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OPTIMAL PRICING WITH INTERMODAL COMPETITION*

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"Optimal Pricing with Intermodal Competition"

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ABSTRACT

The regulation of multiproduct enterprises has created some difficult problems for regulators, particularly where common costs of production are present and where entry may be allowed in one or more of the markets served by the multiproduct firm. This paper concentrates on aspects of economic efficiency in pricing with multiproduct firms and intermodal competition. It extends the work of Baumol and Bradford on efficient pricing with a multiproduct monopoly to the case where intermodal competition is present. A set of rules is developed, showing how second best prices deviate from marginal cost when economies of scale are present. The paper shows why these rules may be difficult to implement in some cases, with a direct application to the case of freight transport, and then suggests a variation in the theory of second best which may be useful given those difficulties.

Introduction

In recent years there has been a growing concern over certain difficulties encountered in the regulation of multiproduct firms. Among the major issues has been the problem of pricing. At least three factors have contributed to the difficulty of the pricing problem in regulated firms. First, it may be the case that when price is set equal to marginal cost in each of the markets served by a firm, profits would be negative. Hence, some deviation of price from marginal cost is required if the firm is to break even. Second, there may be costs of production which are shared by two or more services in the production process, so that it is impossible to assign costs to services in an unambiguous manner. Finally, there may be other firms participating in some of the markets served by the multiproduct firm. In such a case pricing policies may affect market structure, and the two should not be treated independently.

Some of the issues raised where there are shared costs of production have been addressed by Faulhaber [1972] and Zajac [1972]. These papers focus on the question of what constitutes a "fair" price for any service when there are costs of production shared by two or more services. They analyze the effects of some of the many possible alternative guidelines which have been suggested to define "fair" prices. For example, it is sometimes suggested that any price which is no lower than the marginal cost of providing that service is a fair price for that service. Another possible definition of a fair price is any one that generates enough revenue to at least cover all of the costs which can unambiguously be attributed to the provision of that service. These two definitions are generally considered as defining lower bounds on fair prices. The papers also suggest possible upper bounds on fair prices. For example, one could say that a fair price is one which does not require a service to generate more revenues than would be necessary to cover all costs incurred if that service were the only one being provided. More recently Faulhaber [1975] has proposed another definition of fair prices for a service. Suppose that a firm is producing services 1, 2, ..., n in quantities x_1, x_2, ..., x_n, and that the total cost of producing the services is denoted by $C(x_1,x_2, \ldots, x_n)$. Then the price for the i^{th} service, p^i, is fair as long as

$$p^i x_i \geq C(x_1,x_2,\ldots,x_{i-1},x_i,x_{i+1},\ldots x_n) - C(x_1,x_2,\ldots,x_{i-1},0,x_{i+1},\ldots x_n).$$

In other words, as long as the i^{th} service generates revenues which are large enough to cover the difference in costs caused by the production of that service, then p^i is a fair price.

Several observations can be made regarding these alternative definitions of fair prices. First, one could view these approaches to pricing as being based primarily on equity considerations rather than on principles of economic efficiency. In judging whether prices are fair, regulators have historically tended to allocate shared costs first, and then require that the price charged for any service generate revenues which cover the portion of shared costs allocated to that service plus all costs that can be unambiguously attributed to that service. The rather lengthy proceedings of the Federal Communications Commission in the Private Line Docket (FCC Docket 18128/18684) and of the Interstate Commerce Commission(in ICC Docket 34013) address the manner in which shared costs are to be allocated. Importantly, the prices set as a result of this process may bear no direct relationship to economic efficiency.

The issues of efficient pricing for a multiproduct firm have been examined in a classic article by Baumol and Bradford [1970]. Briefly, this research resulted in the development of rules for second best pricing in a firm which would earn negative profits if price were equal to marginal cost in each market. The pricing rules derived are those which maximize economic efficiency (as measured by the sum of producer and consumer surplus) subject to a constraint which allows the firm to break even. [1]

Suppose a firm produces n commodities in quantities $x_1, \ldots, x_n$, and, for simplicity, that the demands for the commodities are independent of one another. Assume also that the cost function for the production process can be represented by $C(x_1, x_2, \ldots, x_n)$. Then the second best prices $(p^1, p^2, \ldots, p^n)$ are those which satisfy Eq. 1 and Eq. 2.

$$R^i \triangleq \left[\frac{p^i - \frac{\partial C}{\partial x_i}}{p^i}\right] \epsilon_{p^i} = \left[\frac{p^j - \frac{\partial C}{\partial x_j}}{p^j}\right] \epsilon_{p^j} \triangleq R^j, \ \forall_{i,j} \tag{1}$$

and,

$$\sum_{i=1}^{n} p^i x_i - C = 0, \tag{2}$$

where ϵ_{p^i} represents the price elasticity of demand in the i^{th} market.

Eq. 2 represents a condition in which the firm is breaking even (total revenues equal total cost). Eq. 1 represents the well-known rule that in each market the amount by which price deviates from marginal cost is inversely related to the price elasticity of demand. The numbers R^i and R^j are sometimes called Ramsey numbers, based on the work of Frank Ramsey [1927] which was suggestive of the work that Baumol and Bradford later performed. The theory has been extended to cover the case in which the demands are interdependent, resulting in a slightly more complicated form for the Ramsey numbers. [2] The basic idea remains unchanged in characterizing second best, namely, the Ramsey numbers are equal in all markets and the firm is earning zero profits.

It is important to note that prices which satisfy Eq. (1) and Eq. (2) generally may not satisfy all of the possible definitions of "fair" prices examined by Zajac and Faulhaber. [3] There is an essential difference between the approaches to pricing taken by regulators and by Baumol and Bradford. Regulators tend to allocate shared costs first, and then judge prices based on that allocation as described above. In the work of Baumol and Bradford, efficient prices are based on marginal cost and conditions of demand. No prior allocation of shared costs is required. (It is possible to determine how shared costs should be allocated in order to reach second best once the efficient prices have been found, but the allocation is done *ex post* instead of *ex ante*.)

