sweep the wing rearwards.

Most wings are also angled up, or down with respect to the horizontal plane of the aircraft. This modification, called a dihedral angle, is used to increase the functional wing length and pitch stability.

The aspect ratio of the wing is closely related to the lifting area, both of which are critical variables in almost any objective function. In the most basic lift equations, the "footprint" of the wing is linearly proportional to the force:

$$L = k * v^2 * A * C_{lift}$$

Of course, complete lift calculations involve substantially more variables, including altitude and surface topography that relate to the aerodynamic boundary layer thickness.

Example models

Before beginning optimization of a wing shape, several assumptions about the basic structure are made. These typically include the number of wings, vertical position on the fuselage, and sometimes airfoil shape. An initial condition, based on lifting line theory, is designed by aerospace engineers as a starting point for the optimization. Then, in most cases, CFD is run to gather fundamental performance values (e.g. lift, drag, pitching moment) which are converted into an objective value. The next iteration makes a set of small changes to the geometry and repeats the process until the algorithm converges on a form with the best objective value. To check against a local solution, some approaches involve starting the model from several different, randomly generated points instead of the human-designed one.

A few different models from the literature are described below to provide some insight into how optimization problems are structured for wing shape.

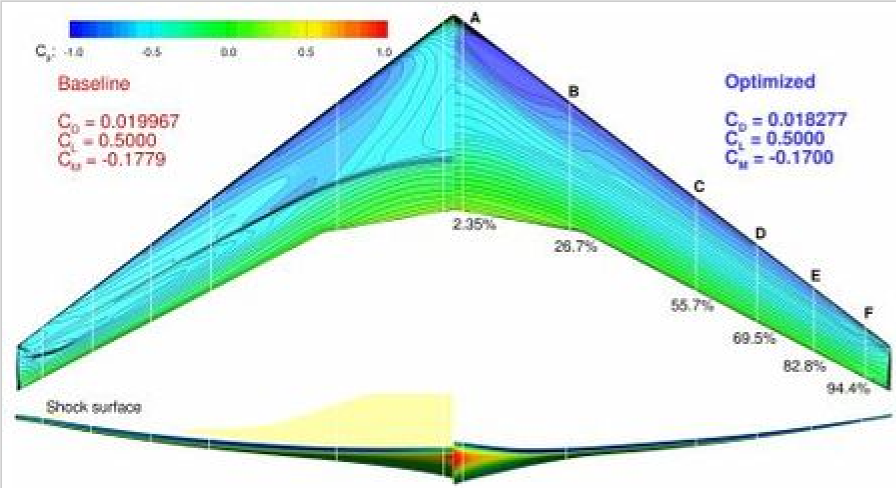
3D Wing Surface Optimization

The large number of potential variables requires careful consideration of computational resources and algorithms. While genetic algorithms are a candidate for global optimization of the shape, their dependence on many iterations (each performing CFD) makes it challenging. Therefore, the gradient-based optimizer with adjoint gradient evaluations was chosen by University of Michigan researchers to guide consecutive iterations[2] (http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19950018914.pdf) .

Minimize Drag Coefficient	$C_D = \frac{2 * DragForce}{fluidDensity * frontalArea * velocity^2}$
Decision Variables...	1. Angle of Attack (1 variable) 2. Surface shape (z-position) (720 variables)
Subject to...	1. Lift coefficient fixed at 0.5 2. Minimum thickness constraint 3. Minimum volume constrained 4. Wing root fixed position at fuselage 5. Trailing edge conformity constraint

Starting with the NASA developed "Common Research Model" wing (CRM), a sparse nonlinear optimizer (SNOPT) algorithm was chosen that employs gradient-based methods in solving large-scale problems. "Smooth, augmented Langrangian merit functions" and the Hessian of the Lagrangian are estimated from a quasi-Newton, adjoint-based method. The adjoint state numerical methods applies the dual problem formulation of the objective and constraints to more quickly converge on solutions.

In essence, this iterative process uses CFD in conjunction with minimization algorithms for a multi-objective model. A starting design is proposed, CFD performed, results analyzed, shape changed, and repeated.