One of the important gaps which has not been filled by the theory of second best is its extension to the case of intermodal competition. As developed by Baumol and Bradford, it applies only to a firm which has a monopoly in each of its markets. Several questions may be posed in this connection. Is the notion of Ramsey numbers useful with intermodal competition? If so, what do the Ramsey numbers look like? What particular kinds of difficulties might be expected in an application of the theory, and what modifications in the notion of second best may be of interest as a result of this line of investigation? We address these questions beginning with the next section, using the regulation of surface freight transportation to facilitate the development.

II. Second Best and Intermodal Competition

The regulation of surface freight transportation has posed perplexing problems for the Interstate Commerce Commission, in large part because of the growth of competition among the various modes of transport, especially since about 1930. Since the regulation of freight transport provides a convenient framework within which to discuss the concept of second best, a brief discussion of some of the economic characteristics of freight carriers will precede the theoretical development.

Modes of transport have often been placed in two categories on the basis of economic traits. According to Pegrum, "railroads and pipelines have the basic economic characteristics of public utilities and are what economists call natural monopolies; motor, water, and air transport exhibit the features of competitive industries." [4] Freight transportation in this country has certain distinctive features which lead us to concentrate on the interactions among rail, motor, and water carriers. First, air carriers primarily provide passenger service. Freight movement generates only about one percent of air transport revenues, which represents a very small amount compared with the freight activity of other modes. [5] Second, pipelines "constitute a highly specialized form of transportation for the movement of products in liquid or gaseous form." [6] Because of this special nature of the service they provide and their apparent economies of scale, the regulation of oil and gas pipelines could be treated separately, under the jurisdiction of either the Interstate Commerce Commission or the Federal Power Commission. [7].

The remaining three modes employ greatly differing technologies to provide services which can be viewed as imperfect substitutes for one another. It should be noted that the issue of economies of scale in railroads is not a closed matter. Several empirical studies have been made to test for the existence of economies of scale, with results that have generally been mixed. For example, Klein [1953] used 1936 data to find statistically significant, though modest economies of scale. However, studies by Borts [1950] and Griliches [1972] have concluded that even if scale economies are present for smaller railroads, they are not prevalent in the larger ones.

It is not the purpose of this paper to critique these empirical studies. Rather, the intent is to examine how second best prices might be set if one of the modes has scale economies (and railroads appear to be the most likely candidate) and the other modes (water and motor) do not. If none of these modes has increasing returns to scale, the basis for any regulation at all should be examined. If any of the modes do have scale economies, then the questions addressed in this paper are appropriate ones to examine.

We now construct a model of intermodal competition using the following assumptions:

1) There are m modes which provide transport services between two points. Only one of these modes (mode 1) is characterized by economies of scale. In other words, if the services provided by mode 1 were all priced at marginal cost, the profits for the firm would be negative.

2) There are many suppliers of transport service in each of the other modes, so that each of the modes 2, ..., m is essentially competitive. It is assumed that with free entry the supply of transport services in each of these modes is perfectly elastic.

3) Each mode may transport any or all of n commodities. Let

i = a modal index, $i = 1, \ldots, m$

j = a commodity index, $j = i, \ldots, n$

x_{ij} = the amount of commodity j transported by mode i.

4) All carriers of mode i provide identical service in the transport of commodity j. Restated, this means that there is intramodal homogeneity in the carriage of a particular commodity.

5) There is intermodal service differentiation. In transporting commodity j, carriers of one mode will provide service which differs from the service of carriers of other modes. This recognizes that motor carriers, water carriers, and railroads may differ in the speed of transport, reliability, and in other aspects of service quality.

5) For our purposes, the demand for transportation of commodity j via any mode is independent of the demand for transportation of commodity k ($k \neq j$) via any mode. Formally, let

$$p^{ij} = p^{ij}(x_{1j}, x_{2j}, \ldots, x_{mj}), \quad i=1, \ldots, m; \quad j=1, \ldots, n$$

where p^{ij} represents the (inverse) demand for transport of commodity j via mode i.

In addition, let

S^{ij} = the price corresponding to the (perfectly elastic) supply curve for mode i in the provision of service j, and

$C^1 = C^1(x_{11}, x_{12}, \ldots, x_{1n};$ factor prices) be the total cost function for mode 1. Factor prices are assumed constant, so reference to them is supressed throughout the rest of this paper.

Let us assume that there are zero income effects associated with the demand functions p^{ij}, so that a measure of the gross benefits from the provision of $(x_{11}, \ldots, x_{1n}; x_{21}, \ldots, x_{2n}; \ldots; x_{m1}, \ldots, x_{mn})$

is defined by G, where,

$$G = \sum_{j=1}^{n} \left\{ \int_{w=o}^{x_{1j}} p^{1j}(w,0,\ \ldots,\ 0)dw + \int_{w=o}^{x_{2j}} p^{2j}(x_{1j},\ w,\ 0,\ \ldots,\ 0)dw + \cdots + \int_{w=o}^{x_{mj}} p^{mj}(x_{1j},\ \ldots,\ x_{m-1,j},\ w)dw \right\} \tag{3}$$

If consumers seek to maximize G when confronted by a set of prices (p^{ij}), then they would choose x_{ij} so that

$$\frac{\partial G}{\partial x_{ij}} = p^{ij}$$

We can now write a function, T, which measures the sum of consumer and producer surplus associated with any level of service:

$$T = G - C^{1} - \sum_{i=2}^{m} \sum_{j=1}^{n} S^{ij}x_{ij} \tag{4}$$

We are now ready to examine the nature of a second best operating point when the regulator is able to select the levels of x_{ij} for all i and j. We note that the question of second best is of interest, since if the regulator attempted to reach first best, we would have:

$$\frac{\partial T}{\partial x_{ij}} = p^{1j} - \frac{\partial C^{1}}{\partial x_{1j}} = 0, \quad \forall_{j} \tag{5}$$

which means that the mode with economies of scale would be earning negative profits. If the regulator wants to set the levels of x_{ij} to maximize efficiency while allowing the mode 1 firm to break even, then formally it would find a solution to the following problem:

$$\max_{(x_{ij}, V_{i,j})} \quad T = G - C^1 - \sum_{i=2}^{m} \sum_{j=1}^{n} S^{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} P^{1j} x_{1j} - C^1 \geq 0 \tag{6}$$

Define L as follows, where λ is the nonnegative Lagrangean associated with the break even constraint:

$$L = G - C^1 - \sum_{i=2}^{m} \sum_{j=1}^{n} S^{ij} x_{ij} + \lambda \left(\sum_{j=1}^{n} P^{1j} x_{1j} - C^1 \right) \tag{7}$$

Among the first order conditions are:

$$\frac{\partial L}{\partial x_{1j}} = P^{1j} - \frac{\partial C^1}{\partial x_{1j}} + \lambda \left(\frac{\partial P^{1j}}{\partial x_{1j}} x_{1j} + P^{1j} - \frac{\partial C^1}{\partial x_{1j}} \right) \leq 0,$$

$$x_{1j} \geq 0, \quad x_{1j} \frac{\partial L}{\partial x_{1j}} = 0; \quad j = 1, \ldots, n \tag{8}$$

and

$$\frac{\partial L}{\partial x_{ij}} = P^{ij} - S^{ij} + \lambda \left(\frac{\partial P^{1j}}{\partial x_{ij}} x_{1j} \right) \leq 0,$$

$$x_{ij} \geq 0, \; x_{ij} \frac{\partial L}{\partial x_{ij}} = 0; \quad i=2, \ldots, m; \quad j=1, \ldots, n \tag{9}$$

Eq. 8 can be rewritten

$$\left[\frac{P^{1j} - \frac{\partial C^1}{\partial x_{1j}}}{P^{1j}} \right] \left[\frac{P^{1j}}{\frac{\partial P^{1j}}{\partial x_{1j}} x_{1j}} \right] = - \frac{\lambda}{1+\lambda} \quad ; \; j=1, \ldots, n \tag{10}$$

where the second term on the left hand side is the reciprocal of the quantity elasticity of demand for x_{1j}. One could think of the expression on the left hand side of Eq. 10 as a modified Ramsey number which will be equal for all values of j, since $-\lambda/(1+\lambda)$ does not vary with j.

However, Eq. 10 must also be satisfied, and here we encounter a potential administrative nightmare. Since the demands for x_{1j} and x_{ij} $(i \geq 2)$ are not independent, the term

$$\lambda \frac{\partial p^{1j}}{\partial x_{ij}} x_{1j}$$

is not zero. In fact, as long as x_{1j} and x_{ij} are weak gross substitutes for one another, this term will have a negative sign. [8] Hence, a second best solution in which x_{ij} is positive would occur only when the markets served by modes 2, ..., m do not clear! Eq. 9 tells us that price would have to exceed marginal cost in those modes. There are two effects working against each other which make this property interesting. Heuristically, there is some loss in efficiency which occurs in the markets served by modes 2, ..., m because price is greater than marginal cost. However, the higher prices in modes 2, ..., m lead to increased demands (and more consumer surplus) for the services provided by mode 1. Eq. 9 implies that the second effect exceeds the first.

In principle one could calculate Ramsey numbers for modes 2, ..., m which would equal the number $-\lambda/(1+\lambda)$ from Eq. 10, although the form of these numbers is more complex than the modified Ramsey number in that equation. Using the assumption of zero income effects, by Hotelling's integrability condition we have that

$$\frac{\partial p^{1j}}{\partial x_{ij}} = \frac{\partial p^{ij}}{\partial x_{1j}}, \quad \forall i,j \tag{11}$$

From Eq. 9 and Eq. 11 it follows that

$$\frac{\dfrac{p^{ij}-s^{ij}}{p^{ij}}}{\left(\dfrac{\partial p^{ij}}{\partial x_{1j}}\right)\dfrac{x_{1j}}{p^{ij}} - \dfrac{p^{ij}-s^{ij}}{p^{ij}}} = -\frac{\lambda}{1+\lambda}\ ; \quad i=2,\ldots,m; \quad j=1,\ \ldots,\ n \tag{12}$$

The numerator of the left hand side represents the amount by which the price of x_{ij} would exceed marginal cost, stated as a fraction of the price itself. A similar expression appears in the second term of the denominator. The first term of the denominator represents the cross elasticity of the inverse demand p^{ij} with respect to the quantity x_{1j}.

The achievement of second best would then require that

(1) Mode 1 earns zero economic profit.

(2) Prices are set so that the modified Ramsey numbers for all modes in all markets are equal to one another. The modified Ramsey numbers for mode 1 are defined by Eq. 10, and for all other modes are as shown in Eq. 12.

(3) Since price exceeds marginal cost in the markets served by modes 2, ..., m, the regulator would have to prevent free entry in those markets.

There can be little doubt that the regulatory scheme just outlined represents an enormous regulatory undertaking. Some might argue that there is a striking similarity between the outlined program and the actual kind of regulation we observe in freight transportation presently. After all, regulators do adjudge the reasonableness of prices (tariffs) for all regulated modes, and in addition control conditions of entry through certificates required of common carriers wishing to provide service over particular routes. One could even argue that through a consideration of "value of service" in pricing, regulators attempt to require higher tariffs on commodities with more inelastic demands, and that this is generally consistent with the guidelines suggested by rules such as those of Eq. 10 and Eq. 12.

However, one would be hard pressed to carry the analogy much further. It would be an understatement to say that the data requirements for the outlined program are great. In fact, the information required on the numerous cross elasticities of demand alone is enough to make the outlined program quite unwieldy.

Unfortunately, even if we were to commit ourselves to the quest for second best, we are likely to encounter other difficulties at least as important as the information requirements. The case of freight transportation serves well to illustrate this point. Suppose that mode 2 represents regulated motor carriage, and that a regulator seeks to limit entry in order to hold price above marginal cost as discussed above. Then the presence of an unregulated sector of the motor carrier industry, as we have in this country, may present an overwhelming problem. For example, if prices are held above marginal costs for regulated carriers, shippers who would otherwise have used regulated motor carriers will have incentives to buy their own trucks for the purposes of hauling their own commodities. Such private haulage is not regulated, and thus could not be prevented by the regulations applying to common carriers. As a result, although the intent of regulation is to proscribe entry, the probable effect would simply be to change the form of entry to circumvent the regulation.