Results from the University of Michigan study. Even when the starting point was a perturbed CRM wing, the resolved solutions were relatively close to one another in shape and merit value suggesting a reliable methodology.

The result was a design with 8.5% less drag and completely shock-free over the surface, indicating a robust process. Granted, the CRM starting point lacks some of the finesse of any wing on the market[3] (http://mdolab.engin.umich.edu/sites/default/files/Lyu_AIAAJ_ASO_2014_preprint.pdf) .

2D Airfoil Optimization

Minimize Drag to Lift Ratio	$C_D/C_L = \frac{\left(\frac{2*DragForce}{fluidDensity*frontalArea*velocity^2}\right)}{\left(\frac{LiftForce}{SmeatonCoefficient*velocity^2*LiftingArea}\right)}$
Decision Variables...	1. Angle of Attack 2. Surface shape (z-position)
Subject to...	1. Minimum allowable lift coefficient (0.3) 2. Minimum and Maximum moment coefficients (-0.1 - 0.0) 3. Maximum allowable AOA 5 degrees 4. Maximum thickness constraints 5. Camber distribution (aka curvature limitations) 6. Minimum leading edge radii

At Penn State University, an evolutionary/genetic algorithm was applied to find optimum airfoil sections. By rendering a 2D problem, the method avoids performing rigorous CFD and solely relies on analytical approaches. To account for the massive computational costs, a fixed number of "evolutions" was taken to be 50. To validate the solution, a number of other starting points were chosen and their final results were compared. Unlike the University of Michigan work, this effort used the aerodynamic "cost" objective function and formulated a minimization problem. Furthermore, this approach is inherently searching for a global solution (for large starting "populations")[4] (http://www.coe.psu.edu/water/images/b/b5/Alpman.pdf) .

Every iteration involves the comparison between two solutions of the same generation, calculating the differences in their decision variables, producing a "mutation", and applying it to each. The objective function is then calculated for each and the winner moves on to "reproduce."

Using the same assumptions about low speed flows and negligible fluid density fluctuations applied by University of Michigan, Penn State Mach number, Reynolds number are fixed to particular values found in normal operating conditions. Therefore, only two independent variables are left to be optimized, angle of attack and shape.

While most of the constraints are directly related to practical requirements for a wing (e.g. allowable lift coefficient, given a known thrust, must be reached in order to lift an expected load), some are simply present to speed up the calculation (e.g. maximum allowable AOA has no physical reason to be bounded, other than the authors' experience with airfoils knowing that exceeding such a limit adds computational rigor and provides no improved solutions).

In the best combination of population size and crossover ratio (relating to how many mutations are tested per generation), the drag/lift ratio was observed to decrease in two major jumps. This behavior is expected while running "greedy" genetic algorithms, since the best solution always wins and only changes when explicitly beat. This differs from other executions of the same algorithm, where mutations are, by default, allowed to continue to the next generation. From the start to finish, the D/L ratio decreased 99.8% while satisfying all constraints. Visually, the changes are obvious as well.