These difficulties lead us to ask if there is not some modified form of the notion of second best which requires less information on the part of regulators, and at the same time avoids the kinds of entry control problems described above for modes where there do not appear to be significant technological barriers to entry, such as economies of scale. One rather interesting candidate for examination would be a regulatory program which allows the modes without economies of scale (i=2, ..., m) to clear, and which concentrates on the prices set by the mode with economies of scale. In terms of administration, regulators would not have to set the n(m-1) tariffs (or quantities) for the modes without increasing returns to scale, and in addition would not concern themselves with the thorny problem of entry control in those modes. The administration of regulation under this scenario would be much simplified.

There are other reasons why such a program, which we will call market-clearing second best, might be of interest. In recent years there has been much debate over the extent to which regulation is actually needed in freight transport. Since 1930 technological changes have made the transport of freight by motor and water carriers economically viable on a large scale. To control the interactions among modes, the hand of regulation has been extended repeatedly. However, it is sometimes argued that the presence of viable alternative modes may mean that with less regulation market forces might work quite well in making many of the resource allocation decisions now made by regulators. The market-clearing second best characterization can be considered as one version of partial regulation (or, alternatively, partial deregulation), since modes without economies of scale are not directly regulated.

The information requirements and welfare properties of such a program will be addressed in the next section.

III. Market Clearing Second Best

There are two ways one could formalize the concept of market-clearing second best. First, a set of market-clearing constraints for modes 2, ..., m could be appended to the problem defined in Eq. 6. [9] Then the optimal prices (or quantities) for mode 1 could be determined. The additional constraints would be

$$p^{ij} - S^{ij} = 0; \quad i=2, \ldots, m; \quad j=1, \ldots, n \tag{13}$$

There would be a Lagrange multiplier associated with the break even constraint (λ), and one associated with each market-clearing condition (μ^{ij}). Unfortunately, this approach does not easily lend itself to the derivation of a set of expressions equal across markets and at the same time avoiding explicit reference to the values of the Lagrange multipliers λ and μ^{ij}. Thus an important advantage of the approach used by Baumol and Bradford is lost.

Fortunately there is a second method which will lead us to a surprisingly simple result. We proceed by using implicit functions instead of the set of market clearing conditions. Based on the set of equalities in Eq. 13, one could express the changes in the quantities x_{ij} produced by modes 2, ..., m in reaction to changes in the quantities x_{1j} for mode 1 as follows:

$$\begin{bmatrix} \partial p^{2j}/\partial x_{2j} & \cdots & \partial p^{2j}/\partial x_{mj} \\ \partial p^{3j}/\partial x_{2j} & \cdots & \partial p^{3j}/\partial x_{mj} \\ \vdots & \vdots & \vdots \\ \partial p^{mj}/\partial x_{2j} & \cdots & \partial p^{mj}/\partial x_{mj} \end{bmatrix} \begin{bmatrix} dx_{2j}/dx_{1j} \\ dx_{3j}/dx_{1j} \\ \vdots \\ dx_{mj}/dx_{1j} \end{bmatrix} = \begin{bmatrix} -\partial p^{2j}/\partial x_{1j} \\ -\partial p^{3j}/\partial x_{1j} \\ \vdots \\ -\partial p^{mj}/\partial x_{1j} \end{bmatrix}$$

$$j = 1, \ldots, n, \tag{14}$$

where all terms as are they have been defined previously. For brevity, let the first matrix on the left hand side of Eq. 14 be denoted by $[B^j]$, the second matrix be denoted by $[dx_{ij}/dx_{1j}]$ and the matrix on the right hand side be written as $[-\partial p^{ij}/\partial x_{1j}]$.

Then

$$[dx_{ij}/dx_{1j}] = [B^j]^{-1} [-\partial p^{ij}/\partial x_{1j}], \qquad j = 1, \ldots, n \tag{15}$$

The market-clearing second best problem can be stated as follows:

$$\max_{(x_{11},\ldots,x_{1n})} T = G - C^1 - \sum_{i=2}^{m} \sum_{j=1}^{n} S^{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} p^{1j} x_{1j} - C^1 \geq 0 \tag{16}$$

where λ will be the Lagrange multiplier associated with the break even constraint. All of the variables x_{ij} $(i \geq 2)$ are now implicit functions of x_{1j}. At an optimum of Eq. 16, the first order conditions are found from the Lagrangian

$$L \triangleq G - C^1 - \sum_{i=2}^{m} \sum_{j=1}^{n} S^{ij} x_{ij} + \lambda\left(\sum_{j=1}^{n} p^{1j} x_{1j} - C^1\right)$$

They are [10]

$$\frac{\partial L}{\partial x_{1j}} = p^{1j} - \frac{\partial C^1}{\partial x_{1j}} + \sum_{i=2}^{m} (p^{ij} - S^{ij}) \frac{dx_{ij}}{dx_{1j}}$$

$$+ \lambda \left[\frac{\partial p^{1j}}{\partial x_{1j}} x_{1j} + p^{1j} - \frac{\partial C^1}{\partial x_{1j}} + \sum_{i=2}^{m} \frac{\partial p^{1j}}{\partial x_{ij}} \frac{dx_{ij}}{dx_{1j}} x_{1j} \right] \leq 0 \; ;$$

$$x_{1j} \geq 0; \quad x_{1j} \frac{\partial L}{\partial x_{1j}} = 0 \; ; \quad j=1, \ldots, n \tag{17}$$

and

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^{n} p^{1j} x_{1j} - C^1 \geq 0; \quad \lambda \geq 0 \; ; \lambda \frac{\partial L}{\partial \lambda} = 0 \tag{18}$$

Since $p^{ij} - s^{ij} = 0$ by the market-clearing conditions, Eq. 17 simplifies to Eq. 19.

$$(1+\lambda)\left(p^{1j} - \frac{\partial C^1}{\partial x_{1j}}\right) = -\lambda x_{1j}\left[\frac{\partial p^{1j}}{\partial x_{1j}} + \sum_{i=2}^{m} \frac{\partial p^{1j}}{\partial x_{ij}} \frac{dx_{ij}}{dx_{1j}}\right] \tag{19}$$

The key to an easy interpretation of Eq. 19 lies in the meaning of the term in brackets on the right hand side. From Eq. 15 and Eq. 19 it follows that

$$(1+\lambda)\left(p^{1j} - \frac{\partial C^1}{\partial x_{1j}}\right) = -\lambda x_{1j}\left[\frac{\partial p^{1j}}{\partial x_{1j}} - \sum_{i=2} \frac{\partial p^{1j}}{\partial x_{ij}} [B^j]^{-1} \frac{\partial p^{ij}}{\partial x_{1j}}\right] ; \quad j = 1, \ldots, n \tag{20}$$

It can be shown that Eq. 20 implies Eq. 21. [11]

$$\left[\frac{p^{1j} - \frac{\partial C^1}{\partial x_{1j}}}{p^{1j}}\right] \epsilon_p^{1j} = -\frac{\lambda}{1+\lambda} \quad ; \quad j=1, \ldots, n \tag{21}$$

where ϵ_p^{1j} is the own price elasticity (not quantity elasticity) of demand for service j in mode 1.

The net result of all of this is that in finding efficient prices for the market-clearing second best problem, the following conditions must be satisfied:

$$(1) \quad \sum_{j=1}^{n} p^{1j} x_{1j} - C^1 = 0 \tag{22}$$

and

$$(2) \quad \left[\frac{p^{1j} - \frac{\partial C^1}{\partial x_{1j}}}{p^{1j}}\right] \epsilon_p^{1j} = \left[\frac{p^{1k} - \frac{\partial C^1}{\partial x_{1k}}}{p^{1k}}\right] \epsilon_p^{1k} \tag{23}$$

for j=1, ..., n; k=1, ..., n

The market-clearing second best prices for the case with intermodal competition are set according to the same rules as the ones developed by Baumol and Bradford for a multiproduct monopoly. The Ramsey numbers defined in Eq. 23 depend only on local information on price, marginal cost, and the price elasticity of demand for the first mode. [12]

Upon reflection, these results do have an intuitive appeal. The pricing rules of Baumol and Bradford [1970] are conceptually appropriate when goods or services produced by mode 1 have demands which are independent of the demands for goods or services produced by other firms. In other words, the multiproduct firm must monopolize all of its markets. However, suppose there are other products whose demands interact with the outputs of mode 1. Then one could describe second best for the whole set of these products, as we have done earlier. The results say that if one mode has economics of scale, it may be efficient (second best) to alter the market-clearing outcomes for other modes, even if those modes serve markets which are potentially quite competitive.

There are several reasons why a regulator may not even attempt to specify a program of total regulation leading to second best. Regulators may perceive the interactions among the demands for products of mode 1 and other modes to be small, or they may simply be unaware of the interaction. They may also recognize the potentially very large information and administrative requirements for such a program, or the difficulties in controlling entry as effectively as would be required. There may be other reasons for which regulators may explicitly decide to let the markets clear for those modes which are essentially competitive. [13] In any one of these cases an interesting candidate for efficient pricing becomes market-clearing second best.

Suppose that a regulator mistakenly thinks that the multiproduct firm has a monopoly in its markets. Then it might determine second best prices by the rules of Baumol and Bradford. However, this is equivalent to a situation in which the regulator explicitly recognizes the interdependence of mode 1 products with the outputs of other (competitive) modes, but decides to allow the markets for other modes to clear. Thus, the connection between market-clearing second best and the rules of Baumol and Bradford is drawn more clearly.

IV. A Comparison of Welfare Properties

To illustrate the basic properties of several interesting operating points with intermodal competition, it is useful to consider a special case for which a graphical exposition is possible. Assume that there are only two modes, mode 1 with economies of scale (as before), and mode 2 which lacks scale economies. Only one basic kind of service is provided by each mode. The service provided by mode 1 is differentiated from the service of mode 2. However, all firms in mode 2 provide a homogeneous service. We have retained the assumption of intermodal service differentiation and intramodal service homogeneity. For the purpose of the illustration, we will characterize mode 2 as having a supply schedule which may or may not be perfectly elastic. In other words, the supply schedule can be written as $S^2(x_2)$, where S^2 represents the price at which x_2 units of service would be provided by producers in mode 2. As long as the mode 2 market clears, we have

$$p^2(x_1,x_2) - S^2(x_2) = 0, \tag{24}$$

which implies that

$$\frac{dx_2}{dx_1} = -\frac{\dfrac{\partial p^2}{\partial x_1}}{\dfrac{\partial p^2}{\partial x_2} - \dfrac{\partial S^2}{\partial x_2}} < 0 \tag{25}$$

The property that $dx_2/dx_1 < 0$ holds when x_1 and x_2 are weak gross substitutes and when the demand for x_2 is more negatively sloped than the supply curve. Since x_1 and x_2 are assumed to be weak gross substitutes (as throughout this paper), and since mode 2 is assumed not to have increasing returns to scale, the inequality in Eq. 25 is implied. A locus of points satisfying Eq. 24 is represented in Fig. 1 by the curve AE. Point A corresponds to the point at which only mode 2 serves the market. The negative slope of AE follows from Eq. 25.

We may also represent the sum of consumer and producer surplus by T,

$$T = \int_{w=o}^{x_1} p^1(w,0)dw + \int_{w=o}^{x_2} p^2(x_1,w)dw - C^1(x_1) - \int_{w=o}^{x_2} S^2(w)dw \tag{26}$$

As a result, isosurplus curves will have the slope

$$\frac{dx_2}{dx_1} = - \frac{p^1 - \frac{\partial C^1}{\partial x_1}}{p^2 - S^2} \tag{27}$$

Note that along the curve AE the isosurplus curves are vertical, since $p^2 - S^2 = 0$. Also, T increases along AE as x_1 increases up to a level of output at which p^1 equals the marginal cost of producing x_1, where T reaches its maximum. Thus, along AE

$$dT = \left(p^1 - \frac{\partial C^1}{\partial x_1}\right)dx_1 + (p^2 - S^2)dx_2 = \left(p^1 - \frac{\partial C^1}{\partial x_1}\right)dx_1 \tag{28}$$

Let point E represent the level of output at which price equals marginal cost for mode 1 along AE. Since T is maximized here, E represents a first best operating point. The isoprofit curves around E shown in Fig. 1 will have values such that $T_E > T_D > T_C > T_B$.