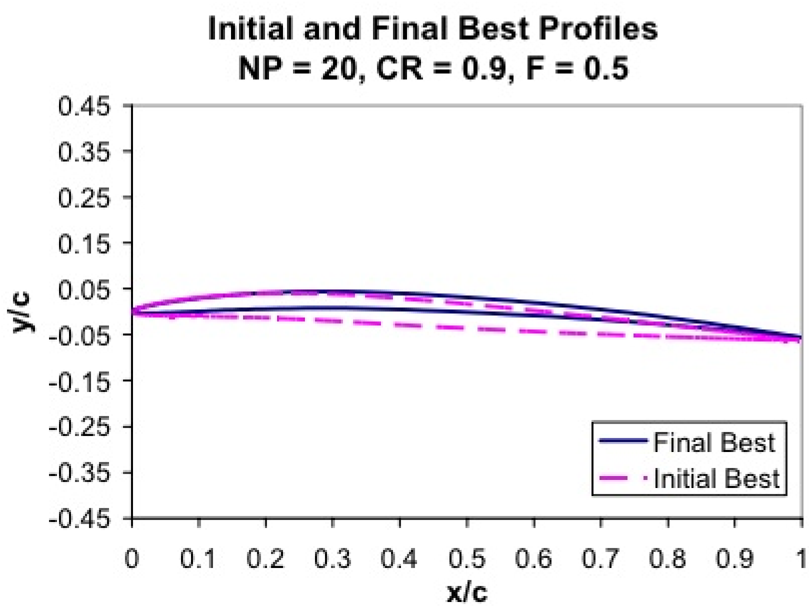


Figure 3. Best airfoils of the initial and final population.

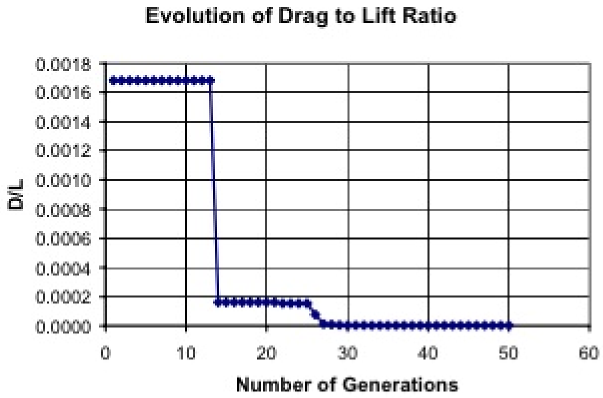


Figure 2. Evolution of D/L

Analytical Basis

Twenty years ago, wing shape optimization methods grew out of control theory. Given the limited computational capabilities at the time, Princeton University researchers applied fundamental Euler's equations with conformal mapping to improve airfoils. A major, simplifying assumption was that the fluids were inviscid (lacking viscosity). This alone makes some of the most sophisticated nonlinear components of drag force linear.

For a perfect gas: $p = (\gamma - 1)\rho[E - \frac{1}{2}(u^2 + v^2)]$ and $\rho H = \rho E + p$ where p, ρ, u, v, E, and H respectively denote pressure, density, velocity along cartesian directions, total energy, and total enthalpy. From this, the Euler flow equations (for inviscid fluids) can be written as: $\frac{\delta w}{\delta t} + \frac{\delta f}{\delta x} + \frac{\delta g}{\delta y} = 0$ where w, f, g respectively denote fluid dynamic vectors of the aforementioned variables in different directions.

To reach a suitable shape, a "cost" function for drag must be developed as a function of direction, pressure, and desired pressures. As such, the Princeton researchers used: $I = \frac{1}{2}\int_C (p - p_D)^2 d\theta$ where I is the cost and p_D is the desired pressure. this mapping function, which relates all the flow over the surface of an airfoil into drag, is then transformed into a variational calculus problem. After significant manipulation and an adjoint approach, the fundamental relations are proposed that: $\delta I = \int_C g \delta f d\theta$ and

$$g = \frac{\delta P}{\delta r} + \frac{\delta Z}{\delta \theta} - (p - p_D) * p = \frac{\delta P}{\delta r} + \frac{\delta Z}{\delta \theta} - Sp$$

Having these equations, the paper proposes a process to optimize inviscid flow over an airfoil surface.

1. Solve the flow equation for w by using steady-state boundary conditions
2. Solve the ODE for S
3. Solve the adjoint equation for the steady-state
4. Evaluate $g = \frac{\delta P}{\delta r} + \frac{\delta Z}{\delta \theta} - Sp$ for a given set of variations
5. Correct the boundary mapping function by $\delta f = -\lambda g$ where $\lambda > 0$

Conclusion

Aerospace represents a rich area for optimization work. Three representative processes for wing-shape optimization are presented here and demonstrate the viability of computation to improve upon analytical design. Most methods begin with a human-designed wing, and use iterations (either genetic or CFD/gradient-based) to reach an optimum. As with many multi-objective problems, formulating the goals is a major concern. Additionally, local minima are quite common in this type of optimization, where the variables and constraints number in the hundreds.

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