The profit for Mode 1 can be expressed as Π^1, where

$$\Pi^1 = p^1(x_1, x_2) - C^1(x_1) \tag{29}$$

The isoprofit curves for Mode 1 will have the slope

$$\frac{dx_2}{dx_1} = - \frac{p^1 + \frac{\partial p^1}{\partial x_1} - \frac{\partial C^1}{\partial x_1}}{\left(\frac{\partial p^1}{\partial x_2} \right) x_1}$$

Since x_1 and x_2 are weak gross substitutes, the sign of the slope will be positive when the marginal revenue for x_1 exceeds the marginal cost of x_1 (for levels of output less than the profit maximizing level, given x_2), and negative when the converse is true. The shapes of these isoprofit curves are shown in Fig. 1. The ordering of the profit levels can be seen by noting that given any level of x_1, the profit of mode 1 will increase when x_2 decreases, i.e.,

$$\frac{\partial \Pi^1}{\partial x_2} = \frac{\partial p^1}{\partial x_2} x_1 < 0 \tag{30}$$

Let us now put all of this together. Fig. 1 is drawn to reflect the case in which it is possible for mode 1 to at least break even for some operating points when the market for x_2 clears. Suppose both modes were unregulated, and that mode 1 chooses the highest isoprofit curve it can attain given that mode 2 will clear. Then this point of no regulation is shown at B.

If a regulator wants to maximize efficiency while allowing mode 2 to clear and mode 1 to just break even, it would choose point C. Point C thus represents market-clearing second best as described in Eq. 22 and Eq. 23.

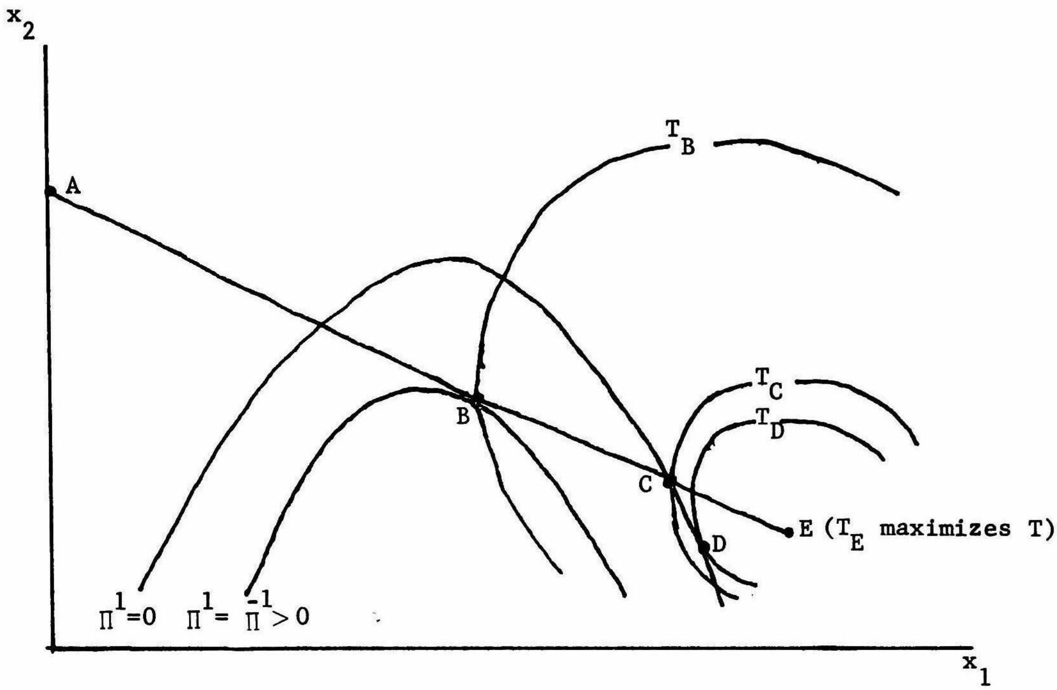

Figure 1: Mode 1 Could Be Profitable When Mode 2 Clears

If a regulator chooses to maximize efficiency while allowing mode 1 to break even, and is willing to undertake the control of quantities (or tariffs) and entry in mode 2, then it would strive to reach the totally regulated second best point, D. At D, the isosurplus curve, T_D, is tangent to the zero isoprofit curve for mode 1. Since the slope of the isoprofit curve for mode 1 is not vertical at that point, D must be located below the curve AE. This points out that at a totally regulated second best solution, the market for mode 2 will not be clearing, and entry control will be necessary.

The relationships between the isoprofit curves for mode 1 and the market-clearing locus, AE, could be other than as depicted in Fig. 1. For example, if mode 1 can not break even at any market clearing price in mode 2, then Fig. 2 is appropriate. There exists no market-clearing second best (point C in Fig. 1) and no totally unregulated point where mode 1 is profitable (such as point B in Fig.1). Mode 1 can only break even (in the absence of a subsidy) when mode 2 is prevented from clearing its market, and the most efficient point of operation where mode 1 breaks even is the second best point, D.

In between the situations shown in Fig. 1 and Fig. 2 is the one in which an unregulated mode 1 would just barely be able to breakeven, such as in Fig. 3. If mode 1 could just earn zero profit in this case, then the market-clearing second best point (C) and totally unregulated point (B) would coincide. In this case the Ramsey numbers of Eq. 23 would be a minus one. [14] This suggests that if it is expected that without any regulation only small economic proftis would be earned by the mode with economies of scale, then an unregulated system would achieve nearly the same efficiency as market-clearing second best, and without incurring the administrative costs of the latter.

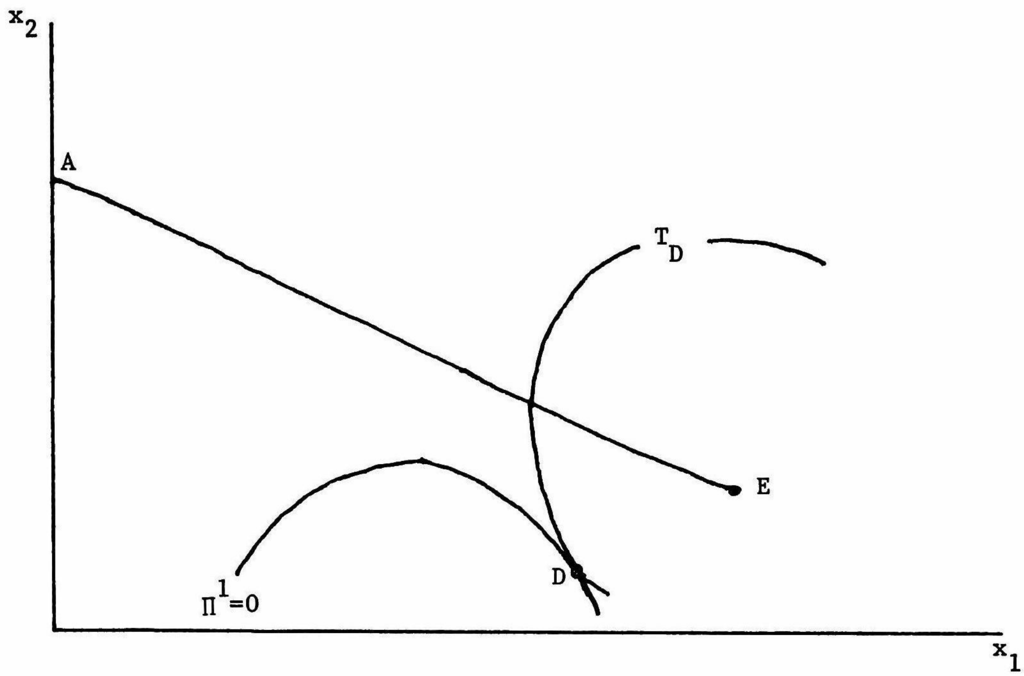

Figure 2: Mode 1 Earns Negative Profit When Mode 2 Clears

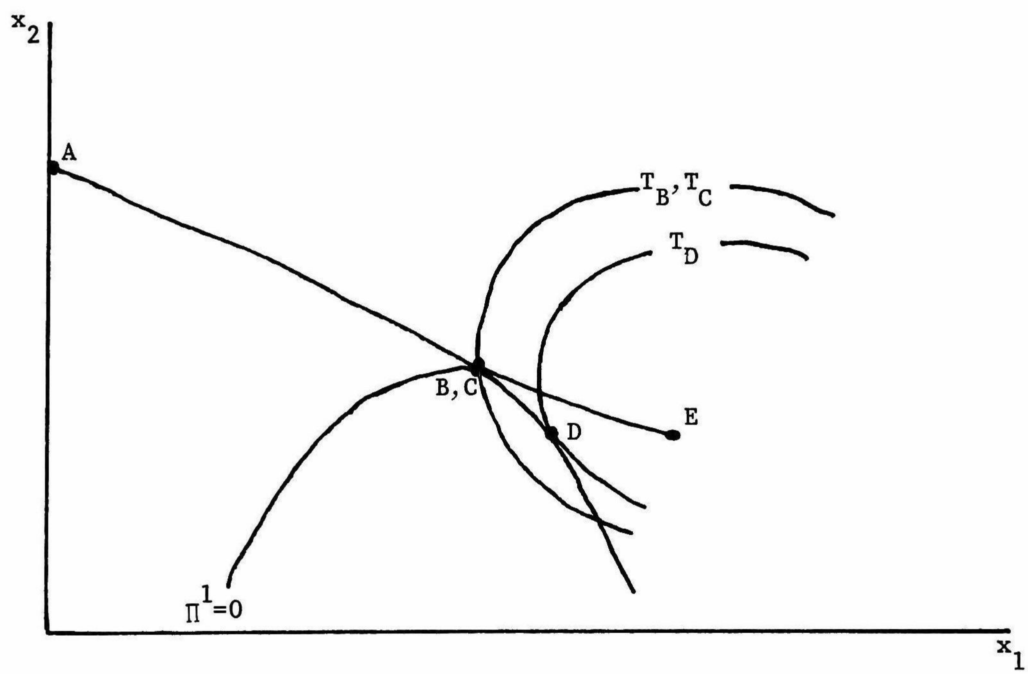

Figure 3: Unregulated Mode 1 Just Breaks Even

V. Conclusions

This paper has shown how the theory of second best can be extended from the work of Baumol and Bradford [1970] to a case in which intermodal competition exists. We have derived rules characterizing second best under a form of intermodal competition which may resemble what we observe in freight transportation in this country. There are at least two major problems which regulators should anticipate if they attempt to reach second best when all modes are regulated. First, there appears to be a large amount of information required to use the rules which are derived. Some of this information may be difficult to obtain, particularly since cross elasticities of demand are important. Second, the achievement of second best may involve a departure of prices from marginal cost even for modes which would be essentially competitive in the absence of regulation. This means that regulators would have to carefully control conditions of entry in markets that may not easily lend themselves to such control, such as with motor carrier freight transportation.

These potential difficulties led to the investigation of a modified form of second best. This form, called market-clearing second best, does not require the direct regulation of modes which appear to be essentially competitive. Prices are specified for a mode with economies of scale, and these prices are designed to maximize efficiency subject to conditions which allow that mode to break even, while the modes without increasing returns to scale are clearing their markets. The rules derived for market-clearing second best turn out to have the same form as the ones developed by Baumol and Bradford for the case without intermodal competition. If total regulation were costless and effective in achieving second best, then second best under total regulation would be more efficient than market-clearing second best. However, if the former is costly to achieve (because of large information requirements) or is otherwise difficult to reach (for example, because of the inability to effectively control entry), then market-clearing second best may become an attractive alternative.

We have compared both of these alternatives to a third one in which there is no regulation of tariffs or entry on any of the modes. Again, if regulation is costless and effective, both forms of second best will achieve greater economic efficiency than the unregulated system, as long as the mode with economies of scale could earn positive economic profits if it were not regulated. However, if the level of positive profits attainable without regulation is near zero, then the efficiency achieved without regulation may be quite close to that reached under market-clearing second best. Once again, the information-gathering and administrative costs associated with partial regulation (market-clearing second best) may be large relative to the case with no regulation. The qualitative nature of the tradeoff between administrative costs and attainable efficiency is clear; however a quantitative determination depends on characteristics specific to an industry. For the case of freight transport, the quantitative determination remains for further work.

Footnotes

[1] In the paper of Baumol and Bradford [1970], the authors asserted that the form of the utility function being maximized was unspecified. However, Mohring [1971] demonstrated that the unspecified utility function actually had the properties of the consumer surplus measure. For more on this measure, see Willig [1976].

[2] One place (and there are quite probably others) in which modified Ramsey numbers are derived for a multiproduct firm with increasing returns to scale and interdependent demands is in Braeutigam [1976].

[3] See the Zajac [1972] paper for some clear examples of this point.

[4] See Pegrum [1973], p. 25.

[5] See Pegrum [1973]. It is noted here that the exclusion of air freight is made here primarily for simplicity. Many of the arguments developed later on could be extended to encompass air freight simply by letting this mode be included as one of the m modes in the model to be developed.

[6] See Pegrum [1973], p. 43.

[7] Moore [1975] suggests the separate regulation of oil pipelines, and recognizes the natural monopoly characteristics of this mode.

[8] See Katzner [1970], Chapter 3. The definition of weak gross substitutes implies that the matrix of cross partial derivatives $\left[\frac{\partial p^{ij}}{\partial x_{kj}}\right]$ for the inverse demands is an N-P matrix, and that the off-diagonal elements will be non-positive.

[9] To state the idea completely

$$\max_{(x_{11},\ldots,x_{1j})} T = G - C^1 - \sum_{i=2}^{m}\sum_{j=1}^{n} S^{ij}x_{ij}$$

subject to $\sum_{j=1}^{n} p^{1j}x_{1j} - C^1 \geq 0$

and $p^{ij} - S^{ij} = 0;\ i=2,\ldots, m;\ j=1, \ldots, n$

[10] In Eq. 17, the terms dx_{ij}/dx_{1j} are equal to the partial derivatives $\partial x_{ij}/\partial x_{1j}$ since the variables x_{ij} are implicit functions of only x_{1j}.

[11] Recall that $[B^j]$ is an (m-1)x(m-1) matrix of the partial derivatives of the inverse demands for commodity j in modes 2, ..., m. Let us border $[B^j]$ with the appropriate terms for product j from the inverse demands of mode 1, and call this new matrix $[A^j]$.

$$[A^j] \triangleq \left|\begin{array}{c|ccc} \frac{\partial p^{1j}}{\partial x_{1j}} & \frac{\partial p^{1j}}{\partial x_{2j}} & \cdots & \frac{\partial p^{1j}}{\partial x_{mj}} \\ \hline \frac{\partial p^{2j}}{\partial x_{1j}} & \frac{\partial p^{2j}}{\partial x_{2j}} & \cdots & \frac{\partial p^{2j}}{\partial x_{mj}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial p^{mj}}{\partial x_{1j}} & \frac{\partial p^{mj}}{\partial x_{2j}} & \cdots & \frac{\partial p^{mj}}{\partial x_{mj}} \end{array}\right| \triangleq \left|\begin{array}{c|c} [W^j] & [V^j] \\ \hline [V^j]^T & [B^j] \end{array}\right|$$

The inverse of $[A^j]$ has as its upper left hand element the (1)x(1) matrix $[Q^j]^{-1}$, where

$$[Q^j]^{-1} = \left\{ [W^j] - [V^j]\,[B^j]^{-1}[V^j]^T \right\}^{-1}$$

$$= \left[\frac{\partial p^{1j}}{\partial x_{1j}} + \sum_{i=2}^{m} \frac{\partial p^{1j}}{\partial x_{ij}} [B^j]^{-1} \frac{\partial p^{ij}}{\partial x_{1j}} \right]^{-1}$$

Thus, the <u>price</u> (not quantity) elasticity of demand for x_{1j} is

$$\epsilon_{p^{1j}} = \frac{[Q^j]^{-1} p^{1j}}{x_{1j}} = \frac{p^{1j}}{[Q^j] x_{1j}}$$

From these last two equations, we have that

$$[Q^j] = \frac{p^{1j}}{x_{1j}\epsilon_{p^{1j}}} = \frac{\partial p^{1j}}{\partial x_{1j}} + \sum_{i=2}^{m} \frac{\partial p^{1j}}{\partial x_{ij}} [B^j]^{-1} \frac{\partial p^{ij}}{\partial x_{1j}}$$

Together, this equation and Eq. 20 from the text imply Eq. 21 in the text.

[12] The more complicated case in which all commodities transported by all modes have interdependent demands could be approached in the same way as for the simpler case developed in this paper, in a manner similar to the extension of the Baumol-Bradford framework to the case of inter-

dependence as described earlier in this paper.

[13] A regulator may also have other equity constraints it wishes to impose on the system. In principle, these constraints could be appended to a model which has as its objective function the maximization of surplus to find efficient prices given these additional constraints.

[14] If mode 1 can just break even as it maximizes its profit without regulation, then under market-clearing second best mode 1 would effectively have to maximize profit in order to satisfy the break even constraint. To further develop the point, under deregulation mode 1 would choose $(x_{11}, \ldots, x_{1n})$ to:

$$\max_{(x_{11},\ldots,x_{1n})} \Pi^1 = \sum_{j=1}^{n} p^{1j} x_{1j} - C^1$$

At an interior optimum, the first order conditions are of the form:

$$p^{1j} - \frac{\partial C^1}{\partial x_{1j}} = -x_{1j}\left(\frac{\partial p^{1j}}{\partial x_{1j}} + \sum_{i=2}^{m} \frac{\partial p^{1j}}{\partial x_{ij}} \frac{dx_{ij}}{dx_{1j}}\right) ,$$

$$\text{for } j = 1,\ldots, n$$

This equation can be rewritten using the last equation of footnote 11 as

$$\left(p^{1j} - \frac{\partial C^1}{\partial x_{1j}}\right) \epsilon_{p^{1j}} = -1$$

It now becomes clear that mode 1 prices will be set so that their deviations from marginal costs will be inversely related to the price elasticity of demand, for both the unregulated and market-clearing second best schemes. As the maximum profit achievable by mode 1 without regulation approaches zero, then λ becomes very large at market-clearing second best. Alternatively, $\lambda/(1+\lambda) \rightarrow 1$ in Eq. 21.

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